AEM 617

FCS: Stab + Control

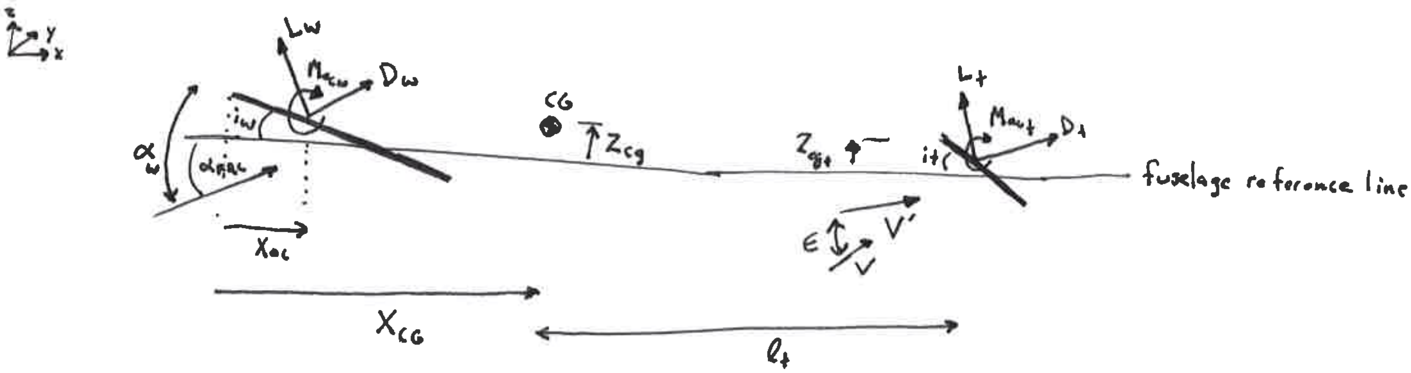
Lecture 9 part 2

Stability and Control

↖ do perturbations of states decrease or increase with time? (Stability)

Can a desired state be reached from the current state? (Control)

Aircraft Longitudinal Static Stability (ref: Flight Stability and Automatic Control, Nelson)



Summation of moments about the CG (wing only)

$$M_{cg} = L_w \cos(\alpha_w - i_w)(X_{cg} - X_{ac}) + D_w \sin(\alpha_w - i_w)(X_{cg} - X_{ac}) + L_w \sin(\alpha_w - i_w)(Z_{cg}) - D_w \cos(\alpha_w - i_w)(Z_{cg}) + M_{ac}$$

Scale by $\frac{1}{2}\rho V^2 S \bar{c}$

$$C_{m_{cg}} = C_{L_w}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)\cos(\alpha_w - i_w) + C_{D_w}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)\sin(\alpha_w - i_w) + C_L \frac{Z_{cg}}{\bar{c}}\sin(\alpha_w - i_w) - C_D \frac{Z_{cg}}{\bar{c}}\cos(\alpha_w - i_w) + C_{m_{ac}}$$

For small angles, $\cos(\alpha_w - i_w) \approx 1$ and $\sin(\alpha_w - i_w) \approx \alpha_w - i_w$ and $C_L >> C_D$

$$C_{m_{cg}} = C_{m_{ac}} + C_L\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) = C_{m_{ac}} + (C_{L_0} + C_{L_\alpha}\alpha)\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right)$$

Wing only static stability

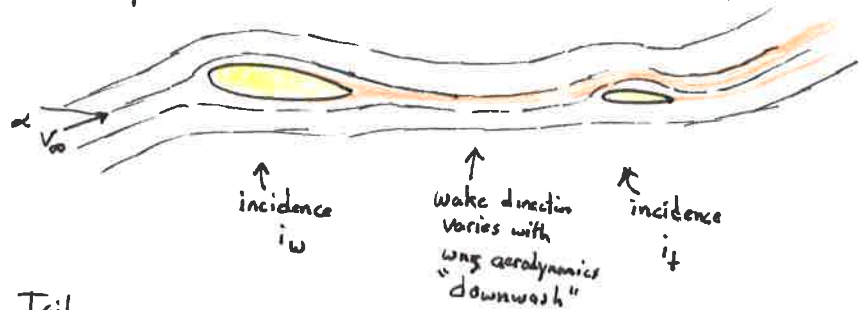
$$C_{m_\alpha} = \frac{dC_m}{d\alpha} = C_{L_\alpha}\left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) \text{ must be } < 0 \text{ for stability}$$

Thus for flying wings, $X_{cg} < X_{ac}$. Also $C_{m_{cg}} = 0$, thus $C_{m_{ac}} > 0$

Flying Wings require the cg forward of the A.C. (pitch stability $C_{m_\alpha} < 0$) plus negative camber ($C_m = 0$).

