

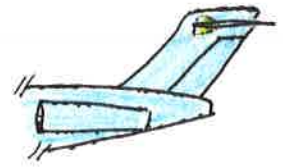
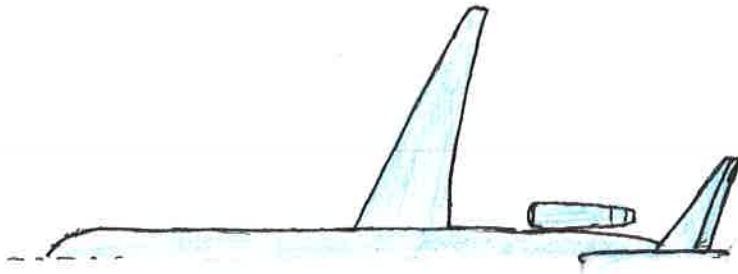
AEM 617

Alaska Air 261

+

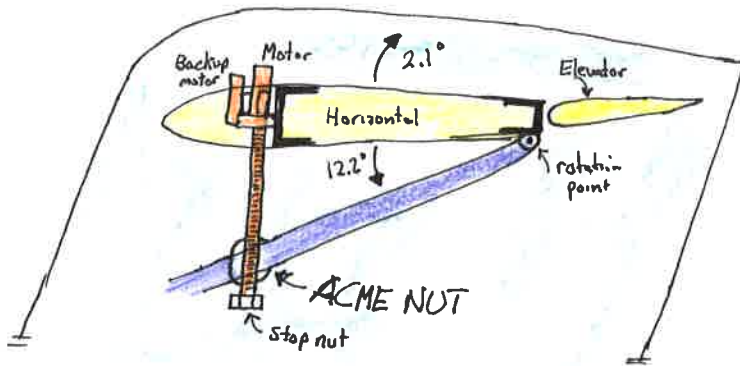
Nonlinear Spring Mass Damper

Alaska Airlines 261 (MD-83)



DC-9 (1965) → MD-80 (1980) → B717 (1999)
 CAR 4b 14CFR 25

- Pitch control from elevator on "t-tail" mounted horizontal. Pitch trim is through a jack screw mounted to the forward horizontal spar.



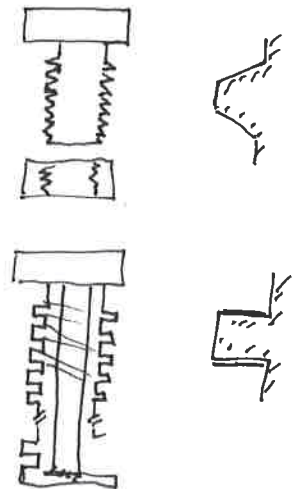
primary: 35 rpm = $\frac{1}{3}^\circ$ /sec
 High torque.
 Alternate: 10 rpm = $\frac{1}{10}^\circ$ /sec

Certified under CAR 4b in 1965,
 not required to be recertified to
 14CFR25!

Previous DC-8 had a dual jack screw

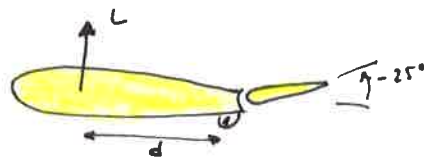
What is the weak link?

- Backup trim motors ✓
- pitch control via elevator ✓
- Stop nut and electrical stops ✓
- Auto pilot out-of-trim indicator ✓
- ACME thread had 2 threads. ✓
- Torque tube inside ACME thread in case of fracture ✓
- Threads + nut designed with 15x factor of safety ✓
- Dual horizontal rotation hinges (attached at spar) ✓
- Removing panels to grease jackscrew takes 4 hrs X



"The NTSB determines that the probable cause of this accident was a loss of aircraft pitch control resulting from the in-flight failure of the horizontal stabilizer trim system jackscrew assembly's acme nut thread. The thread failure was caused by excessive wear resulting from Alaska Airlines' insufficient lubrication of the jackscrew assembly."

Failure Mode Analysis:



Location of A.C of horizontal $\approx 25\% c$

Hinge point at aft span

~~XXXXXXXXXX~~

Load on horizontal pulls stripped threads to mechanical stop at about 3° nose down trim.

Variation in downwash angle from pilot experimenting with flaps and slats (and trim) contributes to extra load on failins screw.

- Lack of Lubrication

↓
Excessively worn acme nut.

