MAE 4223

Aerospace Engineering Laboratory

Cold Gas Thruster

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ABSTRACT

A compressible flow supersonic nozzle using Nitrogen is experimentally tested. A shadowgraph visualization system is setup. Specific impulse calculations are performed from theoretical and experimental data. Compressible flow equations are reviewed.

INTRODUCTION

A supersonic nozzle was experimentally tested for various conditions of compressible flow. The input pressure was varied to allow the nozzle to choke and to create shocks either inside or outside the nozzle exit. Thrust data was recorded. Theoretical calculation are used to verify the experimental data. Specific impulse is calculated.

EQUIPMENT

The test equipment consisted of a supersonic nozzle system and a measurement system.

The supersonic nozzle consisted of a converging-diverging nozzle connected to a compressed Nitrogen source. The nozzle had a throat diameter of 1/8 *in* and a exit diameter of 0.205 *in*. The design Mach number is 2.52 for supersonic flow. Friction losses in the tubing between the low pressure gage and the nozzle were previously estimated at a loss ratio of 0.91 between the pressure gage and the nozzle pressures.

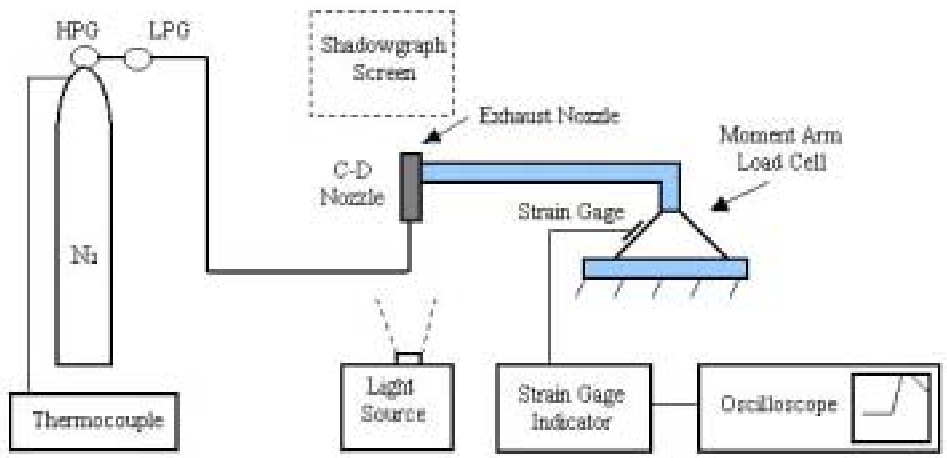


Figure 1. Test Equipment

The measurement system was a moment-arm load cell. The nozzle fired upwards at the end of the 8.5 *in* moment arm. The root end of the moment arm was connected to the base through a group of strain gages.

The measurement system consisted of a strain gage, indicator and oscilloscope. The strain gage measured the strain induced into the load cell due to the weight acting at the end of the moment arm. The strain gage indicator consisted of a Wheatstone bridge amplifying the strain gage signal to levels usable by the oscilloscope. The oscilloscope displayed the resulting transducer output.

PROCEDURE

The measurement procedure consisted of calibration and testing. The load cell was calibrated over the expected experimental range by measuring the transducer output for known test weights. Testing consisted of increasing the low pressure gage to a specified input pressure and recording the resulting transducer output voltage. An increasing series of pressures were tested. These pressures were determined to choke the nozzle and place a shock in certain portions of the nozzle. For all measurements, a pressure, temperature and voltage were recorded.

THEORY

Theory was used for three areas of this experiment. First, the load cell calculations were simplified by assuming a linear relationship. Second, nozzle characteristics are discussed. Finally, a numerical integration method is discussed.

The moment-arm load cell was calibrated to determine a relationship between transducer output and the applied weight. The calibration points are linear due to the particular construction of the load cell. Because of the linearity, the change in applied weight can be described with only a change in output voltage. Thus, the zero point of the calibration setup is useless as long as the load cell remains in a linear transducer output region. Theoretically, this makes the transfer of voltage data to thrust data easier. All zero point load cell calibration will be ignored in this experiment.

