AEM 617

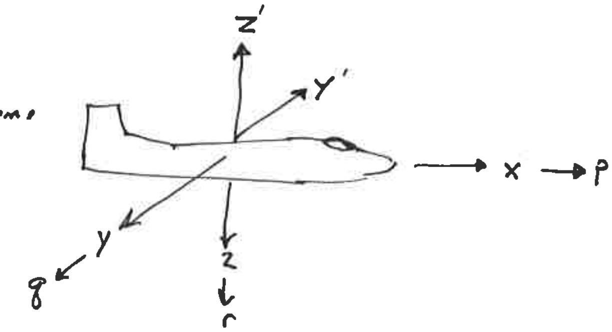
Inertial Navigation

part I.

Inertial Navigation System (INS)

Reference Frames

Body Frame

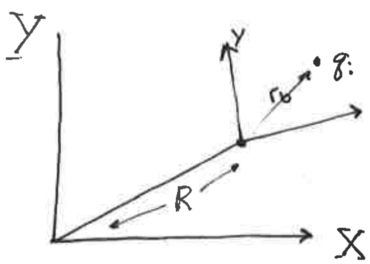


(Non-inertial frame)

Body fixed accelerations + rotations (Strapdown)

x, y, z   p, q, r

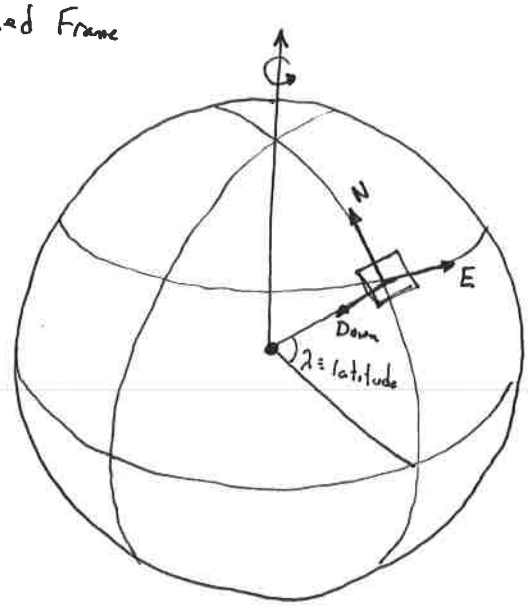
Inertial Frame



$$q_i = R_i + B r_b$$

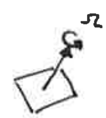
Orientation
- Euler Angles
- Direction Cosines
- Quaternions

Earth Fixed Frame



Earth rotates at 15°/hr

$\Omega_{vertical} = 15 \sin \lambda \left[\frac{deg}{h}\right]$



$\Omega_{horizontal} = 15 \cos \lambda$

- East direction has zero Earth rotation rate. You can find North by finding East with a rate gyro! (Actually easier to find zero rate than maximum rate.)

Realistic Earth Model

The Earth is <u>not</u> an isotropic sphere.

