Lesson 20

Sears Haack

(minimum wave drag shape/area)

and

Whitcomb Area Rule

http://tiny.cc/WhitcombNASA

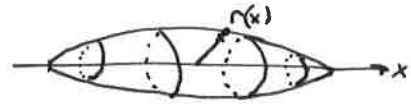
Logistics

Exam #2

Nov 2nd

Take home

Supersonic Singularity Functions in 3D axisymmetric flows



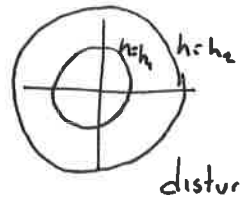
Start from an incompressible flow:

What is the radius, h, from any arbitrary point x, y, z?

$$h_{in}^2 = x^2 + y^2 + z^2$$

or

$$h_{in} = +\sqrt{x^2 + y^2 + z^2}$$



disturbances are propagated evenly in all directions

Apply the subsonic P-G transform:

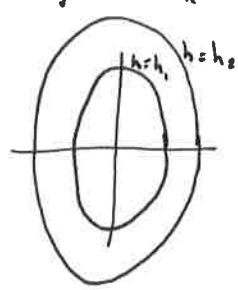
$x^* = x$ but $y^* = \beta y$ and $z^* = \beta z$

$$h_{sub}^2 = x^2 + \beta^2 y^2 + \beta^2 z^2$$

or

$$h_{sub} = \sqrt{x^2 + \beta^2 (y^2 + z^2)} \qquad \text{with } \beta_{subsonic} = \sqrt{1 - M_\infty^2} < 1$$

for a constant h, $y^2 + z^2$ must be larger than x^2 since $\beta^2 < 1$

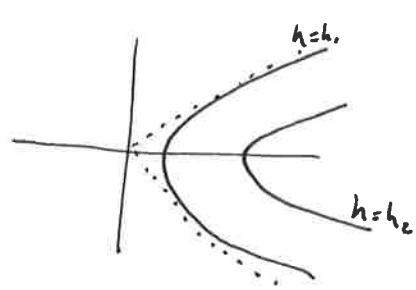


Disturbances are propagated relatively further in non-x directions (i.e. y and z).

Apply supersonic P-G transform.

Define a hyperbolic radius h

$$h_h = \sqrt{x^2 - (M_\infty^2 - 1)(y^2 + z^2)}$$



Point Source

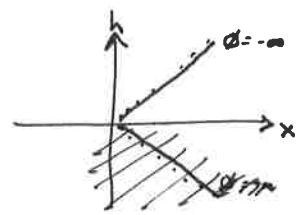
$$\phi_\Sigma = \begin{cases} \dfrac{-1}{2\pi h} & x > \beta\sqrt{y^2+z^2} \\ 0 & x \le \beta\sqrt{y^2+z^2} \end{cases} \qquad \text{where } h = \sqrt{x^2 - (M_\infty^2 - 1)(y^2+z^2)}$$

[Notice that the form differs from an incompressible source

$$\phi_{2D} = \frac{1}{2\pi}\ln r$$

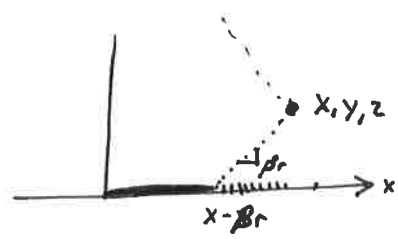
Singular behavior at $h=0$ (i.e. $x = \beta\sqrt{y^2+z^2}$

Not useful except as a building block (with $d\phi$)



Line Source

$$\phi_\Lambda = \int_0^{x-\beta r} \Lambda(x')\,\phi_\Sigma(x-x', y, z)\,dx'$$



Integrate upstream

$$= \int_0^{x-\beta r} \frac{-\Lambda(x')}{2\pi} \frac{1}{\sqrt{(x-x')^2 - \beta^2\sqrt{y^2+z^2}^2}}\,dx'$$

[When Λ is a unit density (i.e. $\Lambda(x) = 1$)

$$\phi_\Lambda = \frac{1}{2\pi}\ln\left(\frac{x-h}{\beta r}\right) \qquad \text{such that } \phi_{\Lambda_x} = -\frac{1}{2\pi h}$$

and

$$\phi_{\Lambda_r} = \frac{1}{2\pi}\left(\frac{1}{h}\frac{\beta^2 r}{x-h} - \frac{1}{r}\right)$$

The objective is to relate the potential function to the surface shape.

$$\Lambda = \frac{dA}{dx}$$

remember that the incompressible case had an identical form!