↓
Found during C check at about 7200 hrs. Scheduled for replacement. (1997)

↓
End-play re-evaluated and barely passed. Jackscrew and nut assembly not replaced.

↓
31 Jan 2000 ≈ 3 hrs before accident. Trim moved to 0.4° TED, Head flown. Autopilot engaged ≈ 30 min prior. Autopilot disengaged and "clunks" heard on CVR. after operating trim!

↓
pitch control lost partially

↓
Flaps + slats extended and retracted.

↓
"loud noise" followed by $\delta_e = -25^\circ$

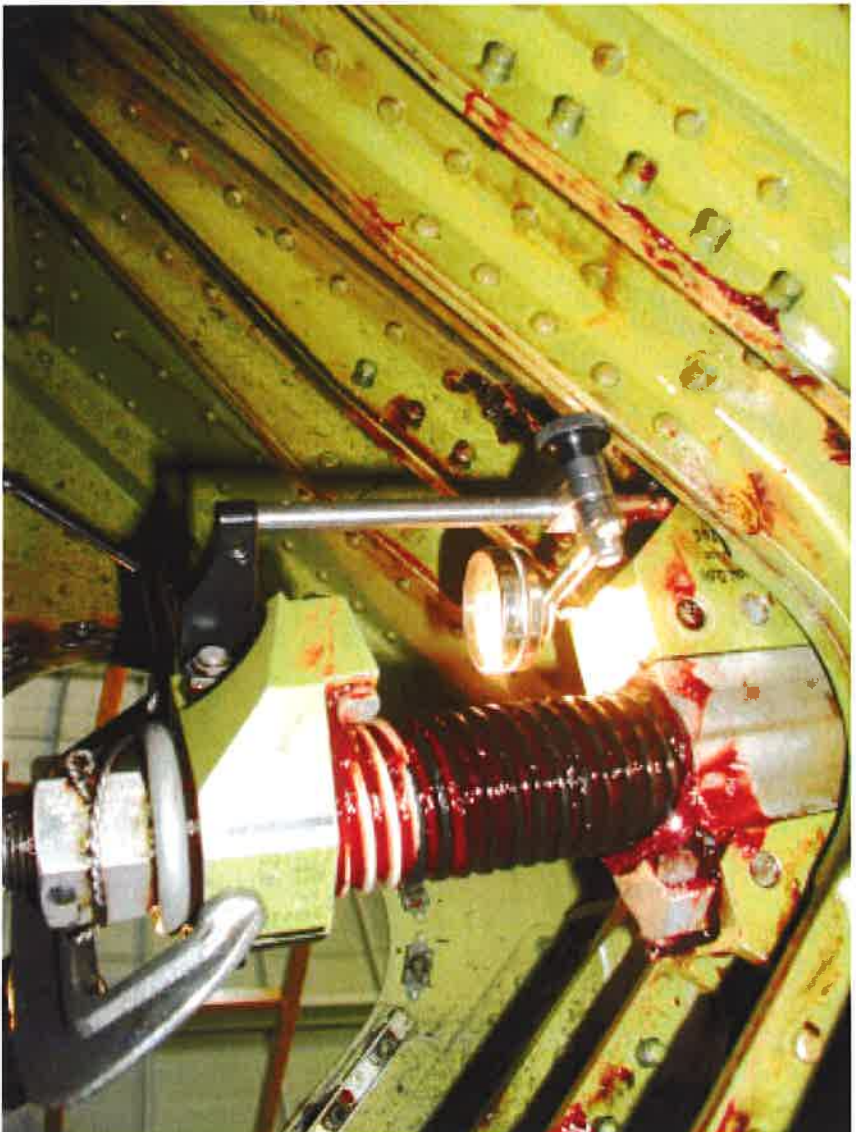


Figure 9. View of a typical dial indicator set up for end play checks.

