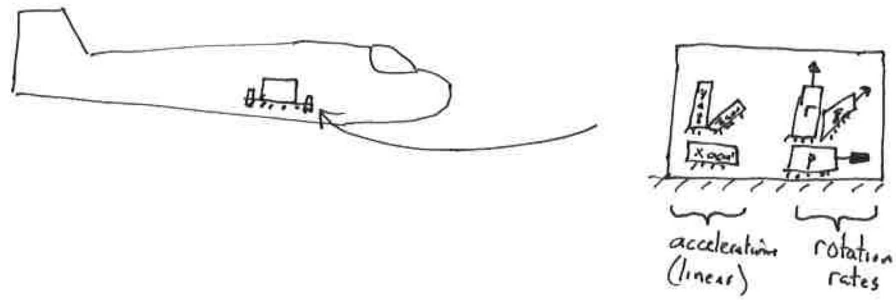
AEM 617

Strapdown INS

2) Strapdown INS

In contrast with the inertial platform, the strapdown INS system is fixed to the body and not to an inertial frame.



Process:

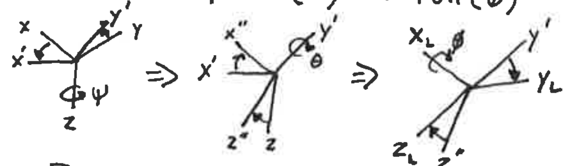
$$\underbrace{\begin{matrix} a_x & p \\ a_y & q \\ a_z & r \end{matrix}}_{\text{Body Frame}} \longrightarrow \underbrace{\text{Orientation Angles}}_{\text{Navigation Frame}} \longrightarrow \underbrace{\phi,\ \lambda,\ h}_{\text{Earth Frame}}$$

Aircraft Orientation

- Euler Angles: An ordered set of rotations

"Navigation"
Global/fixed $\longrightarrow$ local/body

Yaw (ψ) $\rightarrow$ pitch (θ) $\rightarrow$ roll (ϕ)



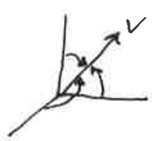
$$\left(\vec{X}'\right) = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\psi}}_{T_\psi^{'g}} \left(\vec{X}_G\right) \quad \text{and} \quad \underbrace{\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}_{\theta}}_{T_\theta^{'g}} \quad \text{and} \quad \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}_{\phi}}_{T_\phi^{'g}}$$

Combine

$$\vec{X}_G = T^{'g} X_L = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} X_L$$

- Can not distiguish between roll and yaw at $\theta = 90°$
- Nice for humans!

- Direction Cosines



See prev lecture.

$$\vec{X}_G = C_B^N X_L \quad \text{and} \quad \dot{C}_B^N = [C_B^N][\omega_B]$$

Non intuitive and becomes non-orthogonal

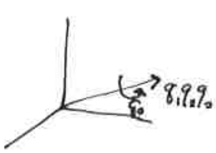
update in time

body frame rotation matrix

$$\omega_B = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

- Quaternions

$q =$ Vector + rotation
$\quad q_1 q_2 q_3 \quad q_0$ with $\|q\| = 1 = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}$



$$\vec{X}_G = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \vec{X}_L$$

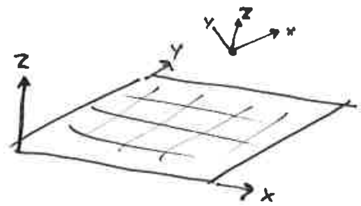
$$\dot{q} = -\frac{1}{2} \underbrace{\begin{bmatrix} 0 & p & q & r \\ -p & 0 & -r & q \\ -q & r & 0 & -p \\ -r & -q & p & 0 \end{bmatrix}}_{\text{body frame rotation rates}} q$$

update in time

Source: Aircraft Flight Dynamics with a non-inertial CFD Code, 2005-0230 O'Neill and Arena

Flat Earth INS with quaternions

- Accelerometers in body frame $a_x \; a_y \; a_z$
- Rotation Rates in body frame $p \; q \; r$



Develop an INS computer:

- Velocities in body frame

$$\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} = \frac{d}{dt}\begin{pmatrix}u\\ v\\ w\end{pmatrix} = \begin{pmatrix}a_x\\ a_y\\ a_z\end{pmatrix} + \overbrace{\underbrace{\left[B^{-1}\right]}_{\text{transform to local body frame}}\begin{pmatrix}0\\ 0\\ g\end{pmatrix}}^{\text{Remove gravity}}$$

$g \approx 32.174 \; ft/s^2$ positive [illegible]

- Position

$$\begin{pmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{pmatrix} = \left[B\right]\begin{pmatrix}u\\ v\\ w\end{pmatrix}$$

