AEM 617

Lesson 4

Aerodynamics
Systems

Lagrangian and Eulerian Frames

$$\underbrace{\frac{D(\,)}{Dt}}_{\substack{\text{particle}\\ \text{frame}\\ \text{"Lagrangian"}}} \equiv \underbrace{\frac{\partial(\,)}{\partial t} + V\cdot\nabla(\,)}_{\substack{\text{Eulerian frame}\\ \text{fixed location}}} = \frac{\partial(\,)}{\partial t} + \frac{d(\,)_i}{dx_j}\frac{dx_i}{dt}$$

Example:

If the lapse rate is $-5°$ per 1000^{ft} and an aircraft is climbing at $1500\ ft/min$, what is the rate of change in temperature on the aircraft?

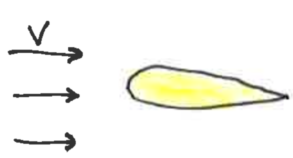
A: We want the temperature along the aircraft moving at $1500\ ft/m$. This is a Lagrangian frame.

$$\frac{DT}{Dt} = \underbrace{\cancelto{0}{\frac{\partial T}{\partial t}}}_{\substack{\text{no change}\\ \text{in temp}\\ \text{at a const}\\ \text{height w/ time}}} + \cancelto{1500\frac{ft}{min}\cdot\frac{dT}{dh}}{\frac{d()_i}{dx_j}\frac{dx_j}{dt}} = 1500\frac{ft}{min}\cdot\frac{-5°}{1000ft} = -7.5°/min$$

Q: If the atmosphere is now cooling at $1°/min$, what is the rate of change in temperature on the aircraft?

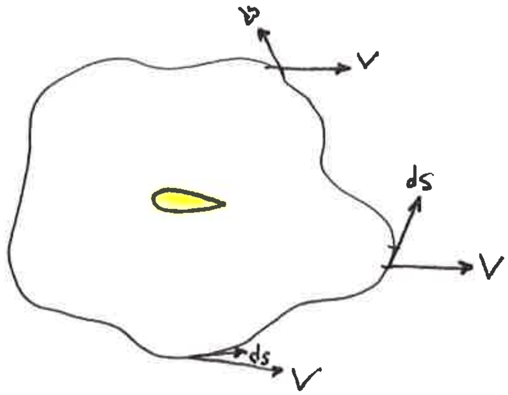
$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{dT}{dh}\cdot\frac{dh}{dt} = -1°/min + \frac{-5°}{1000ft}\cdot\frac{1500ft}{min} = -8.5°/min$$

Use the frame that simplifies the problem



vs

Circulation and Lift



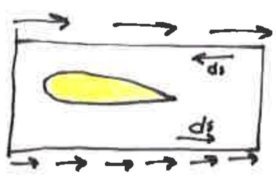
$$\text{Circulation} \equiv \Gamma = -\oint V \cdot ds$$

proportional to the velocity component tangent to a closed curve

Lift is $L = \rho V \Gamma$ ← circulation.
density, freestream velocity

No circulation .... no lift.

So on average, an airfoil generating lift has a net negative $V \cdot ds$. One way this can happen is if the flow above the airfoil is moving faster than the flow below

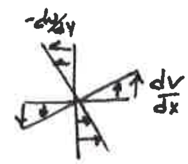


$V \cdot ds$ is negative and larger
$V \cdot ds$ is positive and smaller
$\Big\}\ L > 0$

Vorticity:

The local velocity derivatives represent the pointwise vorticity

$$\omega = \nabla \times V \quad \text{in 2D rect' coord'} = \frac{dv}{dx} - \frac{du}{dy}$$



You can measure vorticity with an object in a flow by watching the rotation.