On May 29, 1984, McDonnell Douglas issued AOL 9-1526, which reported that “two operators have reported three instances of premature removal/replacement of a horizontal stabilizer actuator assembly” and reiterated the OAMP document’s recommendation that all DC-9 operators lubricate the jackscrew assembly at

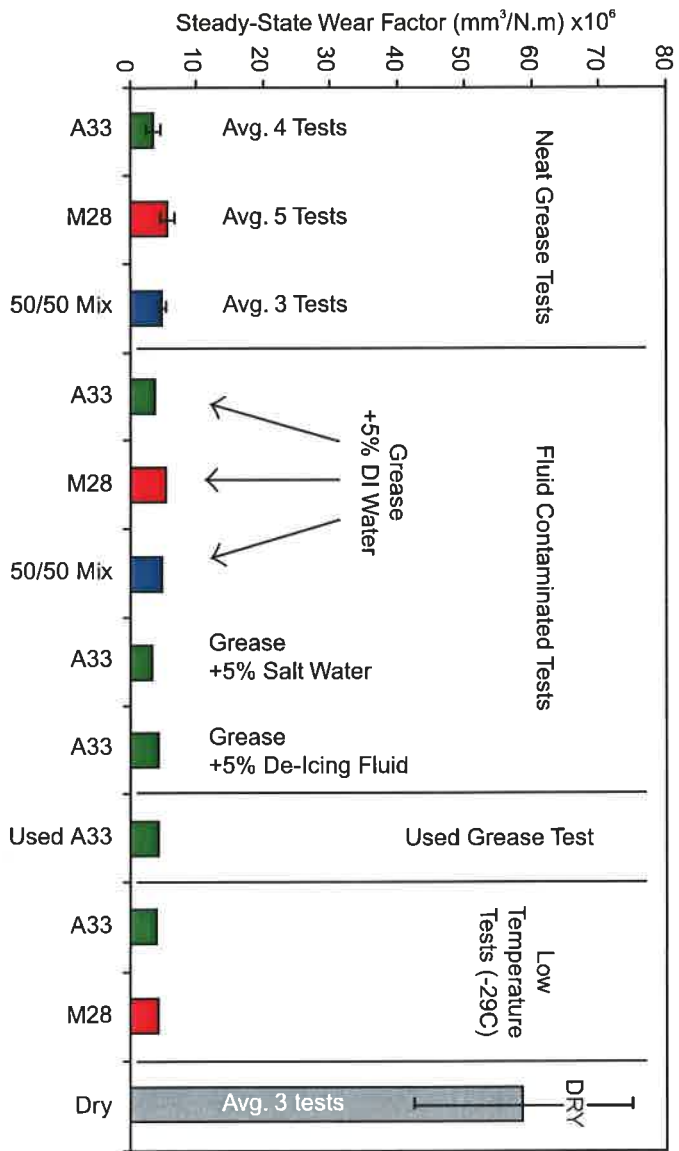


Figure 19. A summary of the results of the high-load wear tests, including the contamination and low-temperature tests.