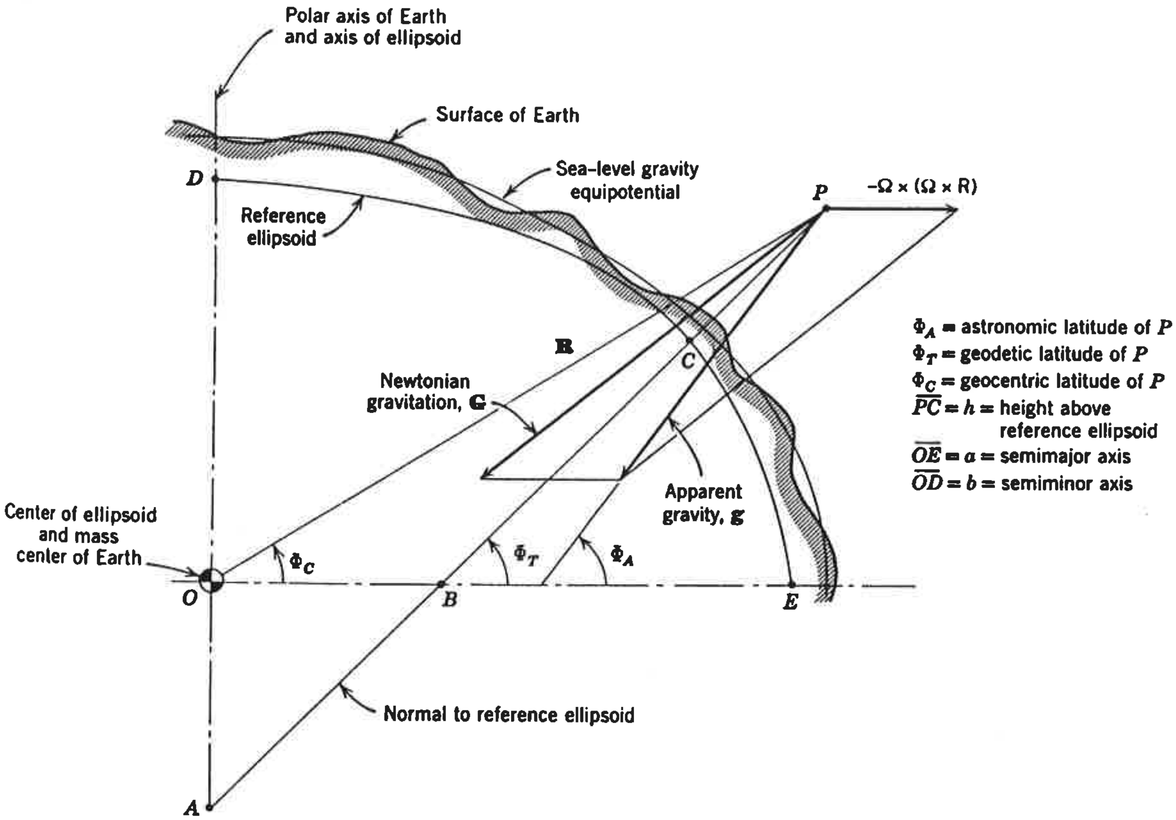
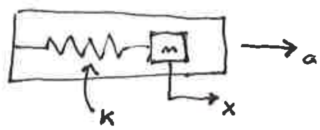


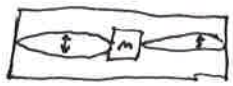
Figure 2.2 Meridian section of the Earth, showing the reference ellipsoid and gravity field.

Sensors

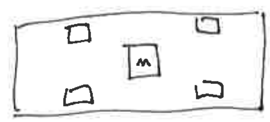
Acceleration (linear)



$$F = ma = kx \Rightarrow a_x = \frac{k}{m} x$$



Vibrating string



~~Rate Gyro~~

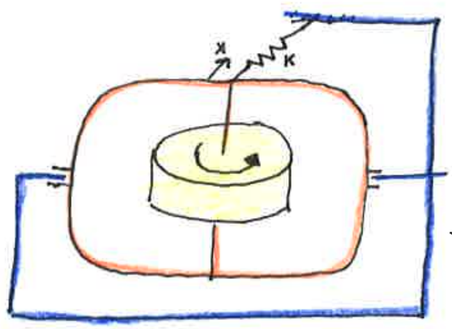
Angular Rate

Gyro: Conservation of angular momentum

$\frac{dL}{dt} = \Sigma T$ (change in angular momentum wrt time is proportional to the applied torque)

The rotation axis remains in an inertial frame direction.

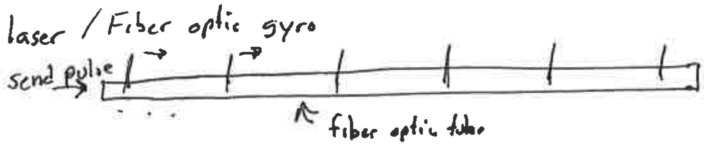
Rate Gyro



$\xrightarrow{\Omega}$ Measure rate in this direction

Applying a rate creates a reaction torque on the gyro which is transmitted to the spring. Measuring the deflection allows for knowing Ω.

Ring laser / Fiber optic gyro



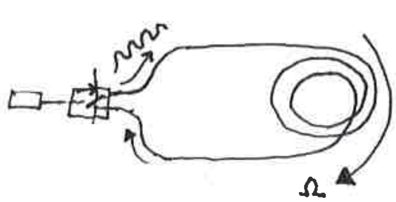
finite time to reach the other side

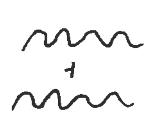
$t = \frac{L}{c}$

If the tube is moving, the pulse travels slightly longer

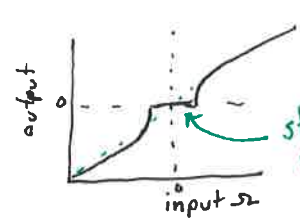
$L = v \cdot t$.