Vorticity and Circulation are related by $\Gamma = -\oint V \cdot ds = -\iint \omega \cdot \hat{n}\, dA$

Since Lift requires Circulation .
and
Circulation is an integrated form of vorticity

Units:

$$\frac{\left[\frac{ft}{s}\right]}{ft} = \frac{1}{s}$$

<u>Lift requires vorticity</u> ... <u>somewhere</u>

Lift

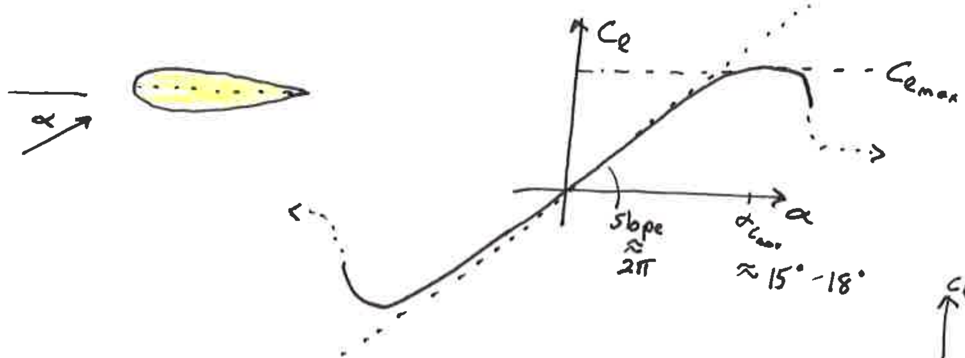
$\boxed{2\pi \text{ corrected for aspect ratio}}$

$$\text{Lift} = \underbrace{C_L}_{\text{lift coeffic.}} \; \underbrace{q}_{\text{dynamic pressure}} \; \underbrace{S}_{\text{area}}$$

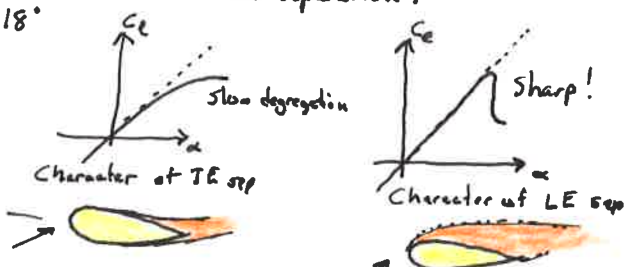
2-Dimensional airfoil

$C_{l_\alpha} \approx 2\pi$

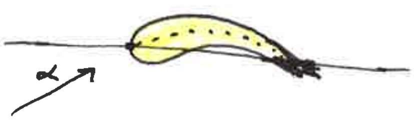
$C_{l_\alpha} \equiv \dfrac{dC_l}{d\alpha}$ in $\dfrac{1}{\text{radians}}$



Lower C_l deviation from $2\pi\alpha$ as α increases is due to flow separation.

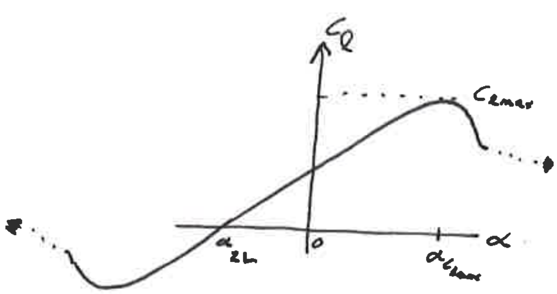


Camber



Mean chord line is through the forward ~~aut~~ LE and rear TE pts.