1.17 Organizational and Management Information

1.17.1 Alaska Airlines, Inc.

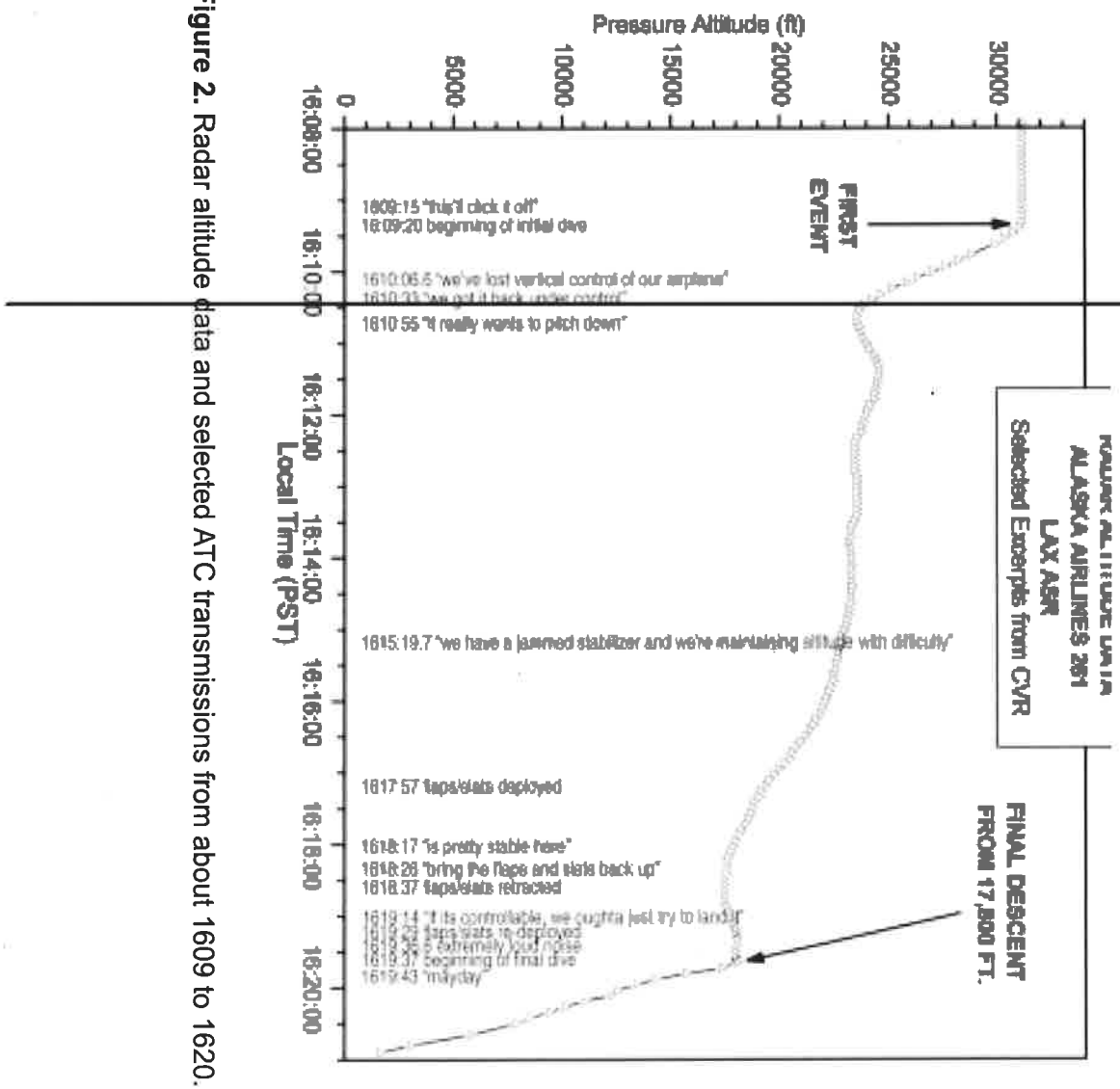


Figure 2. Radar altitude data and selected ATC transmissions from about 1609 to 1620.

Figures showing the four positions of the screw threads versus the nut threads

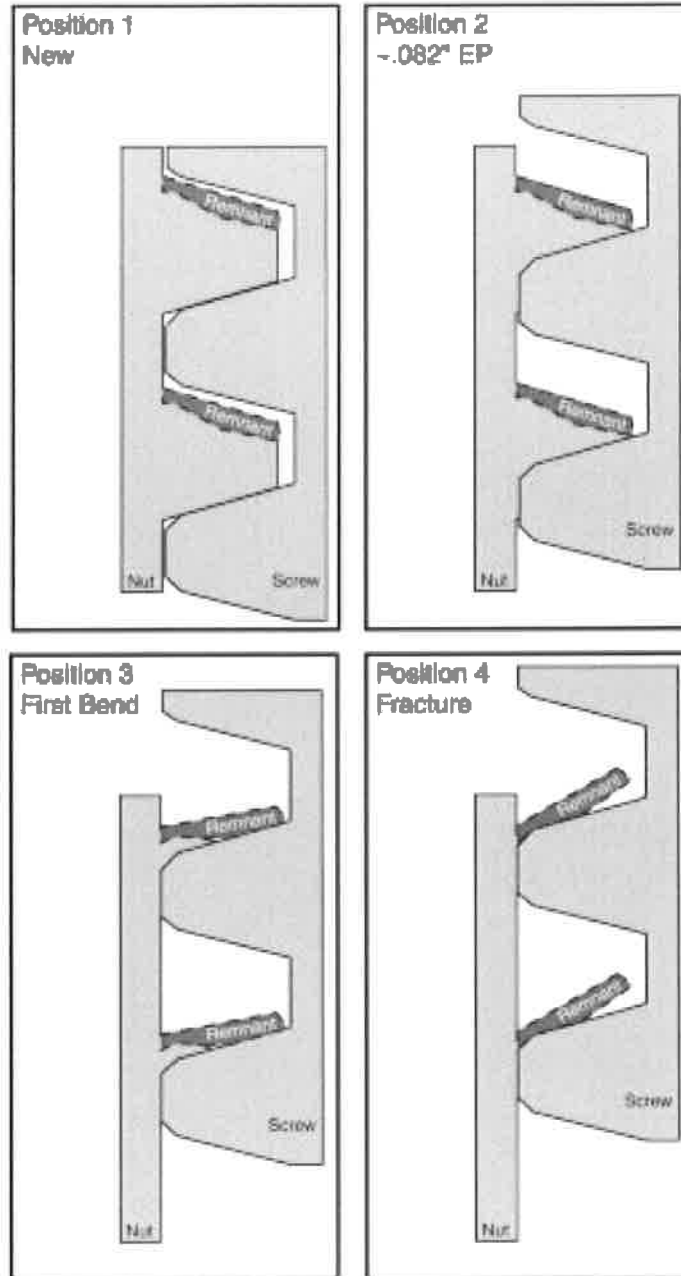
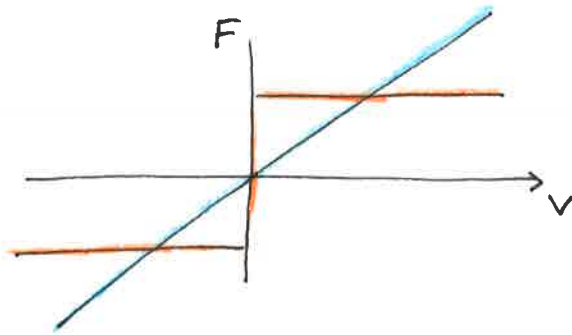
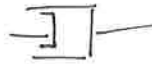