Impulse for a nozzle is defined as the total energy expended during the burn. The impulse's magnitude would give an indication of the change in a satellite's movement.

$$I_b = \int_0^t T dt$$

Specific impulse is defined as the impulse per unit of fuel. Specific impulse of a nozzle is related to the type of propellant.

$$I_{sp} = \frac{I_b}{W_{eff}} = \sqrt{\frac{2\gamma R T_o}{(\gamma - 1) g^2} \left(1 - \frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

Thrust is related to the exit velocity and pressures. From John[1],

$$T = \dot{m} V_e + (P_e - P_a) A_e$$

The exit velocity of a choked nozzle is,

$$V_e = \sqrt{\frac{2\gamma R T_o}{\gamma - 1} \left(1 - \frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

The mass flow rate of a choked nozzle is,

$$\dot{m} = \frac{P_o A_t \gamma}{\sqrt{\gamma R T_o}} \sqrt{\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Numerical integration was required to determine the impulse of the rocket motor from thrust and time data. From the above definition, impulse is the integral of thrust over time. A 2-point trapezoidal integration scheme was selected. This method has the advantage of being easy to implement with irregularly stepped data and of being first order accurate. Integration was performed by multiplying the time step with the average value of the two nearest data points. This method is mathematically described as,

$$I = \int_0^t y(x) \approx \sum_0^t \left(\left(\frac{y_2 + y_1}{2} \right) \cdot (x_2 - x_1) \right)$$

RESULTS

Calculations were performed as described above. The best-fit line calibration curve is given in Figure 2. Thus, the relationship between the applied force and the transducer output is,

$$\Delta T = 4.1007 \cdot \Delta mV$$

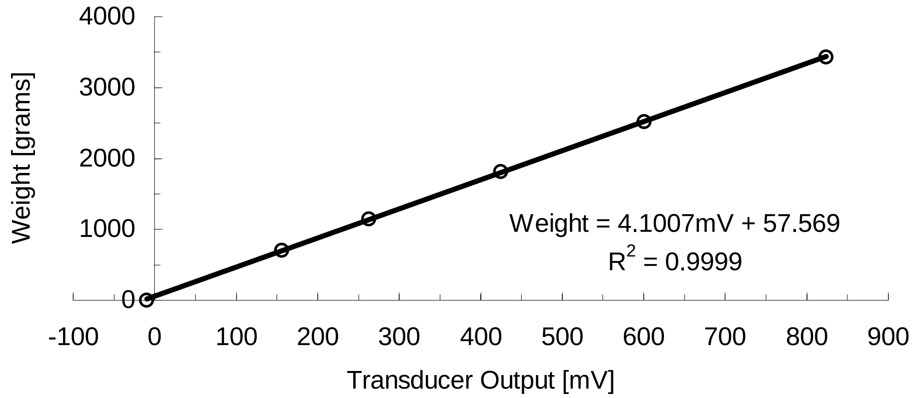


Figure 2. Calibration best-fit line

Experimental uncertainty is estimated by considering the uncertainty in the measurements. From above, the thrust is related to the voltage output by,

$$\Delta T = 4.1007 \cdot \Delta mV$$

The transducer output voltage is assumed to be known within ±0.001 Volt and the calibration weights are known to ±1 gram. Thus, the worst-case estimates of the thrust are,

$$T = 4.1007 (V \pm 1mV) \pm 1 gram$$

For the final 276 psi test, the maximum output voltage was 610 mV so that the maximum estimated thrust is

$$T = 4.1007 (610mV + 1mV) + 1 gram$$

$$T = 5.52 lb$$

Similarly for the 276 psi test, the minimum output voltage was 585 mV so that the minimum estimated thrust is

$$T = 4.1007 (585mV - 1mV) - 1 gram$$

$$T = 5.27 lb$$

Thus the uncertainty for the 276 psi test is,

$$T = 5.395 \pm 0.125 lbf$$

This is an uncertainty of approximately 2 percent.

The nozzle was experimentally tested as described above. Pressure input values were varied to subject the nozzle to different stages of compressible nozzle flow. A thrust versus pressure ratio graph is given in Figure 3. The experimental data is tabulated in Table 2. Shadowgraph photos are given in Photos 1 through 9 for the tested range of input nozzle pressures.