Adding camber offsets the C_l vs α curve such that zero lift occurs at α_{ZL}

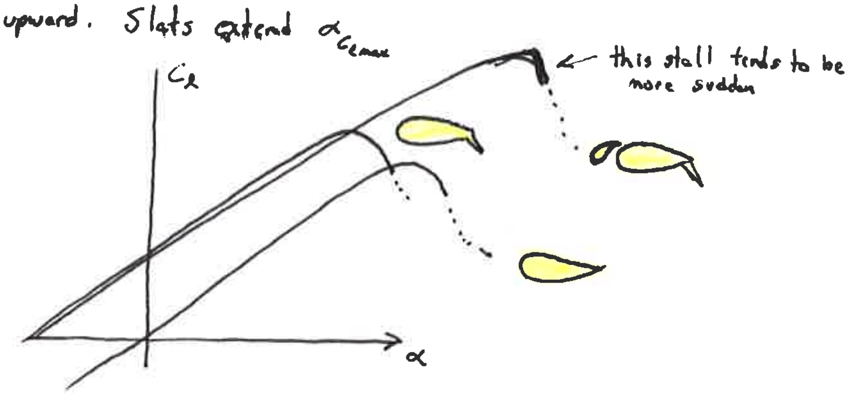


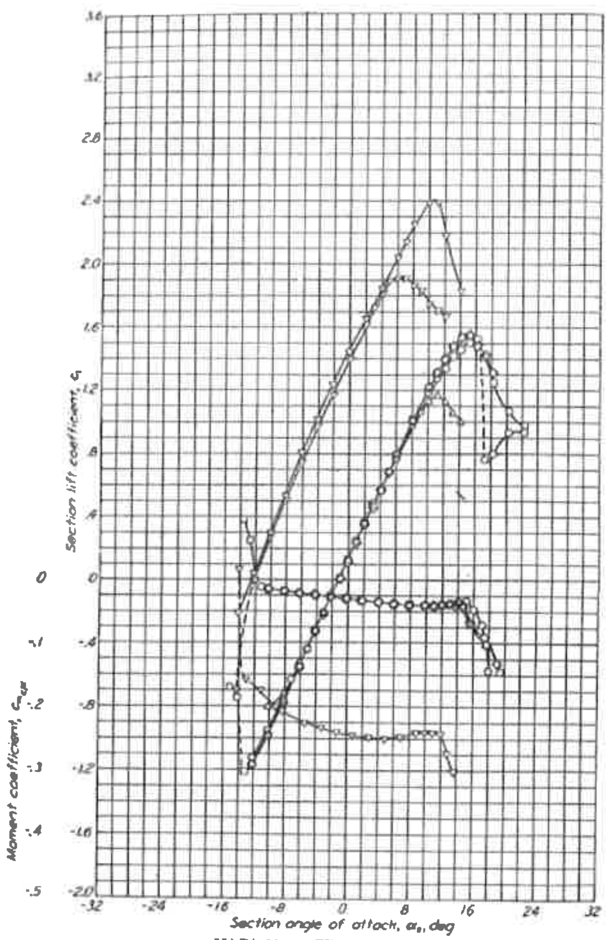
Knowing $C_{l_\alpha} \approx 2\pi$ and α_{ZL}, what is a good estimate for $C_l(\alpha=0)$?

$$C_l(\alpha=0) \approx 2\pi \cdot (-\alpha_{ZL})$$

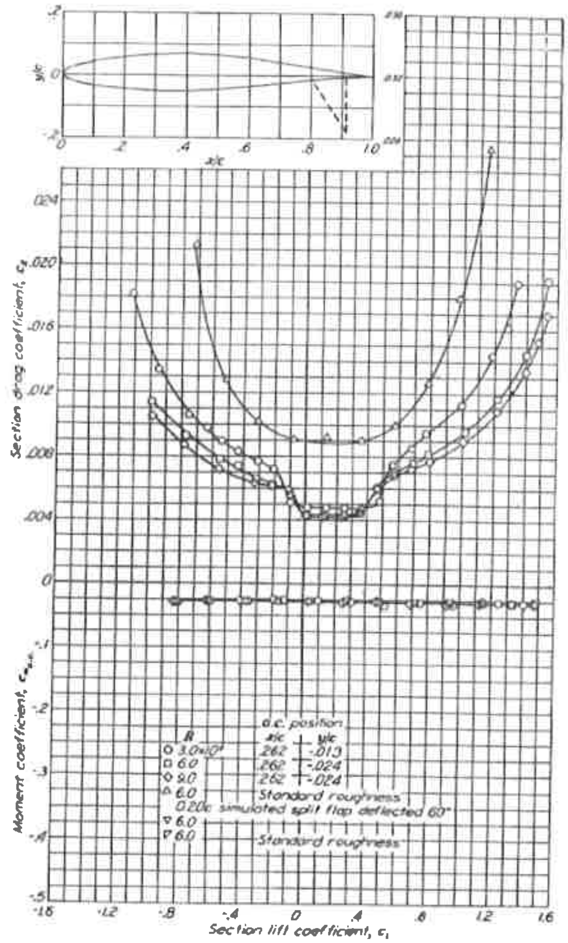
Slats and Flaps

Flaps shift the C_l vs α curve upward. Slats extend $\alpha_{C_{l_{max}}}$





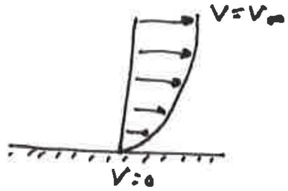
NACA 64₁-212 Wing Section



NACA 64₁-212 Wing Section (*Continued*)