Figure 20. A graphical depiction of the stages of acme nut thread wear to the point of fracture.



Coulomb Friction vs "Dashpot" damping



Coulomb

$$F = c \frac{\dot{x}}{|\dot{x}|} = c \text{sign}(v)$$

Traditional Damping

$$F = c \dot{x} = c \cdot v$$

The traditional damping of a spring-mass-damper 2nd order system is

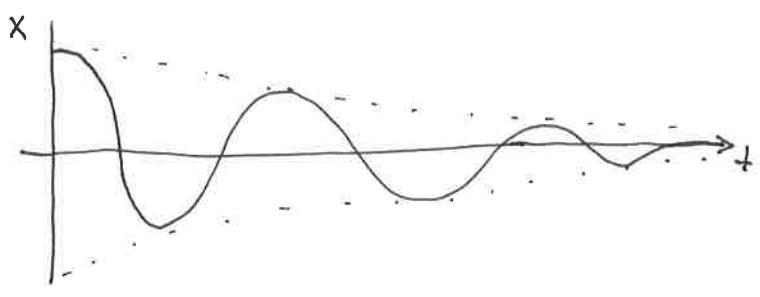
$$m\ddot{x} + c\dot{x} + Kx = 0$$

$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = 0 \quad \text{where } \omega_n^2 = \frac{K}{m}$$

$$\frac{c}{m} = 2\zeta\omega_n$$

Solution:

$$x = X_0 e^{-\zeta\omega_n t} \sin(\sqrt{1-\zeta^2}\omega_n t + \phi)$$



Phase Plane Graphical solution method (Moretti's Modern Vibrations Primer)
CRC Press

Given an arbitrary 2nd order system: $m\ddot{x} + F(x, \dot{x}) = 0$

Convert to phase plane 

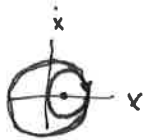
$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v = \frac{1}{2} \frac{d}{dx}(v^2)$$

$$\frac{dv}{dx} v = -F(x, \dot{x}) \Rightarrow \frac{dv}{dx} = -\frac{F(x, \dot{x})}{v}$$

From Moretti, scale $v = y \cdot \omega_{ref}$

$$\frac{dy}{dx} = -\frac{F(x, \dot{x})}{y \omega_{ref}^2}$$

If F were a linear function ^{at Kx} , the phase plane solution is a circle.



Pick an equivalent ODE

$$\ddot{x} + \omega_{ref}^2 x + f(x, v) = 0 = \ddot{x} + F(x, \dot{x})$$

$$\text{thus } F(x, \dot{x}) = F(x, v) - \omega_{ref}^2 x$$

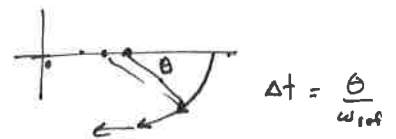
We will consider $f(x, \dot{x})$ stays constant for small time increments.