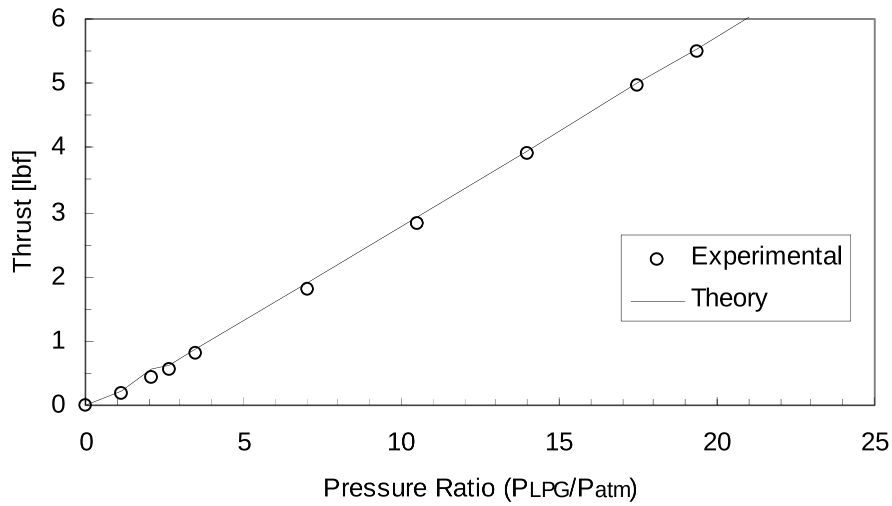


Figure 3. Thrust versus Pressure Ratio

The resulting thrust appears linear with the pressure ratio. Towards the low pressure ratios, the line appears to have more upward curvature. This is expected due to the choking of the nozzle at less than 38 psi. Theory predicted the experimental thrust results nearly perfectly along the entire tested range.

At an input pressure of 16.2 psi, the nozzle is choked. From Figure 1, no density changes are seen in the shadowgraph. This is probably due to the Nitrogen achieving atmospheric pressure.

At a pressure of 30 psi, the nozzle has a normal shock inside the diverging nozzle. The exit velocity is less than Mach 1. From Figure 2, the shadowgraph of the test shows only axial changes in density. This is expected due to the absence of shocks outside the exit. The axial striations are due to refraction at the air-Nitrogen interface. Thus, the shadowgraph setup is working properly.

At 38.2 psi, the nozzle should have a standing normal shock at the exit plane. The Mach number before the shock is 2.52 and 0.51 after the shock. From Figure 3, the axial striations are again seen; however, a large grey line is seen perpendicular to the flow just outside of the exit. This is the standing normal shock.

At 50 psi, the nozzle is supersonic throughout the diverging portion and has a shock outside the nozzle exit. As expected from compressible flow theory, there are a series of diamond-shaped density variations caused by shock waves in Figure 4. These shock waves reflect off the air-Nitrogen interface to form the successive downstream diamond. Likewise, Figures 5 through 7 show similar shock waves.

At 276 psi, the nozzle is operating at the design Mach number. At this flow condition, the entire nozzle is operating isentropically and supersonic. The exit pressure exactly matches the atmospheric pressure. There should be no shock waves or Pradtl-Meyer fans. From Figure 8, we do have P-M fans as seen by the black lines radiating from the nozzle edge. We probably overshot the design input pressure. Likewise for Figure 9, the 300 psi case shows the increase in P-M fans. The shadowgraph looks like the exhaust plume of a high speed aircraft.

The specific impulse was experimentally and theoretically calculated. Theoretical specific impulse is known to be 77 s for Nitrogen. From Table 5, the experimental specific impulse was calculated. It was decided not to include the initial transient startup and shutdown in the specific impulse calculation. Because the thrust profile was nearly linear over a large time period, specific impulse was calculated from a flow rate and thrust derived from Data Sheet 1. The experimental specific thrust was calculated to be 76.9 s.