Combining ϕ_Λ with a previous derivation of drag $D = -\rho V_\infty^2 \iint \phi_z \phi_x \, dS$ gives

$$D = \frac{-\rho_\infty V_\infty^2}{4\pi} \int_0^\ell \int_0^\ell \underbrace{\frac{d\Lambda(x')}{dx'} \frac{d\Lambda(x'')}{dx''}} \ln|x' - x''| \, dx' \, dx''$$

$$\frac{d\Lambda}{dx} = \frac{d^2A}{dx^2}$$ 2nd derivative of Area wrt x.

For slender axisymmetric bodies, C_D drag coefficient is <u>independent</u> of Mach # and depends on the <u>2nd derivative of area</u> squared.

Sears Haack Body (1947)

Apply cosine transform

$$x = \frac{L}{2}\left(1 - \cos\theta\right)$$

Expand the line source as a sine function

$$\Lambda = L \sum B_n \sin n\theta \qquad \text{with} \quad \frac{dA}{dx} = \Lambda$$

Apply to drag equ and simplify (see Aerodynamics of wings and bodies, Ashley and Landahl)

$$D = \frac{\pi}{8}\rho V_\infty^2 L^2 \sum_{n=2}^{\infty} n B_n^2$$

Area distribution (integral of $\frac{dA}{dx} = \Lambda$)

$$A(x) = \int_0^x \frac{dA}{dx}\,dx = \int_0^x L \sum B_n \sin n\theta \, dx = \dots = \frac{L^2}{4}\sum_{n=2}^{\infty} B_n\left(\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right)$$

Volume

$$V = \int_0^L A(x)\,dx = \dots = \frac{\pi L^3}{16} B_2$$

Drag

$$D = \frac{1}{2}\rho V_\infty^2 \frac{128}{\pi}\frac{V^2}{L^4}\left[1 + \frac{3}{2}\left(\frac{B_3}{B_2}\right)^2 + \frac{4}{2}\left(\frac{B_4}{B_2}\right)^2 + \frac{5}{2}\left(\frac{B_5}{B_2}\right)^2 + \frac{6}{2}\left(\frac{B_6}{B_2}\right)^2\right]$$

Since these are squared, always +

Minimum Drag when $B_3 = B_4 = \cdots B_\infty = 0$ and $B_2 \neq 0$

$$D = \frac{1}{2}\rho V_\infty^2 \frac{128}{\pi}\frac{V^2}{L^4}$$

or

$$C_{D_A} = \frac{\frac{1}{2}\rho V_\infty^2 \frac{128}{\pi}\frac{V^2}{L^4}}{\frac{1}{2}\rho V_\infty^2 \frac{L^2}{3}\frac{16V}{\pi L^3}} =$$

$$A\left(\frac{L}{2}\right) = A\left(\frac{\pi}{2}\right)$$

$$= \frac{L^2}{3} B_2$$

or

$$A\left(\frac{L}{2}\right) = \frac{L^2}{3}\frac{16V}{\pi L^3}$$

$$\boxed{C_{D_A} = 24\frac{V}{L^3}}$$

with

$$A(x) = \frac{L^2}{4}\frac{16V}{\pi L^3}\left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right]\Bigg|_{n=2} = \frac{L^2}{4}\frac{16V}{\pi L^3}\left[\frac{\sin\theta}{1} - \frac{\sin 3\theta}{3}\right]$$

