AEM 617

Hydraulic Systems

part 1

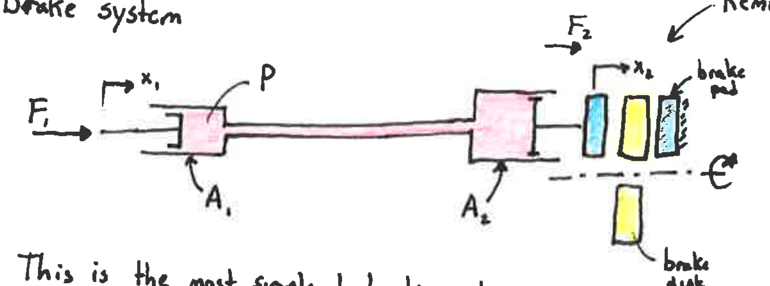
Consider the challenges of FCS (flight Control Systems).

- Large Forces (i.e. $H = C_h \, q \, S \, c$)
- Maintain static position with high precision in presence of large forces
- Rapid control deflections (i.e high power since $F \cdot v = \text{Power}$)
- Widely spaced control surfaces (e.g. ailerons are <u>many</u> feet apart ... same with elevator)
- Control inputs are far from surfaces and need high gain (force ratio)
- Near continuous operation
- Weight constraints
- Redundancy and reliability

A hydraulic system meets these requirements through the use of a pressurized fluid acting on a piston area (or on a motor).

Ex: Brake system

Removes rotational energy through heating pad + disk

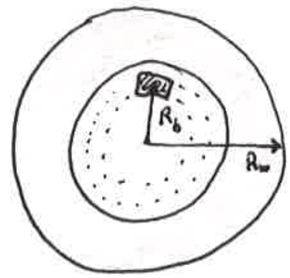


$$F_1 x_1 = F_2 x_2$$

$$F_1 = P \cdot A_1 \qquad F_2 = P \cdot A_2 \qquad \Rightarrow \quad x_1 = \frac{F_2}{F_1} x_2$$

$$x_1 = \frac{A_2}{A_1} x_2$$

This is the most simple hydraulic system. Provides a force gain of A_2/A_1 necessary to give enough force on the brake pads to stop rotation relatively quickly.



Kinetic friction

$$F_f = \mu F_N \cdot N_b$$ # pads

Power loss by brake

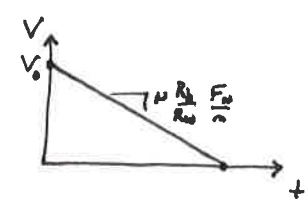
$$P = F_f \cdot V_b = F_f \omega R_b = \mu \omega R_b F_N$$

Energy of aircraft $\equiv E = \frac{1}{2} m V^2 \Rightarrow \frac{d}{dt}(E) = mV\frac{dV}{dt}$

Energy balance $\frac{dE}{dt} + P_{brake} = 0$

or

$$mV\frac{dV}{dt} + \mu \frac{V}{R_w} R_b F_N = 0 \quad \Rightarrow \quad \frac{dV}{dt} + \underbrace{\mu \frac{R_b}{R_w} \frac{F_N}{m}} = 0$$



Kinetic Friction

$\mu \approx 0.3 - 0.4$ for normal brakes

≈ 0.6 for high performance brakes.

Good braking action depends on • brake Kinetic friction

- Ratio of brake to wheel radius (e.g. bicycle brakes)
- Brake normal force

Ex:

Using this brake model, design a brake system for a 2400 lb C-172 to stop the aircraft in 500 ft from 50 mph. Assume the tire is 15 in Ø and the brake pads are 4 inches from the axel.