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations regarding the supersonic nozzle experiment are suggested:

1. Compressible flow in the nozzle was clearly present

2. Compressible flow theory predicted the nozzle performance

3. The shadowgraph easily captured the density variations

4. More precise pressure measuring equipment would allow better control of flow conditions

5. The experimental specific impulse compares favorably with theory.

REFERENCES

1) John, J. E., *Gas Dynamics*, 2^{nd} Ed., Allyn and Bacon, Boston Mass., 1984.

APPENDICES

SAMPLE CALCULATIONS

1. Conversion from Transducer Output to Weight

$$T = Constant \cdot Voltage$$

$$T = 3.8082(1000\,mV)$$

$$T = 2808.2\,gram$$

$$T = \frac{2808.2\,gram}{454\,gram/lb}$$

$$T = 6.185\,lb$$

2. Ideal Exit Velocity from Pressure Ratio

$$V_e = \sqrt{\frac{2\gamma RT_0}{g^2(\gamma - 1)}\left[1 - \frac{P_e}{P_0}\right]^{\frac{\gamma-1}{\gamma}}}$$

$$V_e = \sqrt{\frac{(2)(1.4)\left(1773\frac{ft^2}{s^2R}\right)()}{1.4-1}\left[1 - \frac{P_e}{P_0}\right]^{\frac{1.4-1}{1.4}}}$$

$$V_e = 1000\,ft/s$$

SPREADSHEET TABLES

TABLE 1. Calibration Data

Weight [grams]	Voltage [mV]
0	-9.5
707	156
1144	263
1813	425
2520	601
3427	824

TABLE 2. Experimental Nozzle Calculations

					Patm [psi]
Thrust=	4.1007	mV +	57.569		14.29

Run #	Pressure [psi]	P Ratio P/Patm	Temp [C]	Voltage [V]	Thrust [lbs]
0	0	0	---------	--------	0
1	16.23	1.135759	21.4	7.5	0.195
2	30	2.09937	21.4	35	0.443
3	38.23	2.675297	21.4	48	0.560
4	50	3.49895	21.4	74	0.795
5	100	6.997901	21.3	184	1.789
6	150	10.49685	21.2	298	2.818
7	200	13.9958	21.1	419	3.911
8	250	17.49475	21	535	4.959
9	276.4	19.3422	20.4	596	5.510
10	300	20.9937	20	655	6.043

TABLE 3. Theoretical Nozzle Calculations

R	1773 ft2/s2/R	Patm [psi]	At	
Gamma	1.4	14.29	0.01227	

Run #	PLPG [psi]	P0 [psi]	Pe [psi]	Temp [C]	Temp [R]	V Exit [ft/s]	Flow Rate [slug/s]	Thrust [lbf]
0	0	0	0	---------	-------	0	0	0
1	16.23	14.7826	13.00942	21.4	529.92	1894.250985	0.000128132	0.227001
2	30	27.32459	14.29	21.4	529.92	2307.205206	0.000236842	0.546443
3	38.23	34.82064	1.973212	21.4	529.92	2543.25088	0.000301816	0.616466
4	50	45.54099	2.353612	21.4	529.92	2545.167153	0.000394737	0.858212
5	100	91.08197	4.707225	21.3	529.74	2544.734852	0.000789608	1.891762
6	150	136.623	7.060837	21.2	529.56	2544.302479	0.001184613	2.925313
7	200	182.1639	9.414449	21.1	529.38	2543.870032	0.001579753	3.958863
8	250	227.7049	11.76806	21	529.2	2543.437511	0.001975027	4.992413
9	276.4	251.7506	13.01077	20.4	528.12	2540.840841	0.002185821	5.538128
10	300	273.2459	14.12167	20	527.4	2539.108253	0.002374073	6.025963