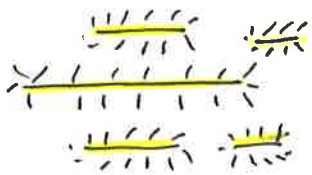
or

$$r(x) = \sqrt{\frac{A}{\pi}}$$

Hayes Method

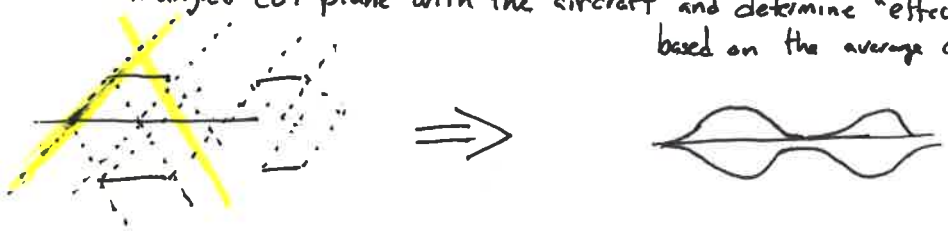
Aircraft are not axisymmetric. Sears Haack is derived axisymmetrically.

Hayes reasoned that there were sources distributed in the aircrafts volume



and these sources could be moved to the x-axis along characteristics

So, intersect an angled cut plane with the aircraft and determine "effective" area based on the average cross section area.



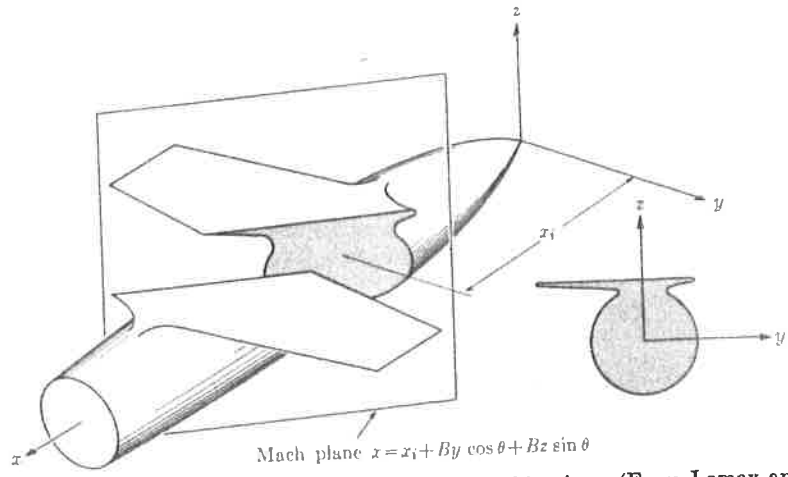
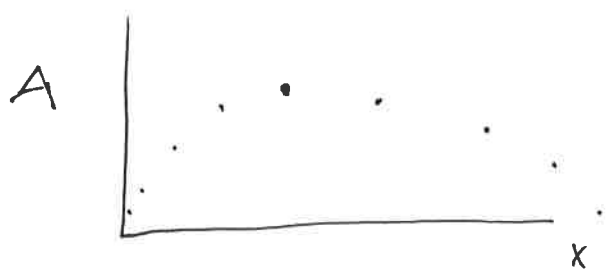


FIG. 9–6. Mach plane cut of a wing-fuselage combination. (From Lomax and Heaslet, 1956. Courtesy of the American Institute of Aeronautics and Astronautics.)

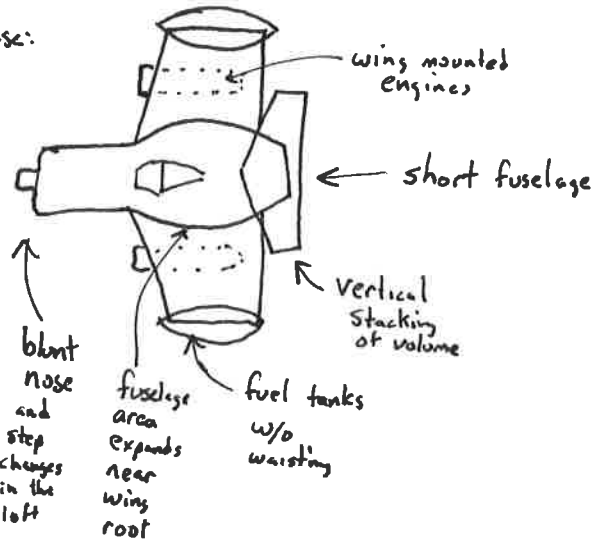
Ashley and Landahl



Supersonic Area Rule

Smooth distribution of area when measured across Mach plane cuts.

Worst case:



Better case