Thus, if we shift the phase plane circle (undamped) to

$$x_{center} = -\frac{f(x, v)}{\omega_{ref}^2} = \frac{\omega_{ref}^2 x - F(x, v)}{\omega_{ref}^2}$$

the solution in the phase plane is just a circle

The time for a given arc is $\Delta t = \frac{\theta}{\omega_{ref}}$



Iterate forward!

Undamped System.

$$m=1 \quad k=1$$

$$m\ddot{x} + kx = 0 \Rightarrow \ddot{x} + \frac{k}{m}x = 0 \Rightarrow \ddot{x} + x = 0 \quad \leftarrow F = x$$

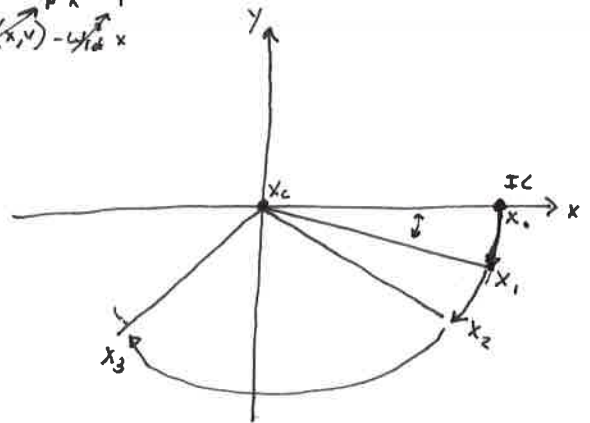
$$1) \text{ pick } \omega_{ref} = \sqrt{\frac{k}{m}} = 1 \Rightarrow F = \underbrace{F(x,v)}_{=0} - \omega_{ref}^2 x$$

2) plot IC

$$3) x_c = \frac{\omega_{ref}^2 x - F(x,v)}{\omega_{ref}^2} = \frac{x-x}{1} = 0$$

$$3.1) x_c = \frac{x-x}{1} = 0 \quad \checkmark$$

Always at $x_c = 0$



Damped System

$$m\ddot{x} + c\dot{x} + kx = 0 \Rightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \quad \text{with } \frac{k}{m} = 1 \text{ and } \frac{c}{m} = 0.1$$

$$1) \omega_{ref} = \sqrt{\frac{k}{m}} = 1 \Rightarrow f = \underbrace{F(x,v)}_{=0.1\dot{x} + 1x} - \underbrace{\omega_{ref}^2 x}_{=1 \cdot x} \Rightarrow \ddot{x} + \underbrace{(0.1\dot{x} + 1x)}_{F(x,v)}$$

2) plot IC

$$3) x_c = \frac{-f(x,v)}{\omega_{ref}^2} = \frac{\omega_{ref}^2 x - F(x,v)}{\omega_{ref}^2} = \frac{-0.1\dot{x}}{1} = -0.1\dot{x}$$

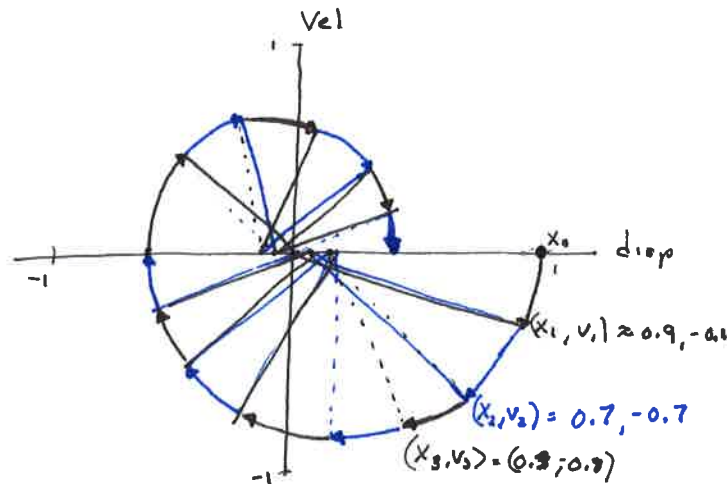
$$3.1) x_c = -0.1 \cdot (-0.2) = 0.02$$

$$3.2) x_c = -0.1 \cdot (-0.7) = 0.07$$

$$3.3) x_c = -0.1 \cdot (-0.8) = 0.08$$

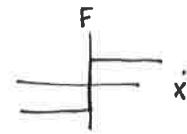
The damped system eventually converges to $(x, \dot{x}) = (0, 0)$.