TABLE 4. Theoretical Specific Impulse

Isp	T0	Pe/P0
	524.88	0.051681

Isp [s]
78.66568

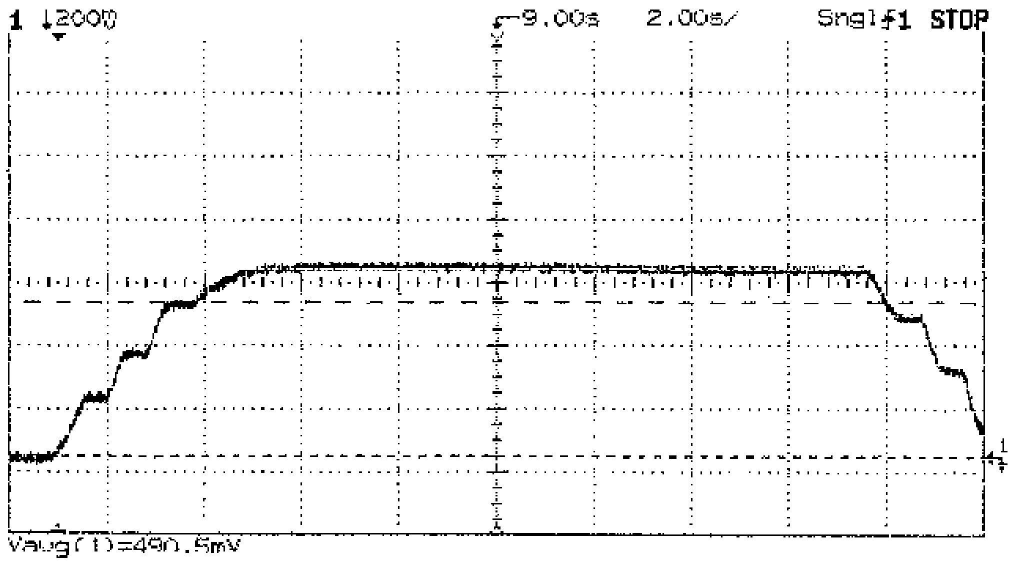
TABLE 5. Experimental Specific Impulse

mV=	5	pixels
Thrust=	4.1007	delta mV
Thrust=	20.5035	pixels

Tank press	P0	Time [s]	Temp [R]	pixels	Thrust [lbf]	Flow rate
276.43	251.7779	0	524.88	122	5.509751	0.002193
276.43	251.7779	10	524.88	117	5.283942	0.002193

Total Flow	0.021928	slug	=	0.701694	lbm
Impulse	53.96846	lbf-s			
Isp	76.91163				

DATA SHEET 1. Nozzle Force Output



	State	Volts/Div	Position	Cplg	BW Lim	Inv	Probe
Chan 1	On	200.0mV	550.0mV	DC	Off	On	1:1
Chan 2	Off	1.000 V	-2.906 V	DC	Off	Off	1:1
Chan 3	Off	100.0mV	0.000 V	DC	---	---	1:1
Chan 4	Off	100.0mV	0.000 V	DC	---	---	1:1

	Mode	Main Time/Div	Main Delay	Time Ref	Delayed Time/Div	Delayed Delay
Horizontal	Normal	2.000 s/	9.000 s	Cntr	---------	-----

PHOTO 1. Shadowgraph 16.23 psi. (Choked Nozzle)

