This spline is a polynomial of order 3 between N points.

$$y(x) = a_i (x - x_i)^3 + b_i (x - x_i)^2 + c_i (x - x_i) + d_i$$

The spacing is

$$h_i = x_{i+1} - x_i$$

At each point, the following should be satisfied

$$h_{i-1} y'_{i-1} + 2 (h_{i-1} + h_i) y''_i + h_i y''_{i+1} = 6 \left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \right)$$

Altogether this requires solving the following

$$\begin{bmatrix} 2(h_1 + h_2) & h_2 & & \\ h_2 & 2(h_2 + h_3) & \ddots & \\ & \ddots & \ddots & h_{n-2} \\ & & h_{n-2} & 2(h_{n-2} + h_{n-1}) \end{bmatrix} \begin{bmatrix} S_2 \\ S_3 \\ \vdots \\ S_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \\ \frac{y_4 - y_3}{h_3} - \frac{y_3 - y_2}{h_2} \\ \vdots \\ \frac{y_n - y_{n-1}}{h_{n-1}} - \frac{y_{n-1} - y_{n-2}}{h_{n-2}} \end{bmatrix}$$

The values of a_i,b_i,c_i,d_i are

$$\begin{aligned} a_i &= \frac{(S_{i+1} - S_i)}{6h_i} \\ b_i &= \frac{S_i}{2} \\ c_i &= \frac{y_{i+1} - y_i}{h_i} - \frac{2h_i S_i + h_i S_{i+1}}{6} \\ d_i &= y_i \end{aligned}$$

For equally spaced points, Hamming uses

$$\begin{bmatrix} 4 & 1 & & \\ 1 & 4 & 1 & \\ & \ddots & \ddots & 1 \\ & & 1 & 4 \end{bmatrix} \begin{bmatrix} y''_2 \\ y''_3 \\ \vdots \\ y''_{n-1} \end{bmatrix} = \begin{bmatrix} 12d_2 - \frac{y''_1}{h} \\ 12d_3 \\ \vdots \\ 12d_{n-1} - \frac{y''_n}{h} \end{bmatrix}$$

Hamming's d_i terms are

$$d_i = \frac{(y_{i+1} - 2y_i + y_{i-1})}{2h^2}$$

This gives an interpolation function

$$\begin{aligned} f_i(x) &= y_i \left(\frac{x_{i+1} - x}{h_i} \right) + y_{i+1} \left(\frac{x - x_i}{h_i} \right) \\ & -\frac{h_i^2}{6} y''_i \left(\frac{x_{i+1} - x}{h_i} - \left(\frac{x_{i+1} - x}{h_i} \right)^3 \right) \\ & -\frac{h_i^2}{6} y''_{i+1} \left(\frac{x - x_i}{h_i} - \left(\frac{x - x_i}{h_i} \right)^3 \right) \end{aligned}$$

Now, we can interpolate betwen the given nodes and points.