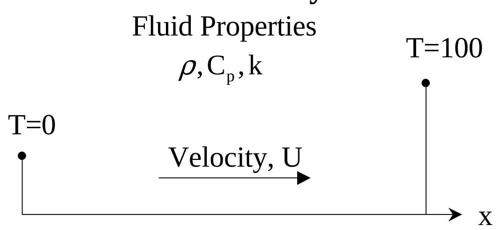
ADVECTION-DIFFUSION PROBLEMS WITH FINITE DIFFERENCE METHODS

March 11, 2002 MAE 6263 Charles O'Neill

PROBLEM DEFINITION

This goal is to represent and solve an advection-diffusion problem with finite difference methods.

Problem Layout



The Peclet number indicates the ratio of advection to diffusion effects. Large Peclet numbers usually result in numerical difficulties. $Pe_{cell} = \dfrac{u \cdot \Delta x}{k}$

MATHEMATICAL DESCRIPTIONS

The governing differential equation is

$$\rho \cdot C_p \cdot u \cdot \frac{\partial T}{\partial x} \cdot = k \cdot \frac{\partial^2 T}{\partial x^2}$$

ANALYTICAL SOLUTION

The analytic solution form depends on the presence of advection. The zero advection solution is linear.

$$T(x) = -\frac{100}{1-e^C} \cdot e^{C \cdot x} + \frac{100}{1-e^C} \qquad \text{when } C \neq 0$$

$$T(x) = 100 \cdot x \qquad \text{when } C = 0$$

$$\text{where } C = \rho \cdot C_p \cdot u \cdot k^{-1}$$

NUMERICAL SOLUTION TECHNIQUE

Numerical solutions are determined through Gauss-Seidel iterations of a discrete update equation

$$T_p = A_W \cdot T_W + A_E \cdot T_E$$

The central difference influence coefficients are,

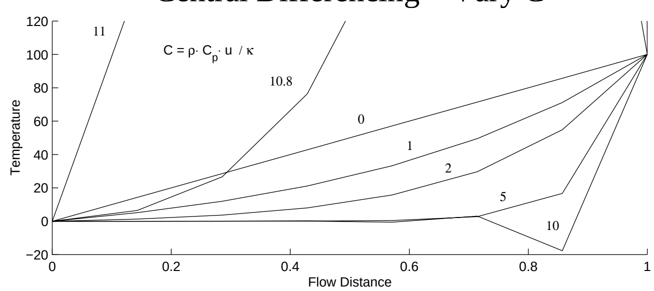
$$A_W = \tfrac{1}{2} \cdot (1 + C \cdot \Delta x) \qquad A_E = \tfrac{1}{2} \cdot (1 - C \cdot \Delta x)$$

The backward difference influence coefficients are,

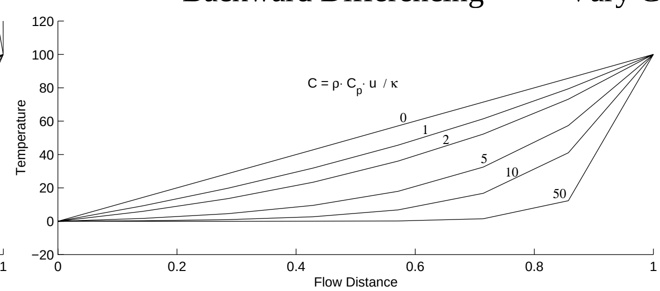
$$A_W = \frac{(C \cdot \Delta x + 1)}{(C \cdot \Delta x + 2)} \qquad A_E = \frac{1}{(C \cdot \Delta x + 2)}$$

RESULTS

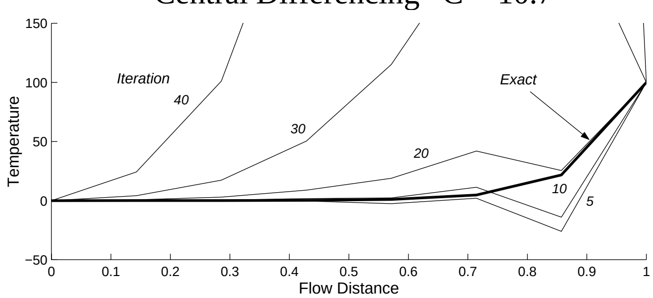
Central Differencing Vary C



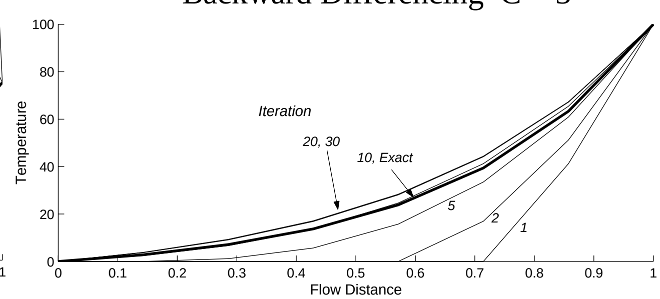
Backward Differencing Vary C



Central Differencing C = 10.7



Backward Differencing C = 3



DISCUSSION

An advection problem was solved with two finite differencing methods. The central difference method diverges for high advection. A non-physical zigzap pattern occurred with the central differencing method. A backward differencing method converges with the final accuracy depends on the grid resolution and iterations.

CONCLUSION

The addition of advection creates computational difficulties. Proper numerical techniques are needed when combining advection and diffusion.***(The two Central Differencing figures are incorrect! 3-15-02)***