Given $V = V_0 - kt$ where $k = \mu \frac{R_b}{R_w} \frac{F_N}{m} \cdot N_b$

$$V = \frac{ds}{dt} \Rightarrow \frac{ds}{dt} = V_0 - kt \Rightarrow ds = (V_0 - kt)\,dt \Rightarrow s = V_0 t - \frac{kt^2}{2}$$

$$S = V_0 t - \frac{kt^2}{2} \quad \text{and} \quad t_0 = \frac{V_0}{k} \Rightarrow S = \frac{V_0^2}{k} - \frac{k V_0^2}{2k^2} = \frac{V_0^2}{k} - \frac{1}{2}\frac{V_0^2}{k}$$

$$S = \frac{1}{2}\frac{V_0^2}{k} = \frac{1}{2} V_0^2 \frac{R_w}{R_b} \frac{m}{F_N} \frac{1}{\mu} \frac{1}{N_b}$$

Mass:

2400 lb ⇒ 75 slugs

Vel:

50 mph ⇒ 73 ft/s

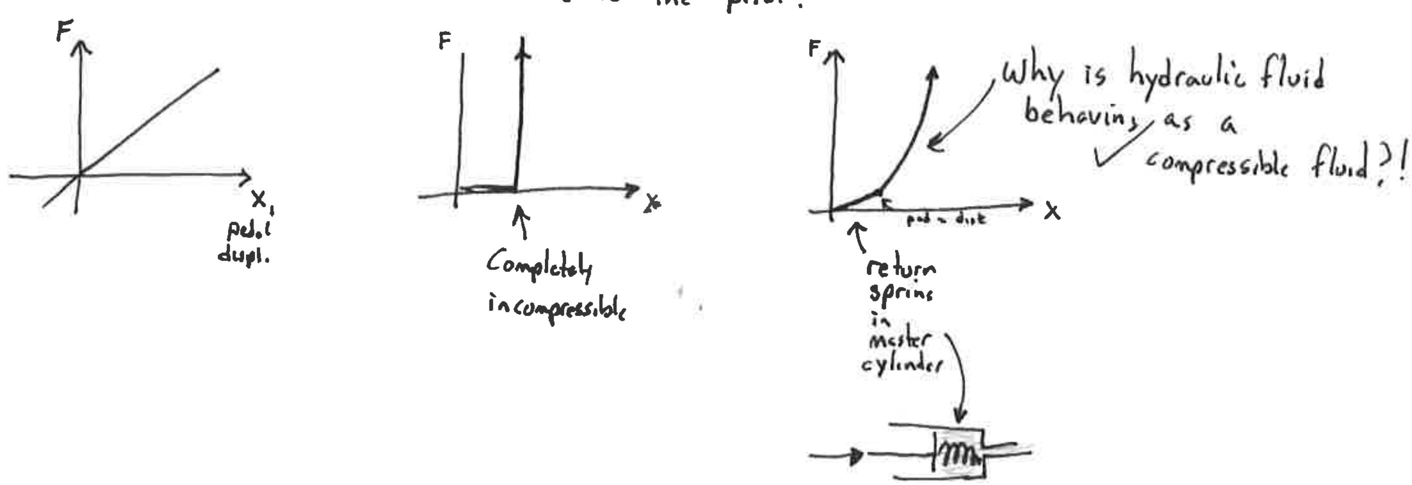
Solution:

$$500\,ft = \frac{1}{2} \cdot \frac{73^2\,\cancel{ft^2}}{\cancel{s^2}} \cdot \frac{7.5\,\cancel{in}}{4\,\cancel{in}} \cdot 75\,\cancel{slugs} \cdot \frac{1}{F_N} \cdot \frac{1}{0.4} \cdot \frac{1}{4} \cdot \frac{\cancel{lbf}\,\cancel{s^2}}{\cancel{slug}\,\cancel{ft}}$$

$$F_N = \frac{1}{2} \cdot 73^2 \cdot \frac{7.5}{4.0} \cdot 75 \cdot \frac{1}{0.4} \cdot \frac{1}{4} \cdot \frac{1}{500} = 468\ lbf$$

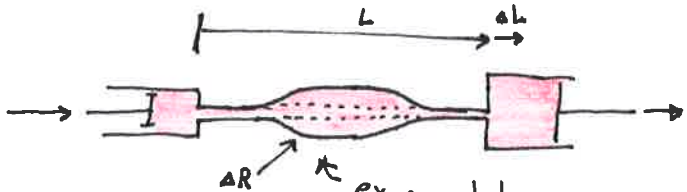
If the pilot can provide 50 lbf to the brake master cylinder, the hydraulic area ratio ≈ 1:10.

Q: What does the brake force feel like to the pilot?



Hydraulic fluid is only <u>very</u> slightly compressible.

However, the large pressures generated can cause the <u>system</u> to show compressibility because the control volume is not fixed.