Tail

A tail operates in the wake of the main wing.



- Tail operates in wing wake + Thrust. The dynamic pressure may be different from the wing's.

$$\eta = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V_w^2} = \frac{q_t}{q_w}$$

- Tail aero α_t

$$\alpha_t = \alpha_w - \epsilon \underbrace{- i_w + i_t}_{\text{fuselage ref angles}}$$

(α_w: flight angle; ϵ: downwash)

- Reference area is <u>wing</u> area S

$$\Delta C_{L_{total}} = \eta \frac{S_t}{S} C_{L_t}$$

Moment about cg from tail is similar to wing derivation.

$$M_t = -l_t L_t \cos(\underbrace{\alpha_{FRL} - \epsilon}_{\text{flow angle}}) + l_t D_t \sin(\alpha_{FRL} - \epsilon) - z_{cg_t} D_t \cos(\alpha_{FRL} - \epsilon)$$
$$- z_{cg_t} L_t \sin(\alpha_{FRL} - \epsilon) + M_{ac_t}$$

Small angles + drop small terms ($L_t \gg D_t$ and $z_{cg_t} \approx 0$) (or $< l_t$)

$$M_t = -l_t L_t = -l_t C_{L_t} q_t S_t$$

Ref' dim' are wing area and chord

$$C_{m_t} = \frac{M_t}{q_w S \bar{c}} = -\frac{l_t S_t}{\bar{c} S} \frac{q_t}{q_w} C_{L_t} = \underbrace{-\frac{l_t S_t}{\bar{c} S}}_{\text{"Tail Volume" Ratio}} \eta C_{L_t} = -V_H \eta C_{L_{\alpha_t}} (\alpha_w - \epsilon - i_w + i_t)$$

(ϵ: $\epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha_w$ approximation)

For an elliptical wing,

$$\epsilon = \alpha_i \approx \frac{C_{l_\alpha}}{\pi AR} \frac{1}{1 + \frac{C_{l_\alpha}}{\pi AR}} \alpha$$

$$\epsilon \approx \frac{C_L}{\pi AR_w}$$

$$\frac{d\epsilon}{d\alpha} = \frac{C_{L_\alpha}}{\pi AR_w}$$

$$C_{m_{cg_t}} = -V_H \eta C_{L_{\alpha_t}} \left(\alpha_w - i_w - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha_w + i_t \right)$$

$$= V_H \eta C_{L_{\alpha_t}} \left(\epsilon_0 + i_w - i_t \right) - V_H \eta C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \alpha$$

Whole airplane (at least wing + tail)

$$C_{m_{cg}} = C_{m_{ac}} + \left(C_{L_0} + C_{L_\alpha} \alpha \right) \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) + V_H \eta C_{L_{\alpha_t}} \left(\epsilon_0 + i_w - i_t \right) - V_H \eta C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \alpha$$

Stability: $C_{m_{cg_\alpha}} = \underbrace{C_{L_\alpha} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right)} - \underbrace{V_H \eta C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha} \right)} < 0$

cg can now be behind the wings aero. center.

from the negative sign, the tail adds stability as long as the change in downwash is not too large.

Static moment:

The aircraft can be trimmed by adjusting the tail incidence i_t.

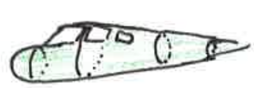
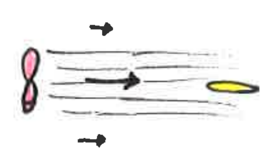
Neutral Point

The point of x_{cg} where $C_{m_{cg_\alpha}} = 0$. Neutral stability.