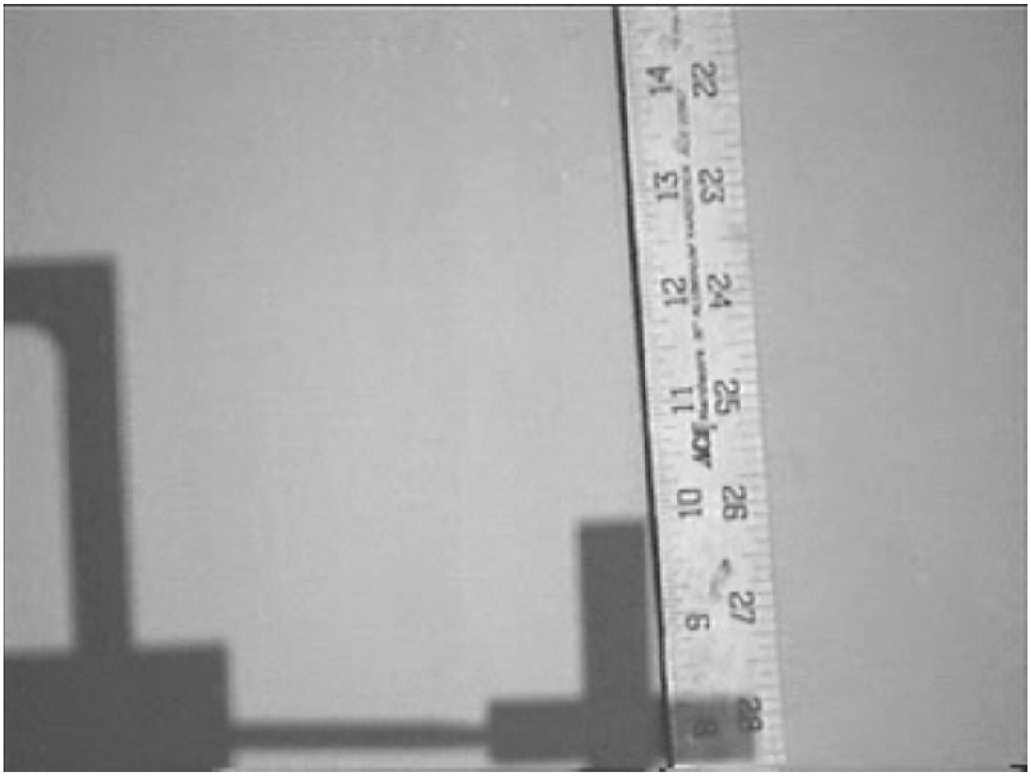


PHOTO 2. Shadowgraph 30 psi. (Normal Shock in Nozzle)

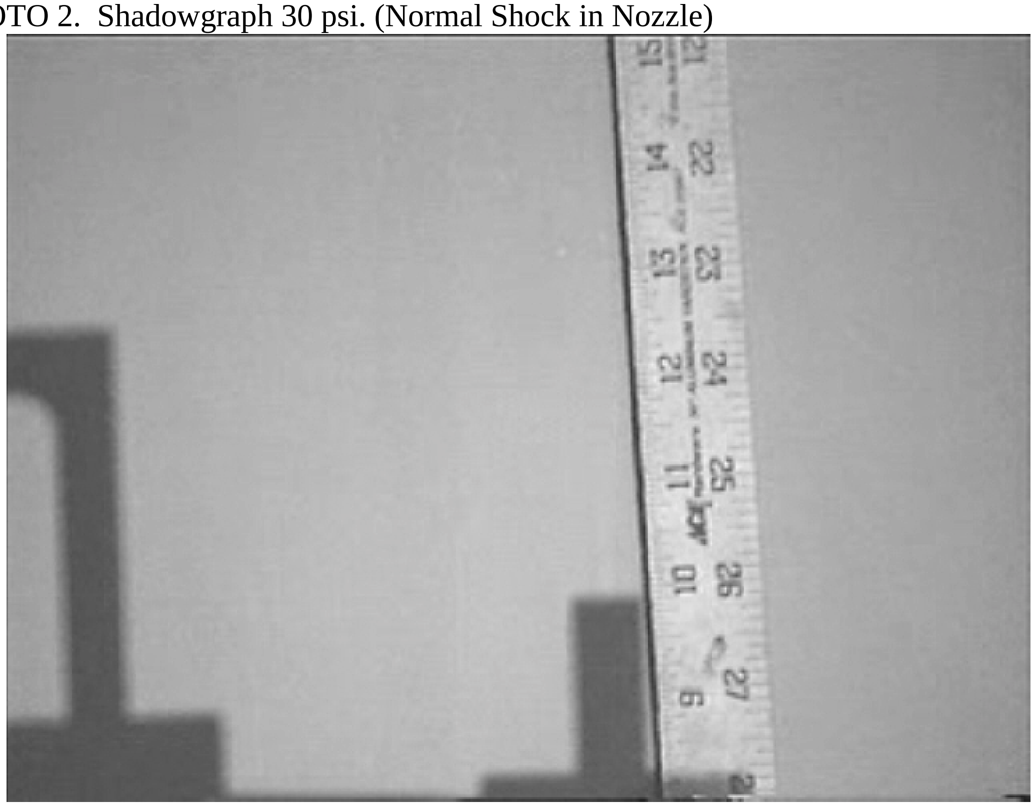


PHOTO 3. Shadowgraph 38.23 psi. (Shock at Exit)