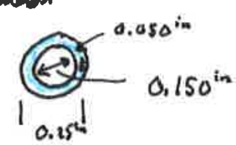
exaggerated expansion of brake lines.

or

Slight lengthening of hydraulic tubes under pressure

Ex: Nylaflow Tubing (Nylon tubing) 10ft

$\frac{1}{4}^{in}$ OD and 0.050^{in} walls



$E \approx 175000$ psi

Hoop stress: (assume thin wall)

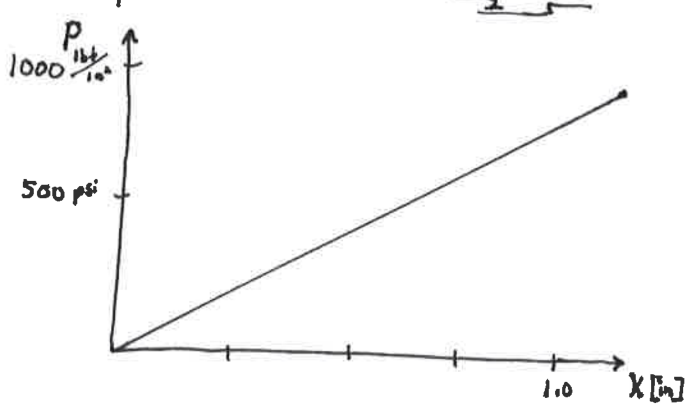
$$\sigma_\theta = \frac{pr}{t} \Rightarrow \epsilon = \frac{\sigma}{E} = \frac{p \cdot r}{tE} \Rightarrow \Delta r = \epsilon r = \frac{p r^2}{tE} \Rightarrow \hat{r} = r + \Delta r$$

Wall stress:

$$\sigma_x R 2\pi t = p \cdot A \Rightarrow \sigma_x = p \pi r^2 \cdot \frac{1}{2\pi r t} = \frac{pr}{2t}$$ half hoop....

$$\epsilon = \frac{\sigma}{E} = \frac{p \cdot r}{2tE} \Rightarrow \Delta L = \epsilon L = \frac{p r L}{2tE} \Rightarrow \hat{L} = L + \Delta L$$

Given a master cylinder area of $\approx 0.2 \text{ in}^2$ (as expected from prev example), the p vs x curve from tubing expansion is



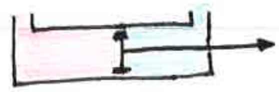
"Soft" Brakes.

1 inch of cylinder travel at 1000 psi with 10 ft of $\frac{1}{4}^{in}$ nylon tubing.

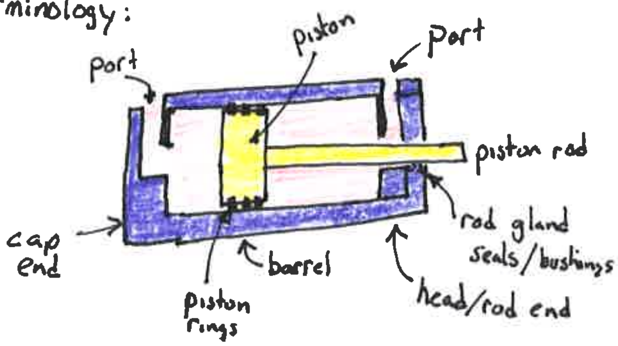
Hydraulic Cylinders

Ram: / Single Acting

Double Acting:

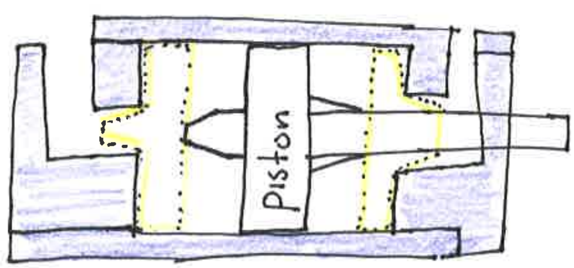


Terminology:

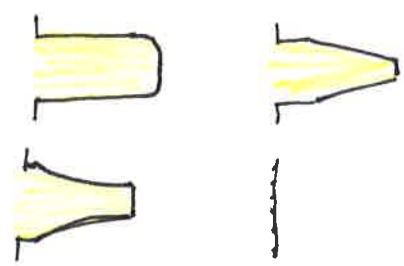


Cushions:

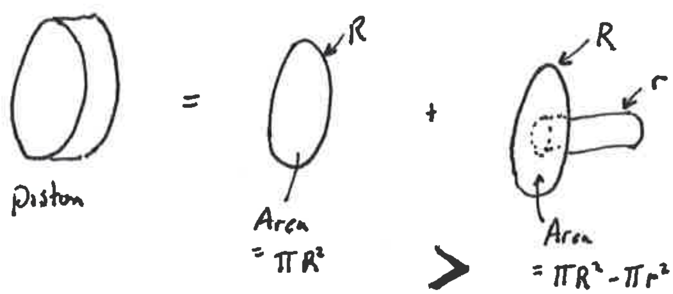
At high flow rates, the piston can slam into the cap end or head end. A tapered spear is added to the piston to reduce the flowrate when the piston approaches the ends.



Cushion spears: Shapes



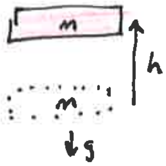
Biased Forces



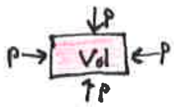
The force in the head direction can be slightly/moderately less for standard rod types (i.e. non-double rods)

Bernoulli's Equ.

Kinetic energy $\equiv K = \frac{1}{2} m V^2$



Potential Energy $\equiv PE = mgh$



Pressure energy $\equiv P_e = p \cdot Vol$

Sum energy:

$$K + PE + P_e = E \cdot Vol$$

$$\frac{1}{2} m V^2 + mgh + p \cdot Vol = E \cdot Vol$$

For an arbitrary volume (i.e. divide by Vol)

$$\frac{1}{2} \frac{m}{Vol} V^2 + \frac{m}{Vol} gh + p = E$$ where $\frac{m}{Vol}$ is density $\equiv \rho$

Bernoulli

$$\boxed{\frac{1}{2} \rho V^2 + \rho g h + p = E}$$

For any two pts, E is constant. (assume no losses)

$$\frac{1}{2} \rho (V_1^2 - V_2^2) + \rho g (h_1 - h_2) + (P_1 - P_2) = 0$$

For our application, $\Delta h = h_1 - h_2$ is usually small,

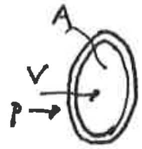
$$\frac{1}{2} \rho (V_1^2 - V_2^2) + (P_1 - P_2) = 0$$

<u>As flow velocity in a hydraulic system increases, pressure decreases</u>

Power

Why is a hydraulic system so effective at power transmission?

$$P_{ower} = F \cdot V$$
velocity



Force at any point / cross section is $F = PA$

$$= \underbrace{P}_{\text{pressure}}\,\underbrace{AV}_{\text{Volumetric flow rate, } \equiv \dot{Q}}$$

Ex: How many hp and kW is flow at $p^{[psi]}$ and $\dot{Q}^{\left[\frac{ft^3}{min}\right]}$?

$$P_{flow}^{[hp]} = p\dot{Q} = \frac{p\ \text{psi}}{} \cdot \frac{Q\ \text{ft}^3}{\text{min}} \cdot \frac{\text{lbf}}{\text{lbf in}^2} \cdot \frac{\text{hp}\ \text{s}}{550\ \text{ft lbf}} \cdot \frac{\text{min}}{60\ \text{s}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} = \frac{p^{[psi]}\,\dot{Q}^{\left[\frac{ft^3}{min}\right]}}{229.2}$$

$$p^{[hp]} = \frac{p^{[psi]}\,\dot{Q}^{\left[\frac{gal}{min}\right]}}{1714}$$

$$p^{[kW]} = \frac{p^{[bar]}\,\dot{Q}^{\left[\frac{L}{min}\right]}}{600}$$

Ex: Hydraulic power at 36 gpm at 3500 psi?

$$p^{[hp]} = \frac{3500^{\ psi} \cdot 36^{\ gpm}}{1714} = 73^{\ hp}$$ ideal ∴ 100% efficiency

Given a $\frac{1}{2}$ in ID, what is the fluid velocity?

$$\dot{Q} = Av \Rightarrow v = \frac{\dot{Q}}{A} = \frac{36\ \text{gal}}{\text{min}} \cdot \frac{4}{\pi\, 0.5^2\ \text{in}^2} \cdot \frac{231\ \text{in}^3}{\text{gal}} \cdot \frac{\text{min}}{60\ \text{s}} \cdot \frac{\text{ft}}{12\ \text{in}}$$

$$= 58\ \frac{ft}{s}$$

Hydraulic Pumps

Hydraulic systems in aircraft are typically relatively high pressure ($\approx$ 3000 psi is common).