Static Margin

The amount $\frac{x_{cg}}{\bar{c}}$ is ahead of the N.P. as a %. $\approx 10\%$ is reasonable

Q: Can other aircraft components impact stability and control?
A: Absolutely Yes

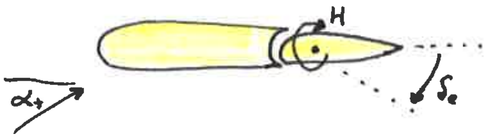
- Fuselage C_{m_α}  A fuselage is wider ahead of the cg.
- Power impacts η as the ratio of dynamic pressure 
- Configuration changes impacting $\frac{d\epsilon}{d\alpha}$ or η

 e.g. Flaps

Stick free vs. Stick fixed

The previous analysis assumed that the tail's lift only depended on the local α_t (in other words, the elevator was fixed).

Actually, the elevator tends to deflect based on the local α_t, since a hinge moment is generated. The stick is free to move.



$$C_h = C_{h_0} + C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e$$

For stick free, $C_h = 0$. Assuming the stick forces are trimmed out, $C_{h_0} = 0$

$$0 = C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e \quad \Rightarrow \quad \delta_e = -\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \alpha_t$$

Since $C_{h_{\alpha_t}}$ and $C_{h_{\delta_e}}$ are usually negative, $\delta_e = -(+)\alpha_t \Rightarrow \frac{d\delta_e}{d\alpha_t} < 0$

The elevator "floats" upward as α_t increases.

If the tail's lift depends on both α_t and δ_e

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e = C_{L_{\alpha_t}} \alpha_t - C_{L_{\delta_e}} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \alpha_t$$

so the lift coeff is

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t \left(1 - \frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \right) = \hat{C}_{L_{\alpha_t}} \alpha_t$$

How does this affect stability?

$$C_{m_{cg_\alpha}} = C_{L_\alpha} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) - V_H \eta \, C_{L_{\alpha_t}} \left(1 - \frac{C_{L_{\delta_e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta_e}}} \right) \left(1 - \frac{d\epsilon}{d\alpha} \right)$$

for most aircraft, this term is less than 1.
Stick free is less stable usually.

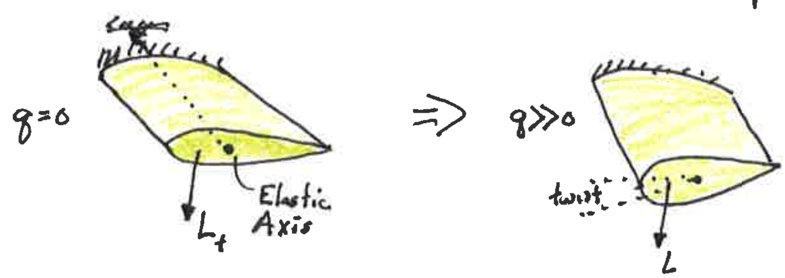
So the term/saying, "she flies hands off" does really have merit!

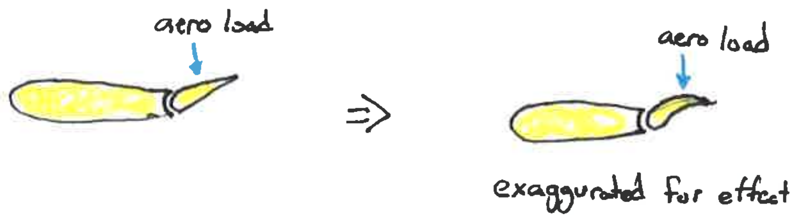
Other factors decreasing longitudinal stability

Aero elastics

The load on the stab varies with V^2 (since $q = \frac{1}{2}\rho V^2$).

There will be deflections due to aero loads, these may increase or decrease stability depending on the structural response.





For light aircraft, the static margin even in an incompressible flow may strongly depend on the flight velocity

Read: tiny.cc/AEM617TailStability

Harry Clements designing the C-180

- Suspected deflection in horz tail → reduced stab at/near V_{ne}
- pencil lead to detect deflection
- Not found!! Why? production changed design!