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COMPLEX VARIABLES AND APPLICATIONS

Sixth Edition

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APPENDIX 2

TABLE OF TRANSFORMATIONS OF REGIONS (See Chap. 8)

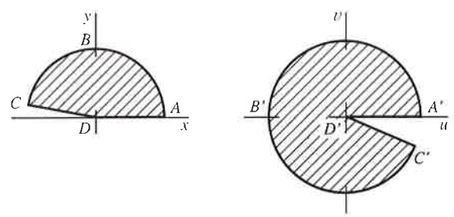

FIGURE 1
$w = z^2$.

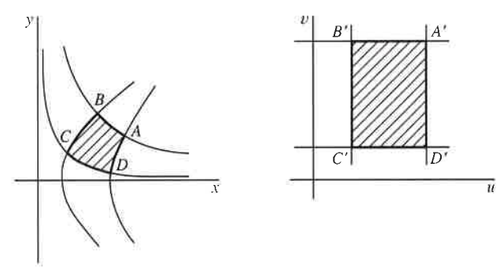

FIGURE 2
$w = z^2$.

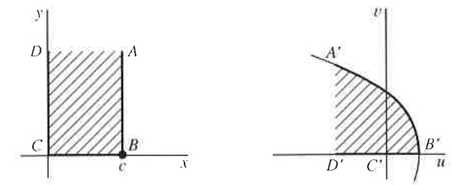

FIGURE 3
$w = z^2$;
$A'B'$ on parabola $v^2 = -4c^2(u - c^2)$.

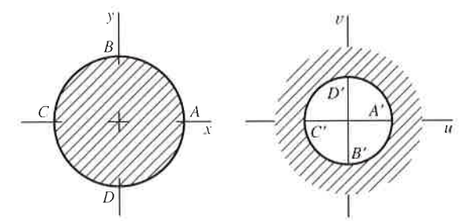

FIGURE 4
$w = 1/z$.

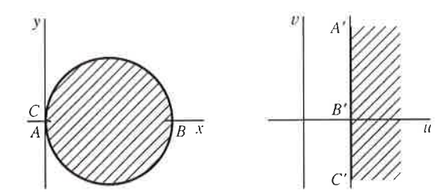

FIGURE 5
$w = 1/z$.

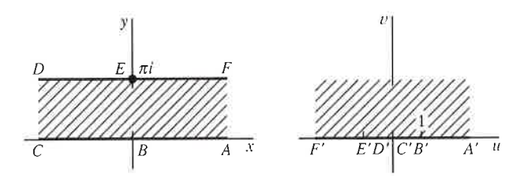

FIGURE 6
$w = \exp z$.

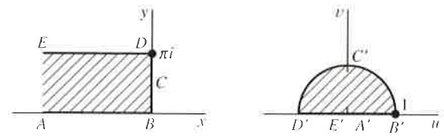

FIGURE 7
$w = \exp z$.

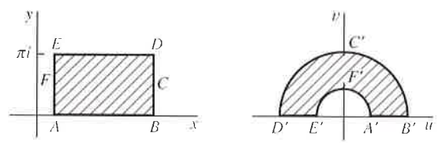

FIGURE 8
$w = \exp z$.

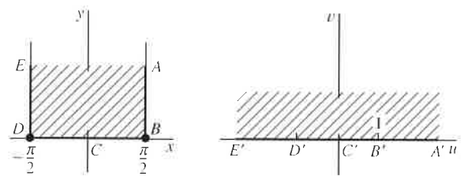

FIGURE 9
$w = \sin z$.

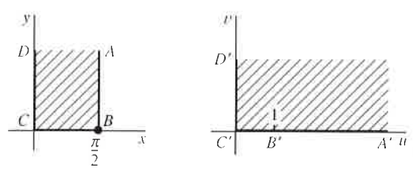

FIGURE 10
$w = \sin z$.

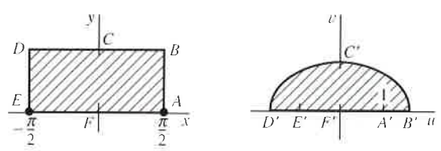

FIGURE 11
$w = \sin z$; BCD on line $y = b$ $(b > 0)$,
$B'C'D'$ on ellipse $\dfrac{u^2}{\cosh^2 b} + \dfrac{v^2}{\sinh^2 b} = 1$.

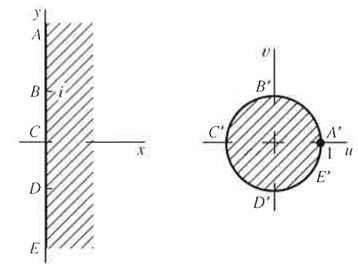

FIGURE 12
$w = \dfrac{z-1}{z+1}$.

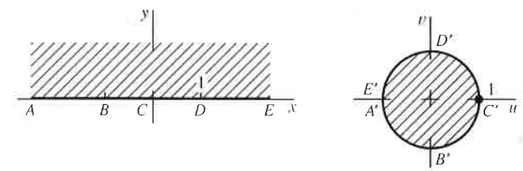

FIGURE 13
$w = \dfrac{i-z}{i+z}$.