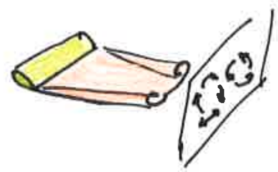
Drag

Surface Friction



+

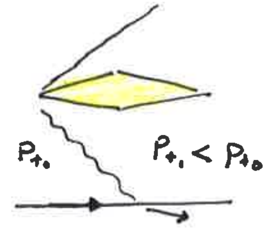
Induced Drag



Rotational energy

+

Wave Drag



In general, drag is complicated to estimate for arbitrary configurations

See:

Fluid-Dynamic Drag, Hoerner

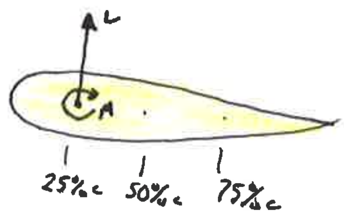
Integrated Forces and Moments on 2D airfoils.

$$L = q C_L S$$

$$D = q C_D S$$

$$M = q C_M S \bar{c}$$

Subsonic Airfoils



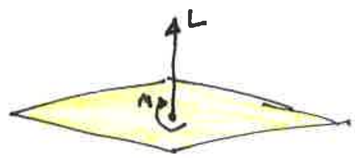
Lift and moment act at the aerodynamic center.

25% chord

$$A.C. \equiv \frac{dC_m}{dC_L} = 0$$

Thus $C_m(\alpha) = C_{m_0}$

Supersonic Airfoil



A.C. at 50% chord

Transonic

Rapid and non-monotonic shift in a.c.

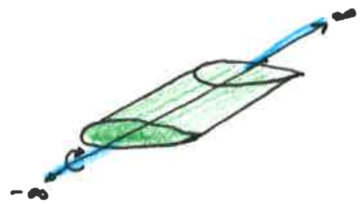
Strongly depends on airfoil shape

3D Wings

From physics, an inviscid vortex must start and end on a solid surface or form a closed loop.

Possibilities:

1)



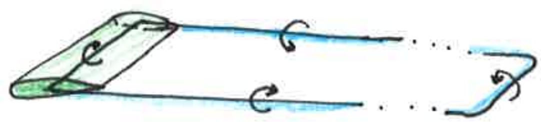
Vortex extends off wingtips forever.

$$C_L = \int C_\ell \, dy \rightarrow \infty$$

Impossible

X

2)

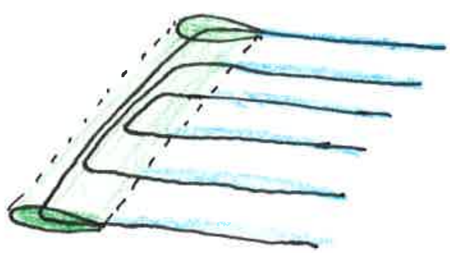


- Finite lift
- Velocity at tip is <u>infinite</u>

$$V \propto \Gamma \cdot \frac{1}{r}$$

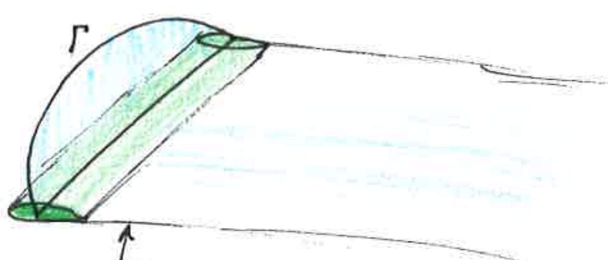
Reasonable mental model, but still wrong!

3)



- Discrete vortices distributed along span
- Velocity at tip $= 0$ since $\Gamma_{tip} = 0$
- Trailing vortices <u>induce</u> α at wing which varies with span location.

4)



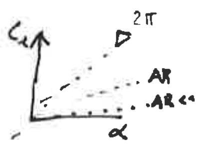
notice that the tip vortex has strength of zero!