$$\omega_0 < \omega_n$$



$$t = \frac{\theta}{\omega_{ref}} = \theta$$

Coulomb Friction



$$m \ddot{x} + c \operatorname{sign}(\dot{x}) + kx = 0$$

$$\ddot{x} + \frac{c}{m} \operatorname{sign}(\dot{x}) + \frac{k}{m} x = 0 \Rightarrow \ddot{x} + \underbrace{0.1 \operatorname{sign}(\dot{x}) + 1}_{F(x, \dot{x})} x = 0$$

1) Pick $\omega_{ref} = \sqrt{\frac{k}{m}} = 1 \Rightarrow f(x, \dot{x}) = 0.1 \operatorname{sign}(\dot{x}) + 1x - x = 0.1 \operatorname{sign}(\dot{x})$

2) IC.

3) $x_c = -\frac{f}{\omega_{ref}^2} = \frac{-0.1 \operatorname{sign}(\dot{x})}{1} = -0.1 \operatorname{sign}(\dot{x})$
 $= 0$

3.1) $x_c = -0.1 \operatorname{sign}(-0.4) = 0.1$

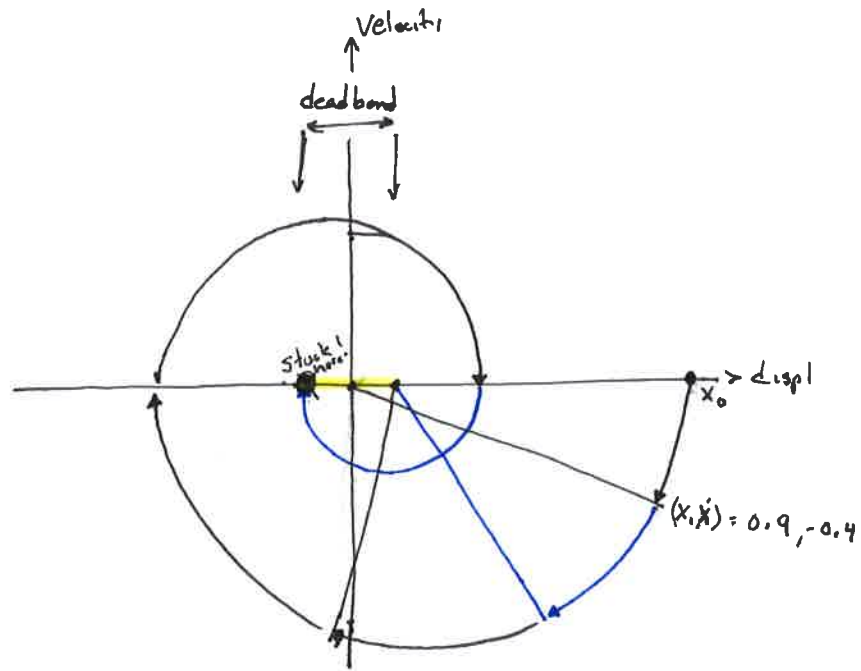
Notice x_c is constant when $\dot{x} < 0$

3.?) $x_c = -0.1 \operatorname{sign}(+) = -0.1$

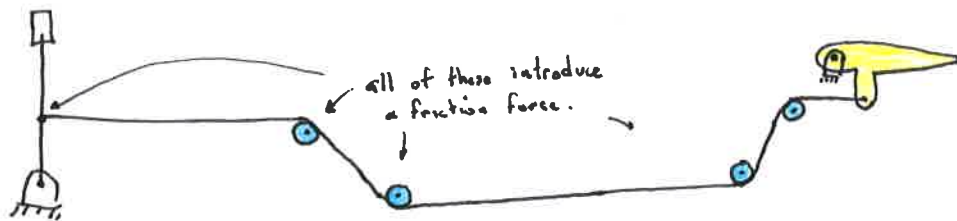
3.?) $x_c = -0.1$

Stuck at $x = -0.1$

Depending on the IC, the system converges to either $x = 0.1$
 or
 $x = -0.1$



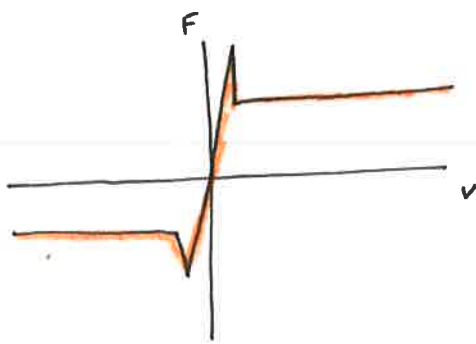
What does this mean for a FCS?