Thus, positive displacement pumps are the norm.

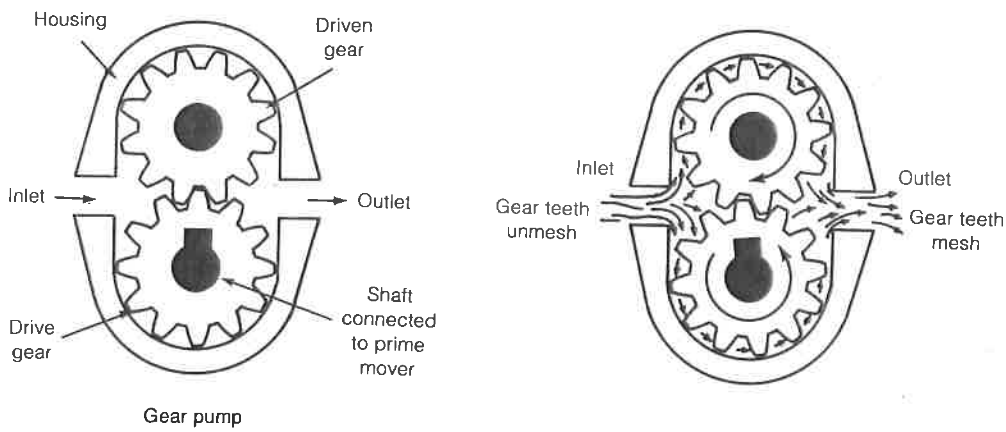


Figure 16.10.1 Gear pump schematic. *Source: Parker-Hannifin Corp., Cleveland, OH.*

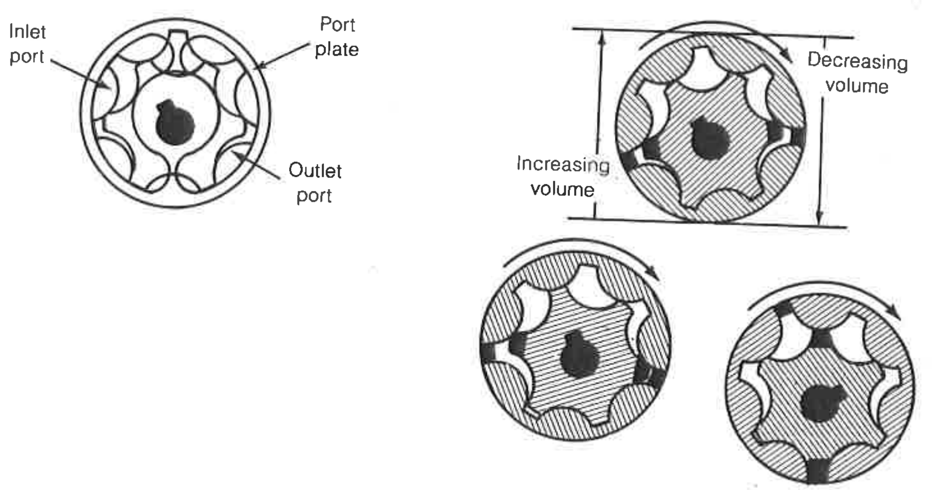


Figure 16.10.2 Schematic of the elements of a gerotor pump. *Source: Parker-Hannifin Corp., Cleveland, OH.*