Measure the interference to obtain length difference



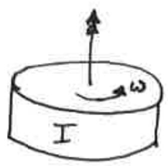


combine constructively or destructively depending on Ω rate

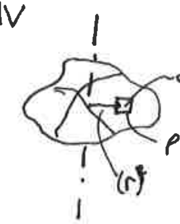


stuck near low Ω.
Solution? Rotate the whole part!

Rate Gyro Analysis



Angular momentum $= L = I\omega$ with $I = \int_V r^2 \rho \, dV$



For a wheel (solid),

$$I = \int_0^h \int_0^{2\pi} \int_0^R (r^2 \rho) \, r\,dr\,d\theta\,dz = h\,2\pi \int_0^R r^3 \rho \, dr$$

$$= h\,2\pi \frac{R^4}{4} \rho \qquad \text{clumbersome} \ldots$$

Mass $= \rho \cdot$ Volume $= \rho \pi R^2 h$

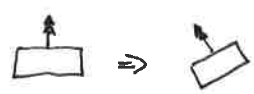
Thus

$$I = \frac{\cancel{h} \cancel{2\pi} \cancel{R^4}^{R^2} \cancel{\rho}}{\cancel{4}_2} \cdot \underbrace{\frac{m}{\cancel{\rho} \cancel{\pi} \cancel{R^2} \cancel{h}}}_{\text{mass, density}} = \frac{m}{2} R^2$$

- L at 20000 rpm for 3 inch disk of 1 lb $\rightarrow$ $L = \frac{1\,\cancel{lbf}\ \text{slug}}{32.174\,\cancel{lbf}} \cdot \frac{1}{2} \cdot 3^2\,\cancel{in^2} \cdot \frac{20000\,\cancel{rev}}{\cancel{min}} \cdot \frac{2\pi\,rad}{\cancel{rev}} \cdot \frac{\cancel{min}}{60\,s} \cdot \frac{ft^2}{144\,in^2}$

$$= 2\ \frac{\text{slug-ft}^2}{s}$$

- In a polar frame, the angular momentum is $L = (L_0 \hat{r}, \theta_0 \hat{\theta})$
- Slightly tilting the wheel/gyro

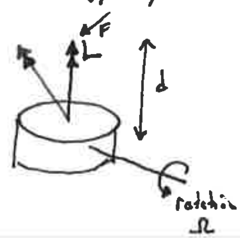


gives

or $dL = (0\hat{r}, L_0 d\theta \hat{\theta})$

or $\frac{dL}{dt} = (0, L_0 \frac{d\theta}{dt} \hat{\theta})$ but $\frac{dL}{dt}$ = applied torque vector

- For the rate gyro,



but the gimbal is connected with a spring. $F = Kx$ for an applied torque of $d \times F = dF = dKx$

Combine $\frac{dL}{dt} = \Sigma T \Rightarrow \frac{dL}{dt} = L_0 \overbrace{\frac{d\theta}{dt}}^{\text{roll rate } \Omega} = dKx \Rightarrow$ Solve for x.

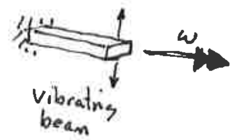
$$\boxed{x = \frac{L_0 \Omega}{Kd}}$$

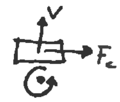
Measure the deflection of a gyro to measure the rate

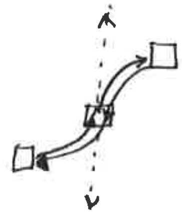
MEMS Devices

- Coriolis

In a rotating frame, the coriolis force is $F = -2m(\omega \times v)$







Measure the off axis vibration to calculate the rotation rate.

Mounted in a <u>small</u> chip: cheap, small, light, low power, not high accuracy.

	GG1320AN (Laser Gyro)	GG5300 (MEMS 3xGyro)
Size	88 mm × 88 mm × 45 mm	50 mm × 50 mm × 30 mm
Weight	454 g	136 g
Start-Up Time	< 4 s	< 1 s
Power	15 Vdc, 1.6 watts nominal 5 Vdc, 0.375 watts nominal	5 Vdc, < 800 mA
Operating Temperature Range	−54 °C to 85 °C	−45 °C to 85 °C
Angular Random Walk	$0.0035°/\sqrt{h}$	$0.2°/\sqrt{h}$
Bias Stability	0.0035°/h	$< 70°$/h

Table 1: Specifications for the Honeywell GG1320AN and GG5300 gyroscopes.