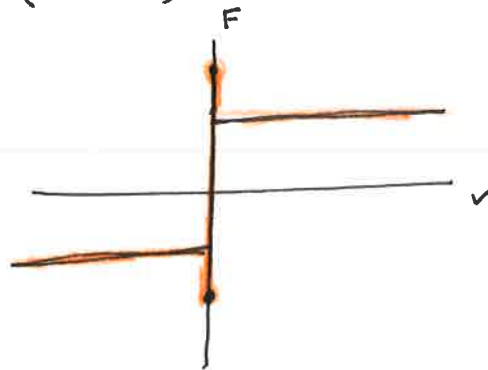
Depending on the last direction of travel, the static position can have multiple positions.

Maintaining a stick position / force on one side ^{at the deadband} is critical for flight tests.

Coulomb Friction with Static Friction (Stiction)



or



pick this one

$$m \ddot{x} + \begin{cases} 0.5 & v=0 \\ 0.1 & \text{otherwise} \end{cases} \text{sign}(v) + kx = 0 \Rightarrow F = \begin{cases} 0.5 & v=0 \\ 0.1 & \text{other} \end{cases} \text{sign}(v) + kx$$

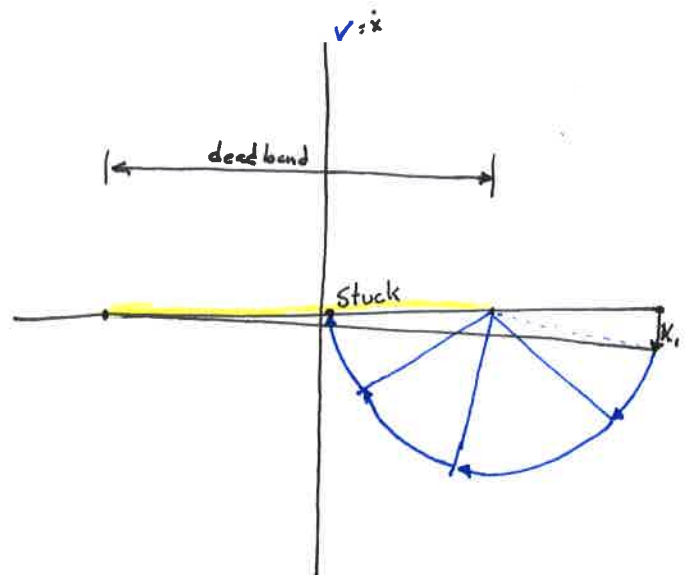
1) $\omega_{ret} = 1 \Rightarrow f(x, \dot{x}) = \begin{cases} 0.5 & v=0 \\ 0.1 & \text{other} \end{cases} \text{sign}(v)$

2) IC (also $\text{sign } v \geq 0$)

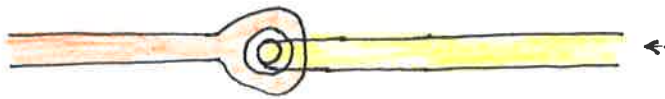
3) $x_c = -\frac{f}{\omega_{ret}^2} = -0.5$

3.1) $x_c = 0.5$

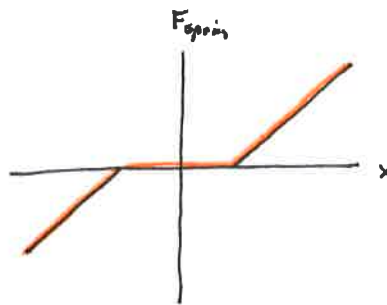
3.2) $x_c = -0.5$



Connections with play



Model with a spring



$$F_{\text{spring}} = \begin{cases} k(x - \epsilon) & x > \epsilon \\ 0 & -\epsilon < x < \epsilon \\ k(x + \epsilon) & x < -\epsilon \end{cases}$$

$$m\ddot{x} + F_{\text{spring}} = 0$$

$$x_c = \begin{cases} \epsilon & x > \epsilon \\ x & -\epsilon < x < \epsilon \\ -\epsilon & x < -\epsilon \end{cases}$$