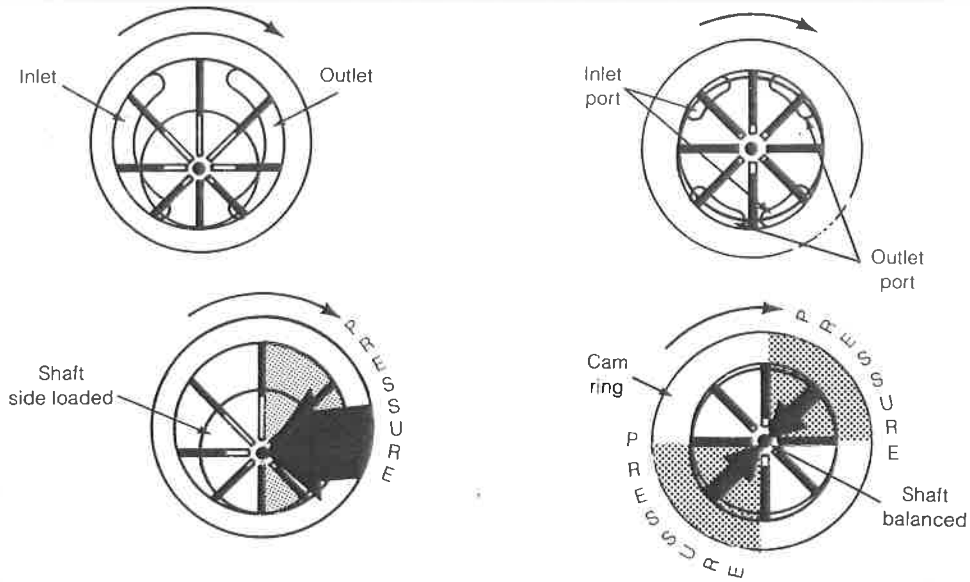


Figure 16.10.4 Cross-sectional schematic of an unbalanced vane pump (*left*), and of a balanced vane pump (*right*). *Source: Parker-Hannifin Corp., Cleveland, OH.*

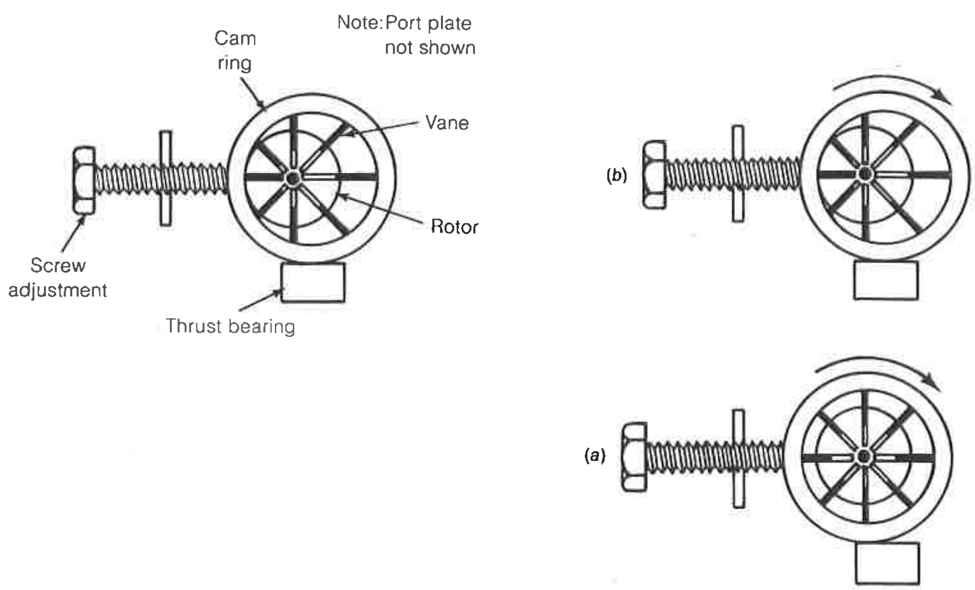


Figure 16.10.5 Cross-sectional schematic of a variable vane pump. *Source: Parker-Hannifin Corp., Cleveland, OH.*