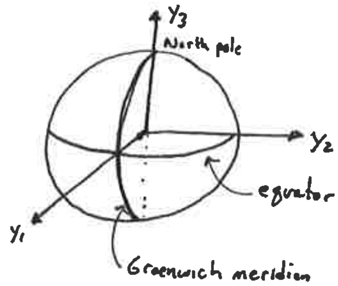
Source:
An Intro' to inertial Navigation
— Oliver Woodman
TR 696 Cambridge

Honeywell GG 1320 AN $$$$

MEMS Gyro $10

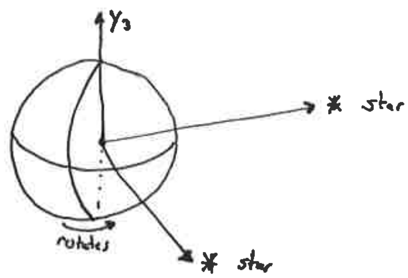
Coordinate Frames

- Earth-centered Earth fixed (ECEF)



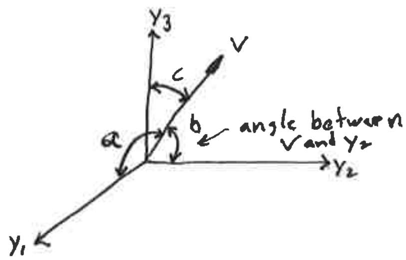
Coordinate system rotates with Earth

- Earth Centered Inertial



Newtons laws valid in this frame.
By definition, good for celestial navigation

- Direction Cosines



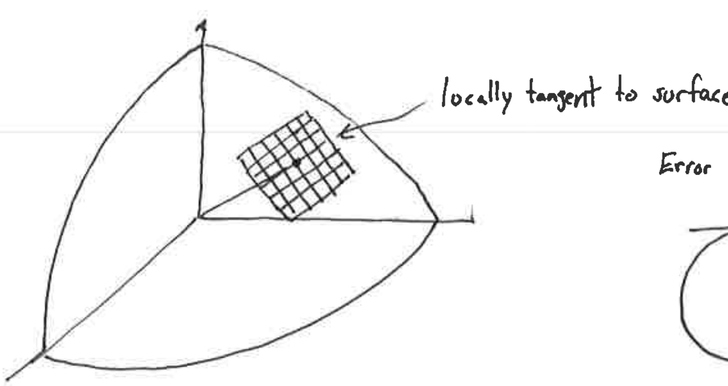
$$V = V_{y_1}\hat{e}_{y_1} + V_{y_2}\hat{e}_{y_2} + V_{y_3}\hat{e}_{y_3}$$

$$\alpha = \cos a = \frac{V \cdot e_{y_1}}{|V|} = \frac{V_{y_1}}{(V_{y_1}^2 + V_{y_2}^2 + V_{y_3}^2)}$$

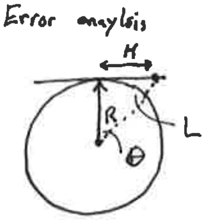
$$\beta = \cos b = \frac{V \cdot e_{y_2}}{|V|} = \cdots$$

$$\underbrace{\gamma}_{\text{direction cosines}} = \underbrace{\cos c}_{\text{direction angles}} = \frac{V \cdot e_{y_3}}{V} = \cdots$$