- Orientation

$$\begin{pmatrix}\dot{q}_0\\ \dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{pmatrix} = \frac{d}{dt}q = -\frac{1}{2}\left[\Omega_q\right]\begin{pmatrix}q_0\\ q_1\\ q_2\\ q_3\end{pmatrix} = -\frac{1}{2}\begin{bmatrix}0 & p & q & r\\ -p & 0 & -r & q\\ -q & r & 0 & -p\\ -r & -q & p & 0\end{bmatrix} q$$

Complete set of states: $s = (X, Y, Z, u, v, w, q_0, q_1, q_2, q_3)$

10 states

$$(\dot{s}) = \underbrace{\begin{bmatrix}0 & \left[B\right]_{3\times3} & 0\\ 0 & 0 & 0\\ 0 & 0 & \left[-\frac{1}{2}\Omega_q\right]_{4\times4}\end{bmatrix}}_{\text{Non linear}}(s) + \begin{bmatrix}0\\ B^{-1}\\ 0\end{bmatrix}\begin{pmatrix}0\\ 0\\ g\end{pmatrix} + \begin{bmatrix}0\\ \begin{pmatrix}a_x\\ a_y\\ a_z\end{pmatrix}\\ 0\end{bmatrix}$$

Numerical solution

Adams-Moulton 4th order

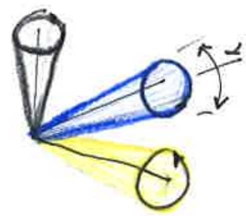
$$S(t+\Delta t) = S(t) + \frac{\Delta t}{24}\left(55\cdot\dot{S}(t) - 59\,\dot{S}(t-\Delta t) + 37\,\dot{S}(t-2\Delta t) - 9\,\dot{S}(t-3\Delta t)\right)$$

Renormalize q at every step. $q^{new} = \dfrac{q^{old}}{|q^{old}|}$

$\Delta t \approx 100^{Hz}$ or more

Coning Errors

Coning ≡ Axis of rotation harmonically moving in space.



<u>No</u> net rotation, Coning rate Ω_c . Frequency f_c

Coning frequency could be ≈ Δt in quaternion update

Error in angles associated with coning (source: Avionics Navigation Systems. Kayton + Fried)

$$\epsilon_c = \left(1 - \frac{\sin(2\pi f_c \Delta t)}{2\pi f_c \Delta t}\right) \quad \text{with} \quad \Omega_c = 4\pi f_c \sin^2\left(\frac{\alpha}{2}\right)$$

"drift"

Rather than updating the quaternion update cycle faster, the sensor can be pre-processed at a higher rate.

C ≡ Correction Vector

$\Delta\theta$ ≡ Sensor (rate gyro) input

// Slow Quaternion update ≈ 100 Hz

Outer loop: $\Delta\phi = \delta\phi + C$

$\delta\phi = 0$ // reset to zero

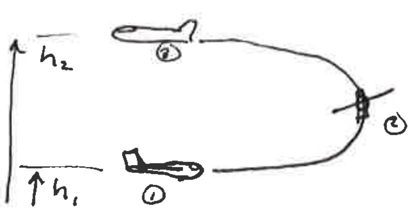
$C = 0$

Inner loop: // Fast update ≈ 1000 - 2000 Hz

$$C = C + \delta\phi \times \Delta\theta_n + \frac{1}{12}\Delta\theta_{n-1} \times \Delta\theta_n$$

$$\delta\phi = \delta\phi + \Delta\theta_n$$

This is <u>not</u> a trivial issue with aircraft INS. Consider a chandelle



180° turn with altitude gain

- Bank
- pull up
- Level wings

⇒ Coning about y axis

if 30 seconds and 30° bank

$$\Omega_c = 4\pi \cdot \frac{1}{30} \cdot \sin\left(\tfrac{30}{2}\right)^2 \cdot 3600\,\tfrac{s}{hr} \cdot 57.3\,\tfrac{deg}{rad} \approx 21000\,\tfrac{deg}{hr} \,!!$$

Sculling Errors

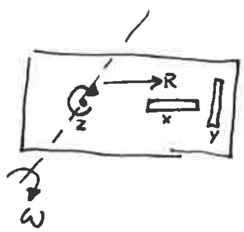
Simultaneous acceleration and rotation with discrete sensor timesteps. leads to errors.

Use coning algorithm to reduce errors.

Sensor Size Error (i.e. lever arm)

So far, we have assumed that the sensors are <u>exactly</u> colocated at one pt.

Reality: Nope!



X accelerometer will see additional acceleration of $\begin{cases} a = \frac{V^2}{R} \\ \quad = \omega^2 R \end{cases}$

Y will see acceleration with $\dot{\omega}$

These offsets <u>must</u> be computed and corrected in the INS computor.

Sensor Orthogonality

The sensors will never be perfectly mounted