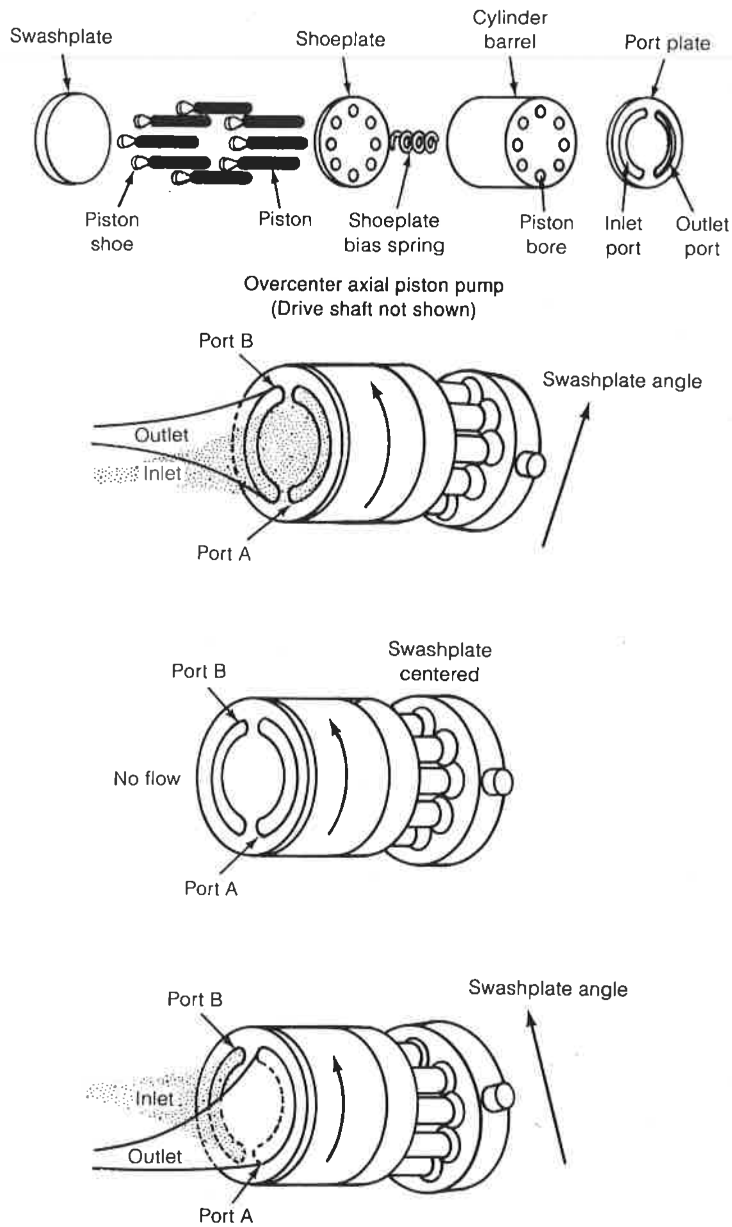


Figure 16.10.6 Exploded view of the elements of an axial piston pump (top) and assembled (bottom) to show operation of the pistons as the cylinder barrel rotates. *Source: Parker-Hannifin Corp., Cleveland, OH.*