- Tangent plane



Error analysis



Radius $\approx$ 3960 mi $\approx$ 4000 mi

Along the circle: $L = R\theta$

Along the plane: $\tan\theta = \frac{H}{R}$

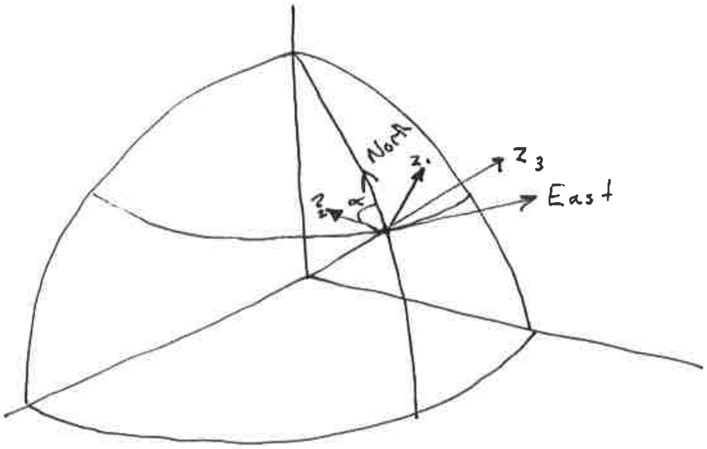
Solve $\tan\frac{L}{R} = \frac{H}{R}$

$$\tan\frac{L}{R} \approx \frac{L}{R} + \frac{1}{3}\left(\frac{L}{R}\right)^3 + \frac{2}{15}\left(\frac{L}{R}\right)^5 + \cdots$$

$$\text{Error} = \tan\frac{L}{R} - \frac{H}{R}$$

$\approx$ 1 mile error at 100 miles

- Geodetic Wander Azimuth



Z_1 and Z_2 in tangent plane

Z_3 upward

Z_2 angled α from north.

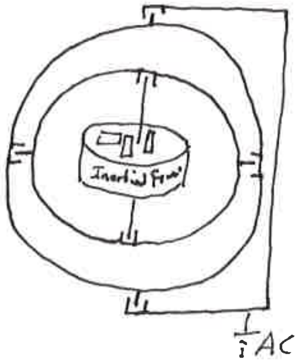
- Others

...

Two types of INS platforms.

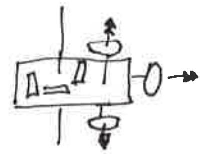
1) Inertial Gyroscopic Platform

- Passive

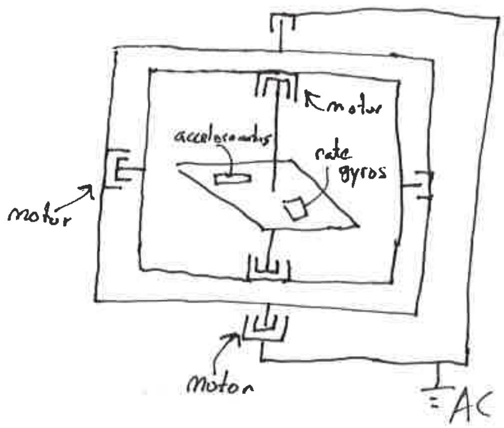


Issues?
Drift, Power, Bearing stiction.