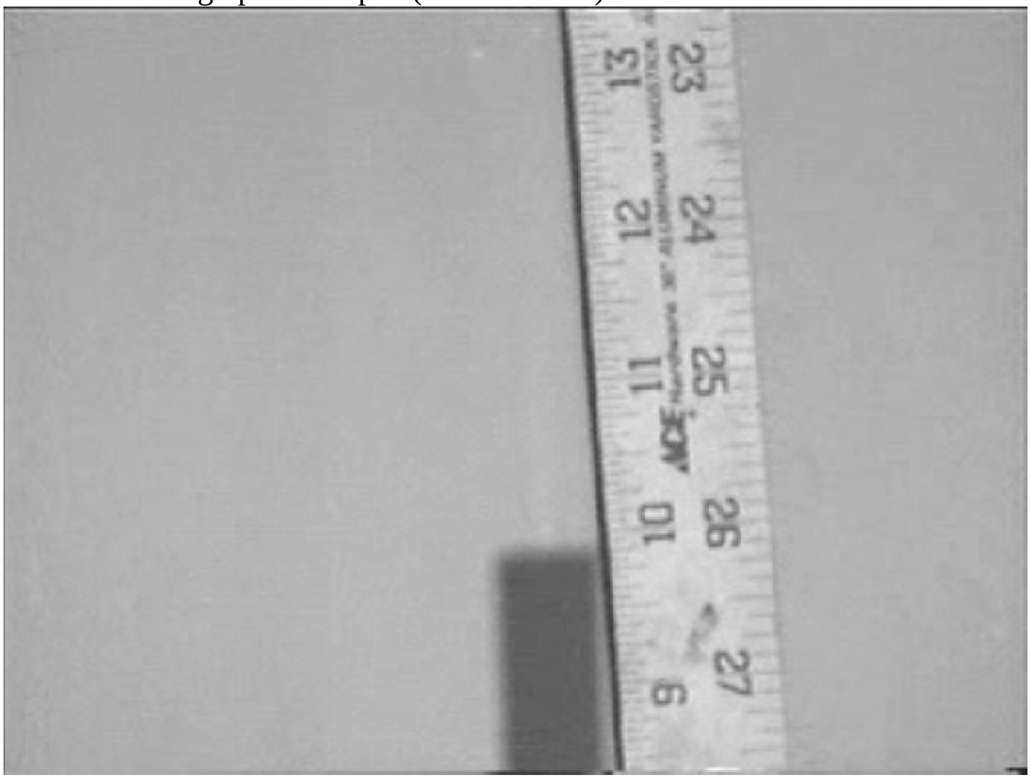


PHOTO 4. Shadowgraph 50 psi. (Shock aft of Exit)

PHOTO 5. Shadowgraph 100 psi. (Shock aft of Exit)