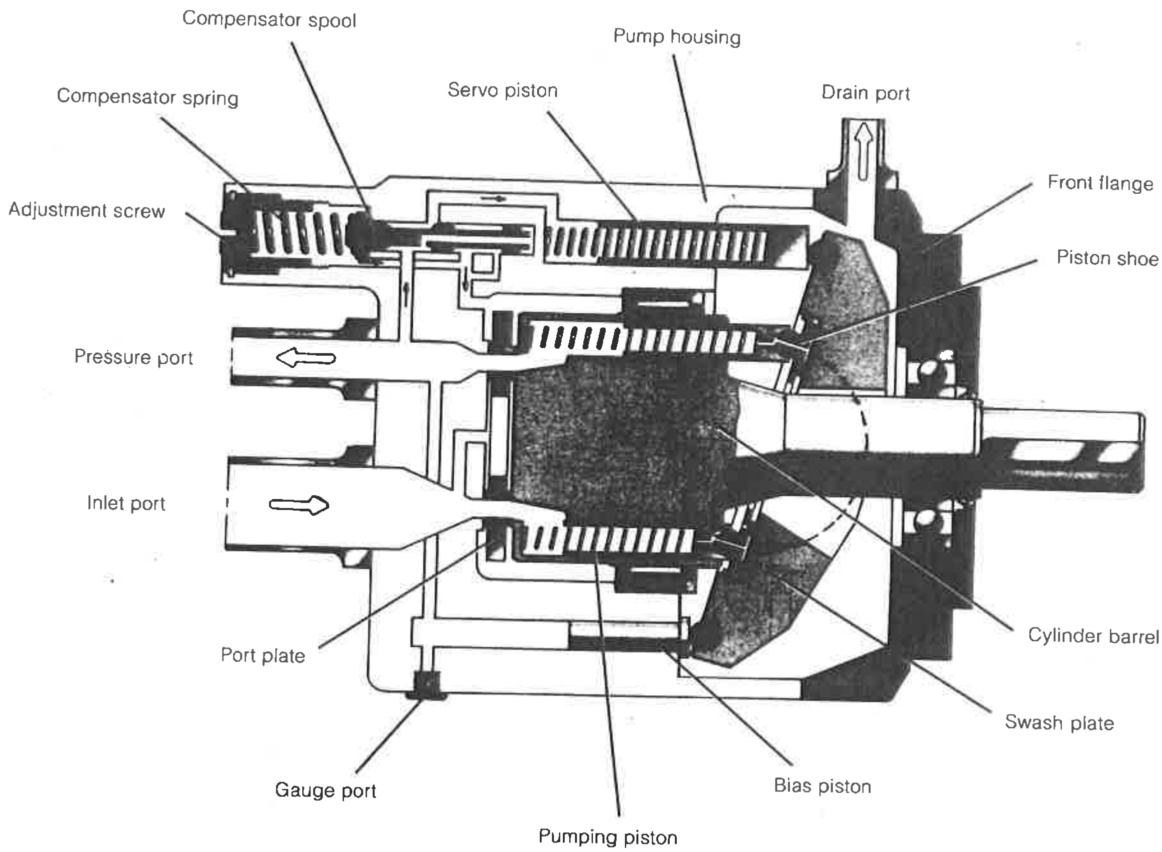


Figure 16.10.7 Cross-sectional drawing of an axial piston pump with compensation. The servo piston controls the angle of the swash plate. *Source: Parker-Hannifin Corp., Cleveland, OH.*

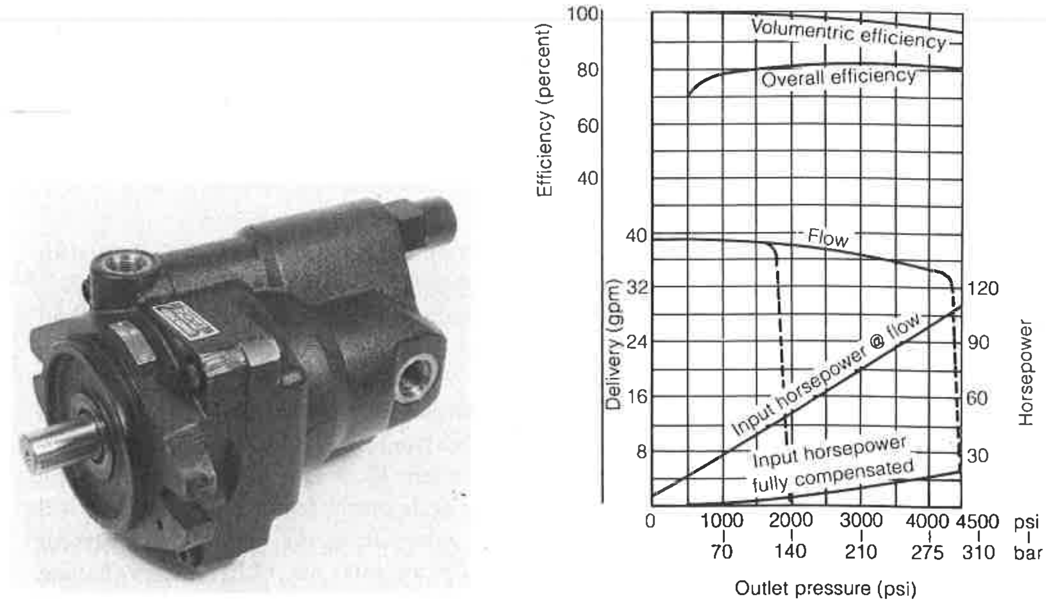


Figure 16.21.2 Performance curves for an axial piston, variable volume, compensated hydraulic pump. *Source: Parker-Hannifin Corp., Cleveland, OH.*