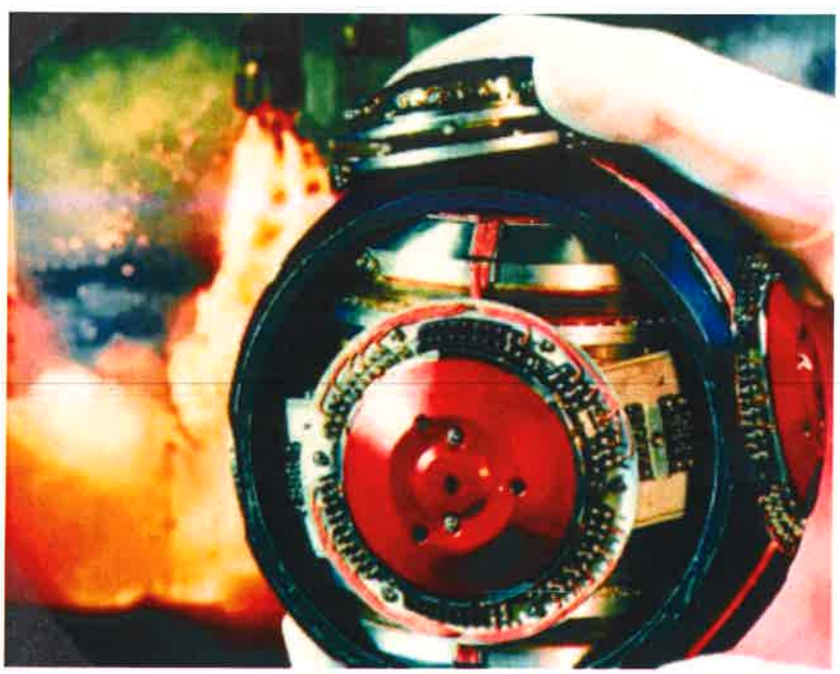
Alternative platform with gyros



- Active

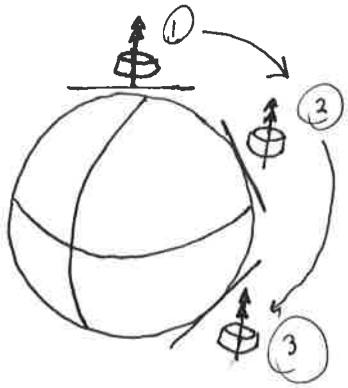


- Gyros are coupled with motors on the gimbals to create a feedback mechanism to ensure the platform remains in an inertial frame.
- Gyros are "null seeking" to drive error to zero. "Integrating gyros" means they track the integral of ω.

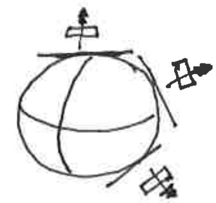


Source:
Inertial Navigation – Forty Years of Evolution A.D. King GEC Review Vol 13 No. 3 1998

An inertial platform maintains an orientation with respect to the stars (i.e. an inertial frame) So navigation on a planet will vary the relative orientation of the platform with respect to the planet surface.



To reduce complications with a non normal accelerometer measurement, we would prefer the gyro to remain normal to the surface.



Schuler Tuning

What is the theoretical orbital period of a zero-altitude object?

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{3960\,\text{mi}\cdot\frac{s^2}{32.174\,\text{ft}}\cdot\frac{5280\,\text{ft}}{\text{mi}}} = 5065\ \text{s}$$

$$\approx 84.4\ \text{min}$$

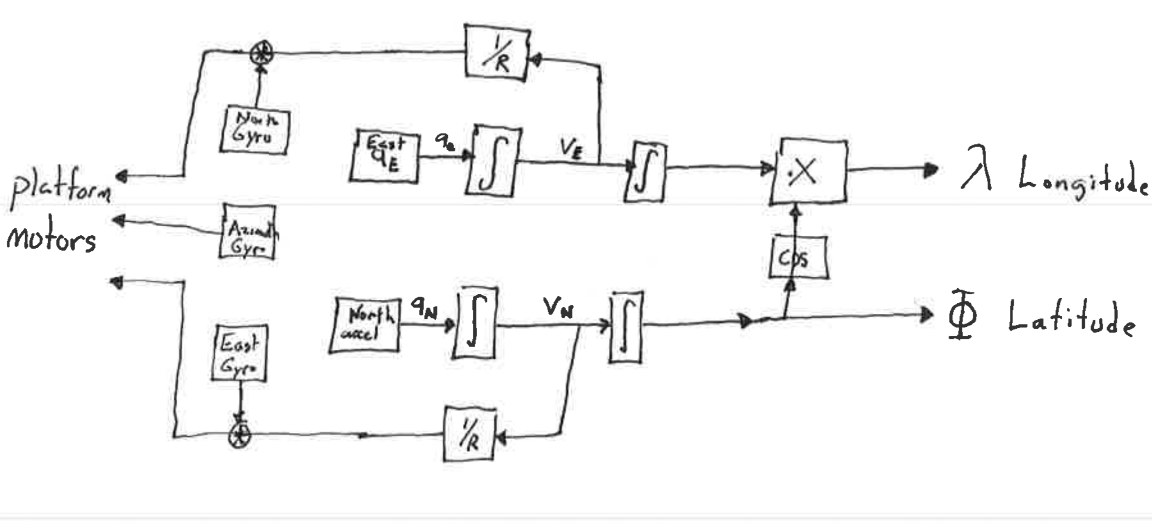
This is also the period of a pendulum of length R!!

- Solution! Add a pendulum to the platform of length 3960 miles!! The platform will always point up now!
- Practical solution: Add a feedback on velocity to the platform rotation rates.