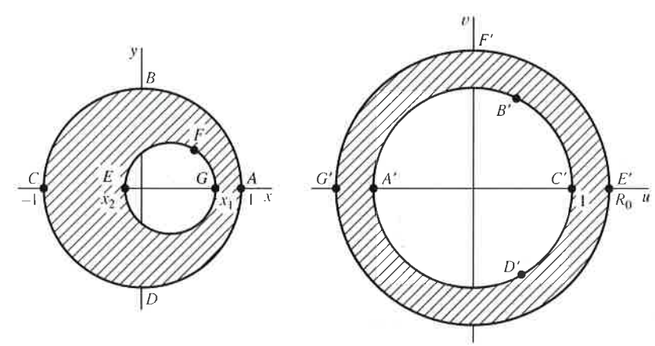

FIGURE 14
$w = \dfrac{z-a}{az-1}$; $a = \dfrac{1 + x_1x_2 + \sqrt{(1-x_1^2)(1-x_2^2)}}{x_1 + x_2}$,

$R_0 = \dfrac{1 - x_1x_2 + \sqrt{(1-x_1^2)(1-x_2^2)}}{x_1 - x_2}$ $(a > 1$ and $R_0 > 1$ when $-1 < x_2 < x_1 < 1)$.

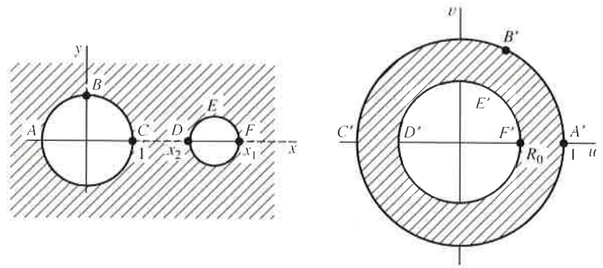

FIGURE 15

$w = \dfrac{z-a}{az-1}$; $a = \dfrac{1 + x_1x_2 + \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1 + x_2}$,

$R_0 = \dfrac{x_1x_2 - 1 - \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1 - x_2}$ $(x_2 < a < x_1$ and $0 < R_0 < 1$ when $1 < x_2 < x_1)$.

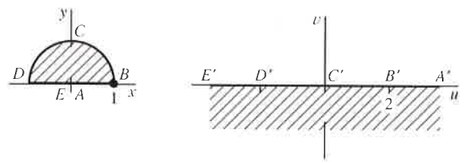

FIGURE 16

$w = z + \dfrac{1}{z}$.

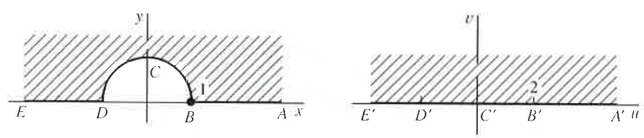

FIGURE 17

$w = z + \dfrac{1}{z}$.

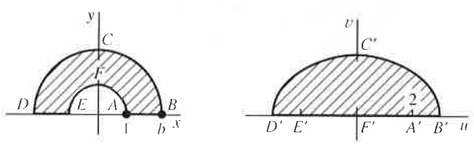

FIGURE 18

$w = z + \dfrac{1}{z}$; $B'C'D'$ on ellipse $\dfrac{u^2}{(b+1/b)^2} + \dfrac{v^2}{(b-1/b)^2} = 1$.

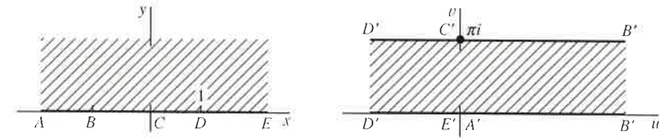

FIGURE 19

$w = \text{Log}\,\dfrac{z-1}{z+1}$; $z = -\coth\dfrac{w}{2}$.

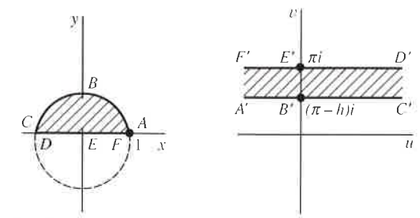

FIGURE 20

$w = \text{Log}\,\dfrac{z-1}{z+1}$;

ABC on circle $x^2 + (y + \cot h)^2 = \csc^2 h$ $(0 < h < \pi)$.

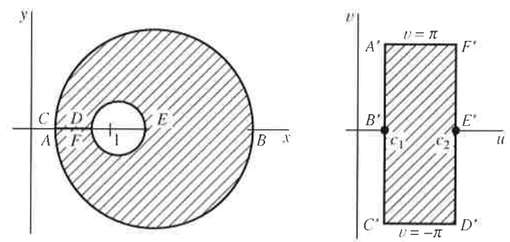

FIGURE 21

$w = \text{Log}\,\dfrac{z+1}{z-1}$; centers of circles at $z = \coth c_n$,

radii: $\operatorname{csch} c_n$ $(n = 1, 2)$.