- Continuous distribution of vorticity along span

✓

Analysis of Prandtl Lifting Line Theory:

The elliptical lift distribution is optimal for minimizing induced drag.



$$C_{L_\alpha} = \frac{C_{l_\alpha}}{1 + \frac{C_{l_\alpha}}{\pi AR}}$$

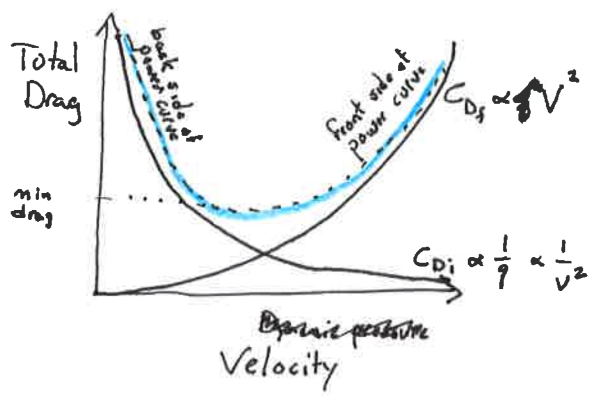
Reducing AR reduces lift slope away from 2D value.

$$C_D = \frac{C_L^2}{\pi AR e}$$

where $e = 1$ for elliptic only

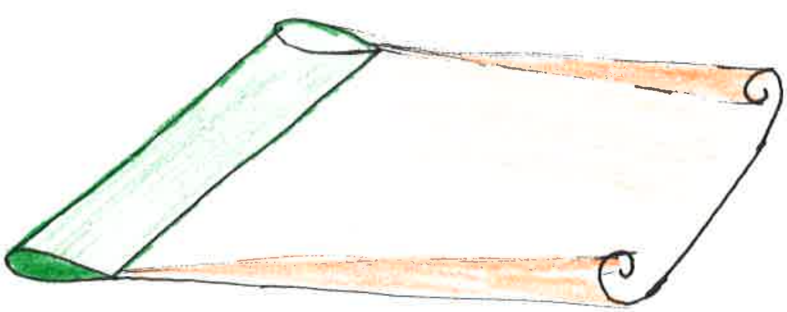
Induced drag depends on lift squared and the inverse of AR.

$$= \frac{\left(\frac{L}{b}\right)^2}{q^2 \pi e} = C\,\frac{(\text{Span loading})^2}{(\text{dynamic pressure})^2}$$



Non-Elliptical distributions have an "e" value are essentially a ratio of actual to elliptical performance

$$e_{\text{non-elliptical}} < e_{\text{elliptical}} = 1$$



Wake rollup

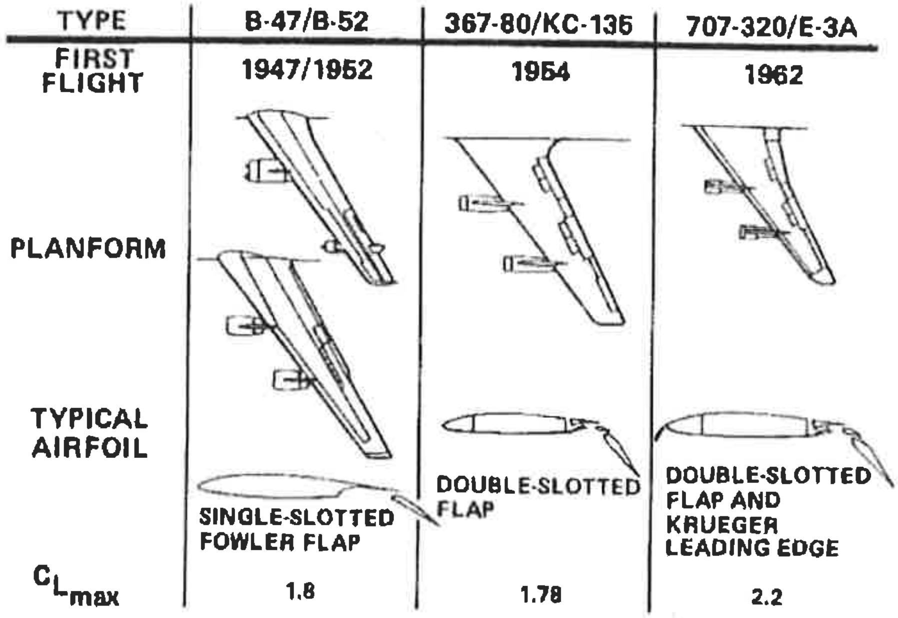


Figure 26.4 - Trends in Boeing transport high-lift development.
Source: AGARD CP-365, paper no. 9