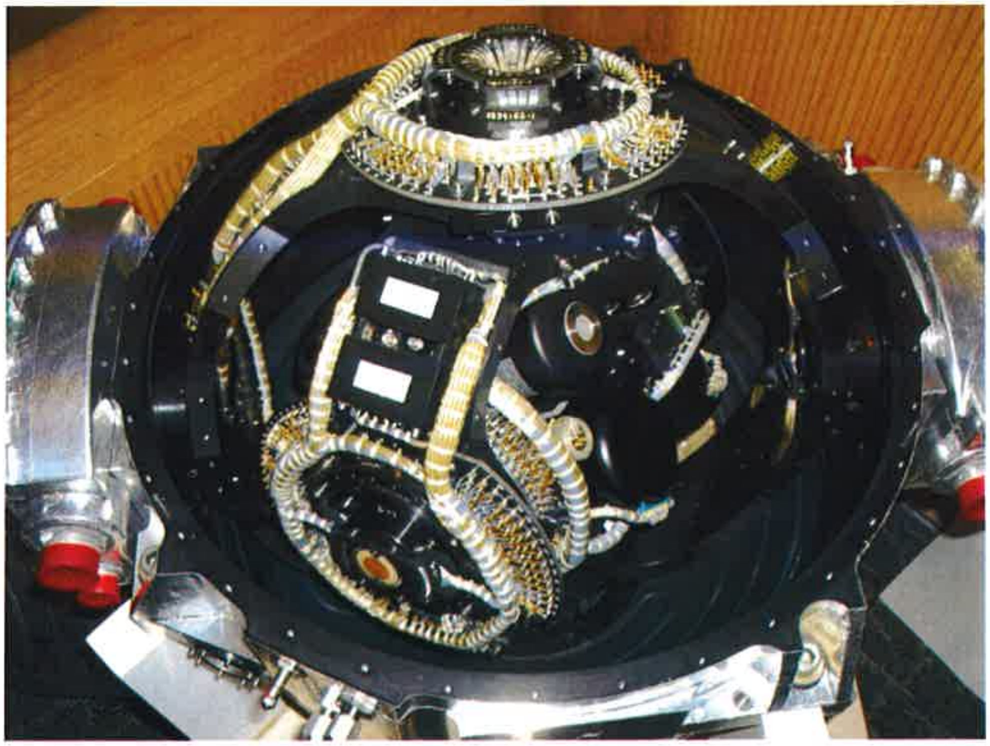
$$\dot{\theta} = \frac{V}{R}$$ added to platform.

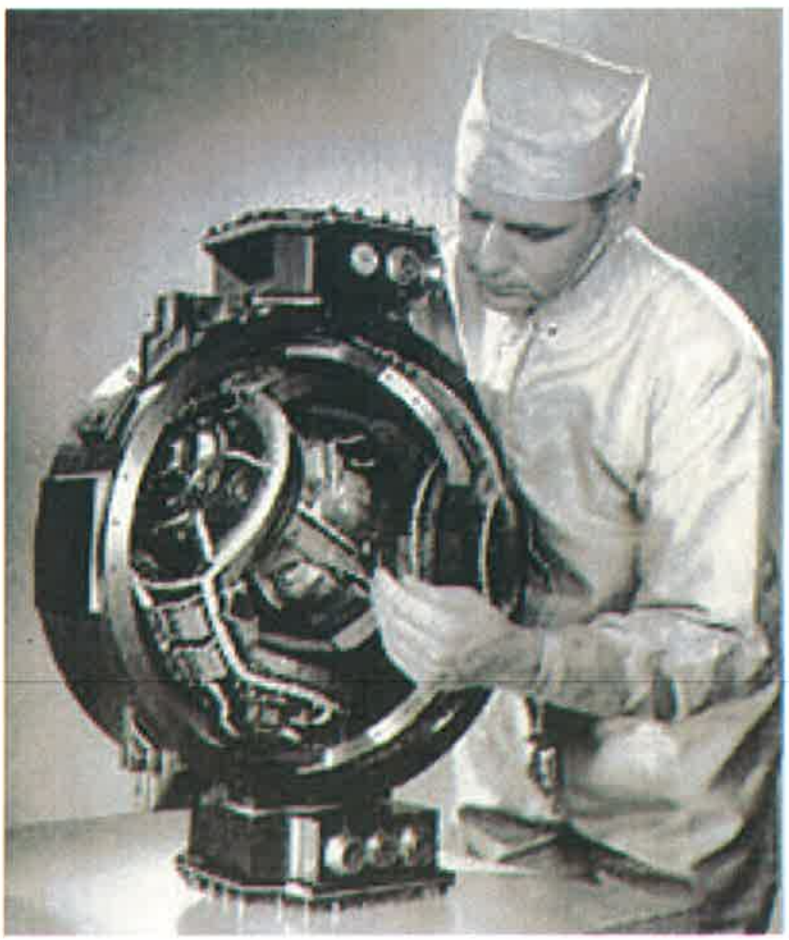
R ← radius of the planet!!