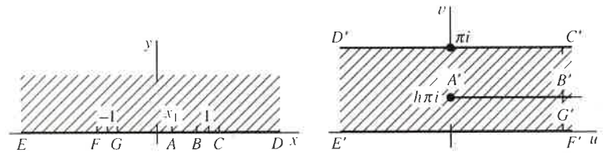

FIGURE 22

$w = h \ln \dfrac{h}{1-h} + \ln 2(1-h) + i\pi - h\,\mathrm{Log}(z+1) - (1-h)\,\mathrm{Log}(z-1);$

$x_1 = 2h - 1.$

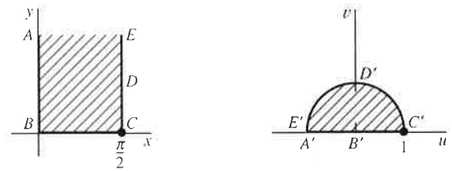

FIGURE 23

$w = \left(\tan \dfrac{z}{2}\right)^2 = \dfrac{1-\cos z}{1+\cos z}.$

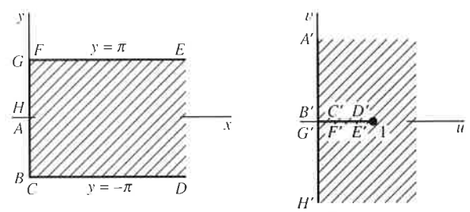

FIGURE 24

$w = \coth \dfrac{z}{2} = \dfrac{e^z+1}{e^z-1}.$

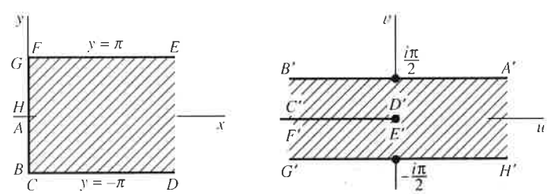

FIGURE 25

$w = \mathrm{Log}\left(\coth \dfrac{z}{2}\right).$

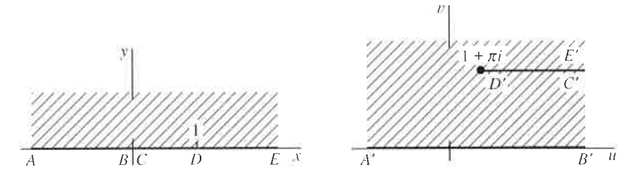

FIGURE 26

$w = \pi i + z - \mathrm{Log}\, z.$

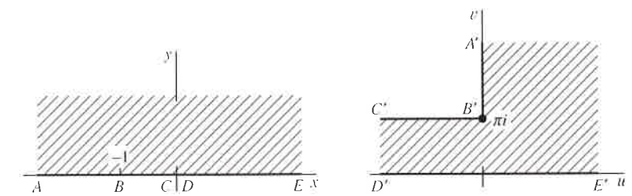

FIGURE 27

$w = 2(z+1)^{1/2} + \mathrm{Log}\,\dfrac{(z+1)^{1/2}-1}{(z+1)^{1/2}+1}.$

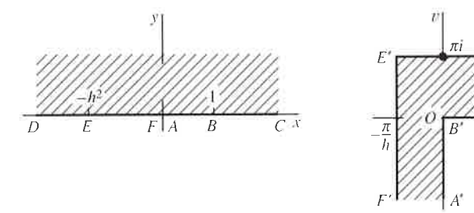

FIGURE 28

$w = \dfrac{i}{h}\,\mathrm{Log}\,\dfrac{1+iht}{1-iht} + \mathrm{Log}\,\dfrac{1+t}{1-t};\ t = \left(\dfrac{z-1}{z+h^2}\right)^{1/2}.$

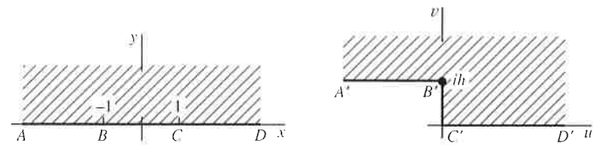

FIGURE 29

$w = \frac{h}{\pi}[(z^2 - 1)^{1/2} + \cosh^{-1} z]$.*

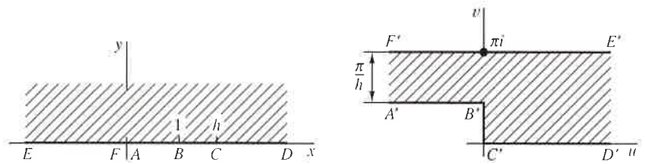

FIGURE 30

$w = \cosh^{-1}\left(\frac{2z - h - 1}{h - 1}\right) - \frac{1}{h}\cosh^{-1}\left[\frac{(h + 1)z - 2h}{(h - 1)z}\right]$.

* See Exercise 3, Sec. 99.

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