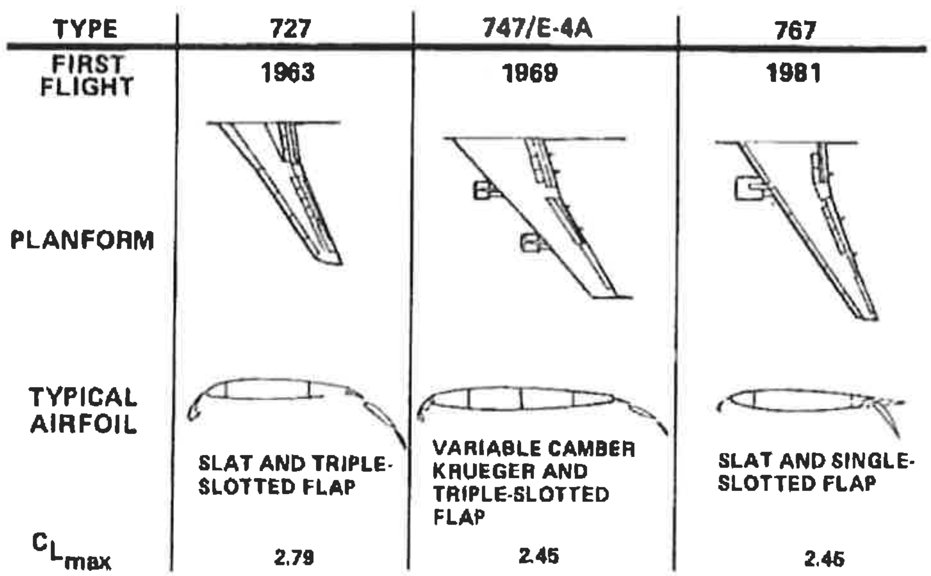


Figure 26.5 - Trends in Boeing transport high-lift development - continued
Source: AGARD CP-365, paper no. 9