INS block diagram (Inertial platform)



Saturn V INS





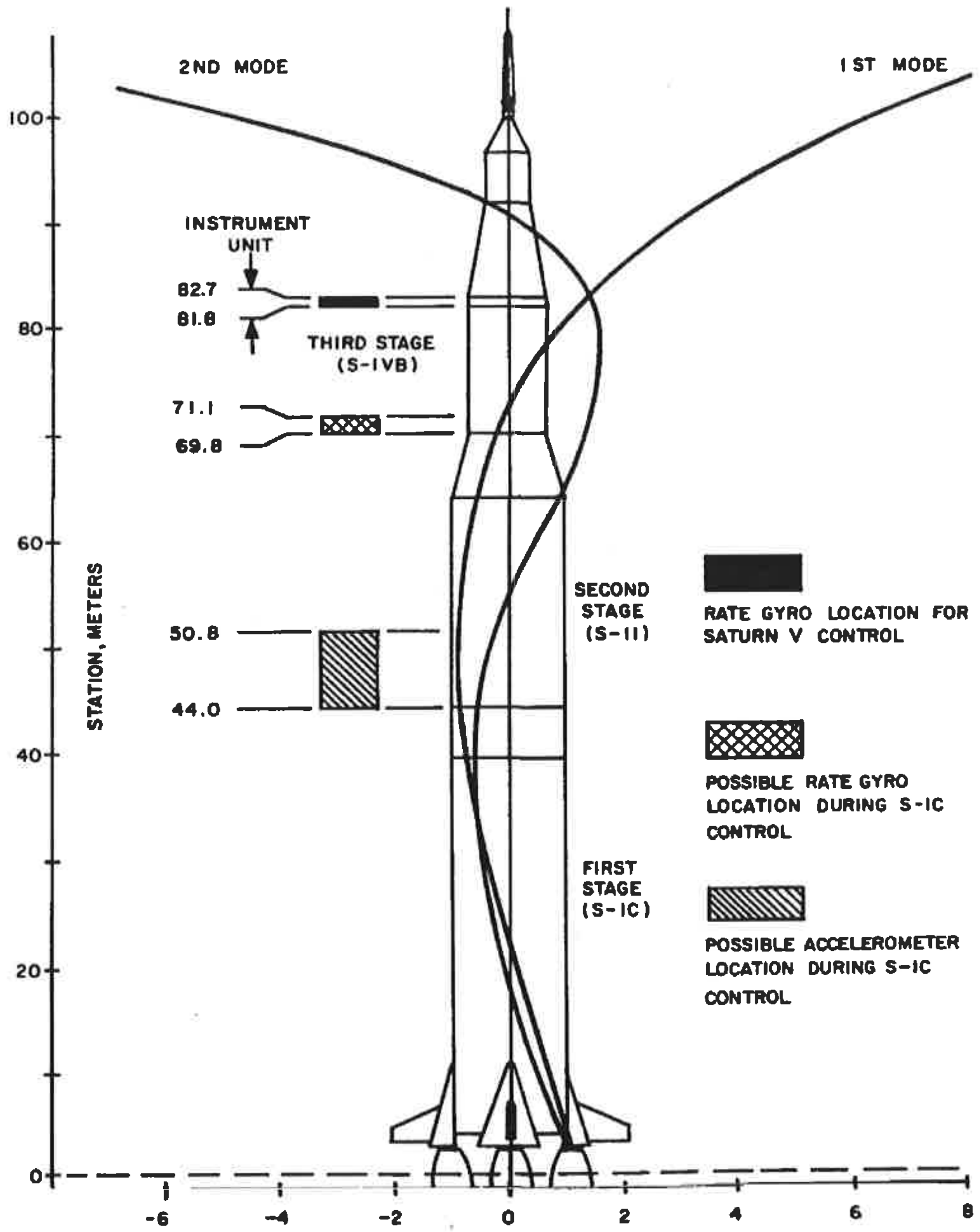


FIGURE 8. SHAPE OF THE FIRST AND SECOND BENDING MODES.

Marconi FIN1330