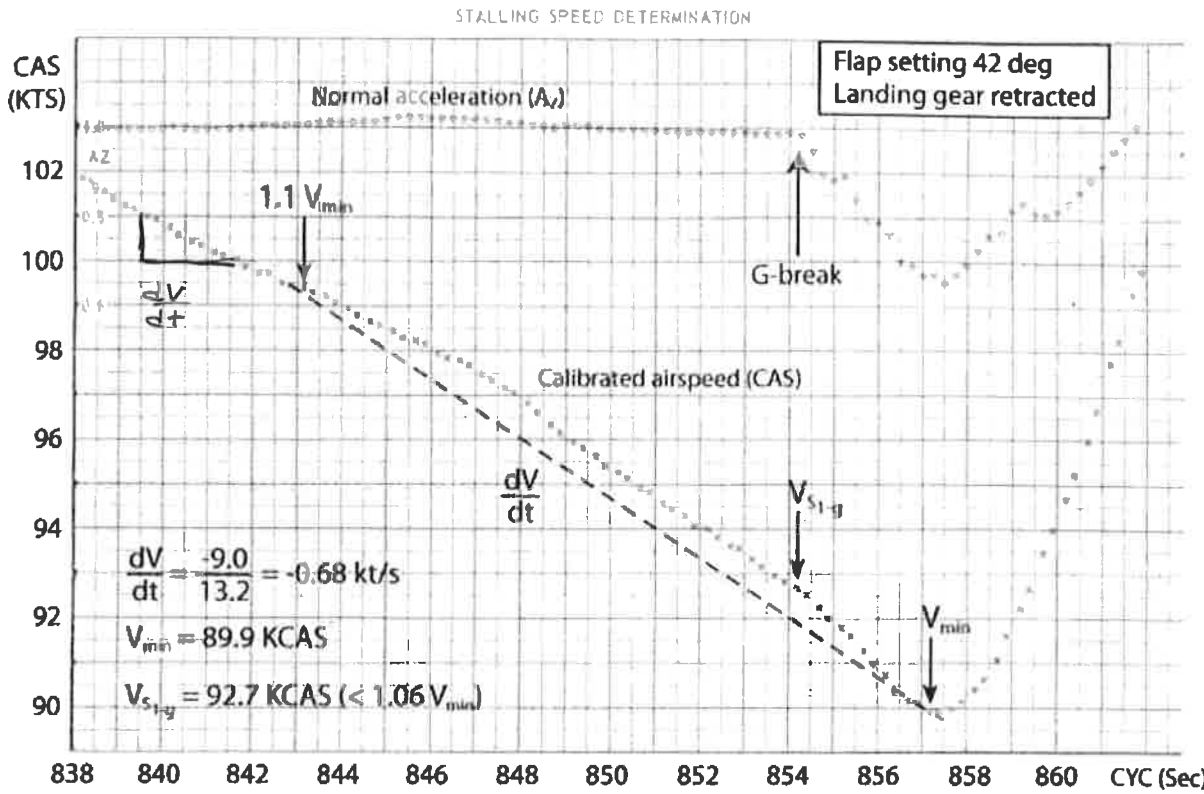
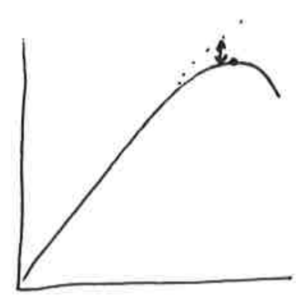
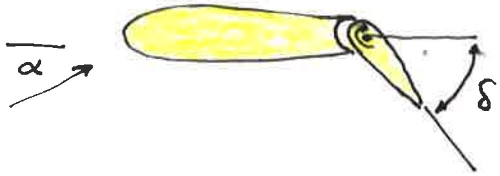


Figure 26.2 - Stall speed determination of a Fokker F-28 Mk 4000.
Source: Fokker Report H-28.40-27.001



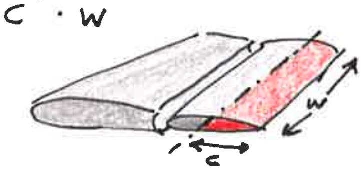
Hinge Moments (Necessary for the upcoming flight control system portion of the class)



A moment is necessary to maintain the control surface at δ.

$$C_h = f(\alpha, \delta, Re, gap, etc) \quad \text{and possibly time}$$

$$H = C_h \cdot q \cdot \underset{\text{area aft of hinge}}{S} \cdot \underset{\text{chord aft of hinge}}{c} = C_h \cdot q \cdot c^2 \cdot w$$



The pilot is connected to the surface in some way

$$\delta = f(\underset{\delta_s}{\text{stick angle}})$$

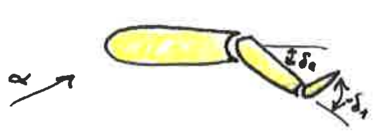
Gearing Ratio

$$F \ell_s \delta_s = H \delta_e \Rightarrow F = \left(\frac{\delta_e}{\ell_s \delta_s}\right) H_e$$

How much force can a pilot exert? How long? $H \propto V^2$; pilot is limited human!

Constraints? Stick position, structural stresses, . . .

Trim Tab



$$C_h = C_{h_0} + \underbrace{C_{h_\alpha}}_{\frac{dC_h}{d\alpha}} \alpha + \underbrace{C_{h_{\delta_e}}}_{\frac{dC_h}{d\delta_e}} \delta_e + \underbrace{C_{h_{\delta_t}}}_{\frac{dC_h}{d\delta_t}} \delta_t$$

For the pilot to have zero stick force, $C_h = 0$

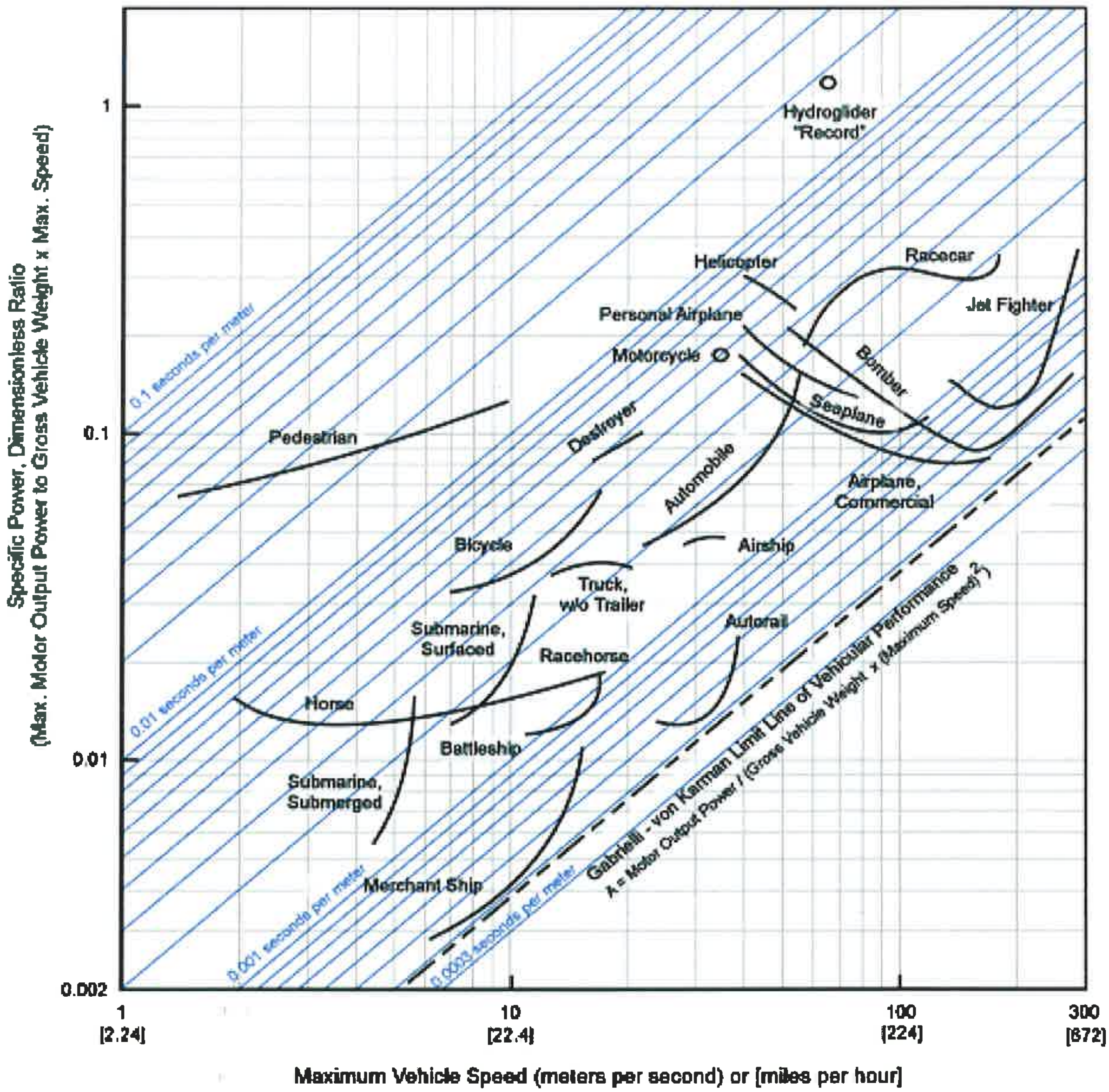
We (as pilots) can adjust δ_t to ensure $C_h = 0$ for a particular δ_e required at a particular α.

Stick free:

$$C_h = 0 = \overset{\text{trimmed } 0}{\cancel{\left(C_{h_0} + C_{h_{\delta_t}} \delta_t\right)}} + \underbrace{C_{h_\alpha}}_{\text{usually negative}} \alpha + \underbrace{C_{h_{\delta_e}}}_{\text{usually negative}} \delta_e \Rightarrow \delta_e = -\frac{C_{h_\alpha}}{C_{h_{\delta_e}}} \alpha$$

The elevator "floats". TEU w/ α increase

Von Karmen – Gabrielli Limit



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