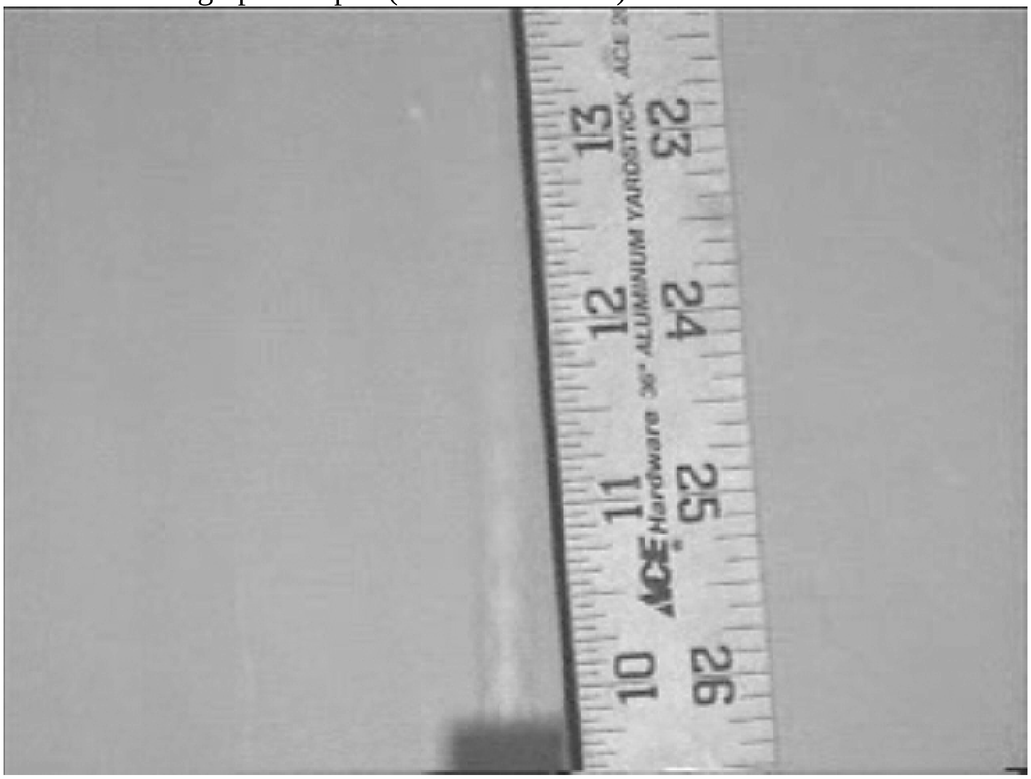


PHOTO 6. Shadowgraph 150 psi. (Shock aft of Exit)

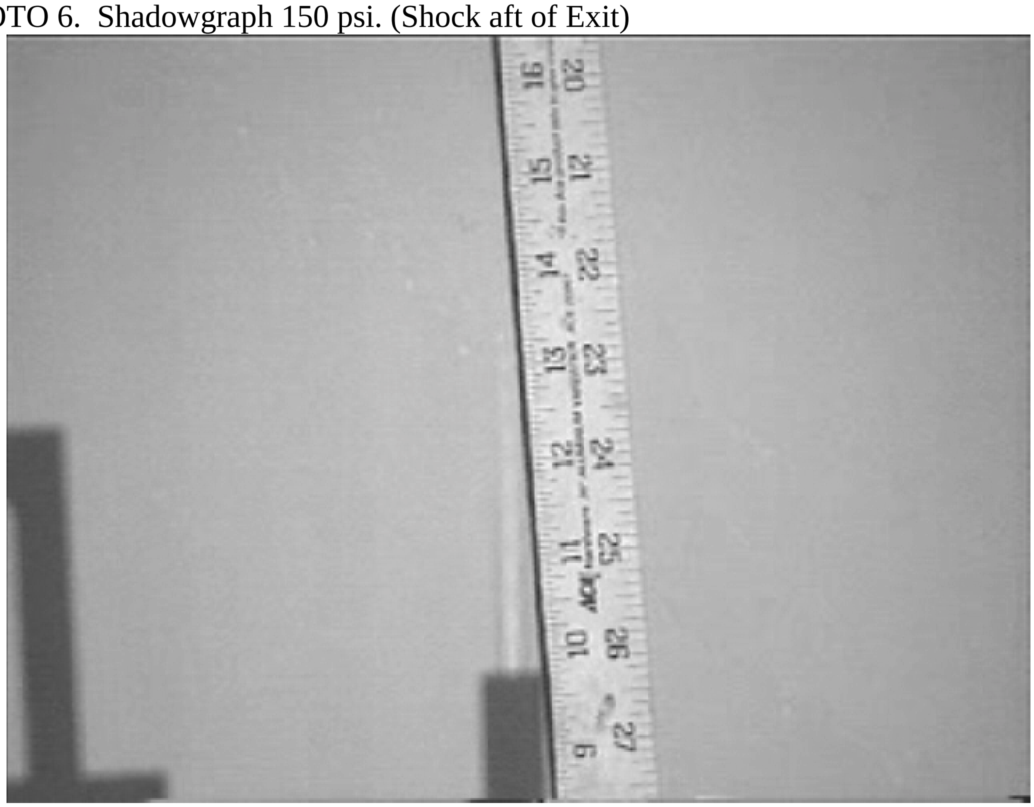


PHOTO 7. Shadowgraph 200 psi. (Shock aft of Exit)

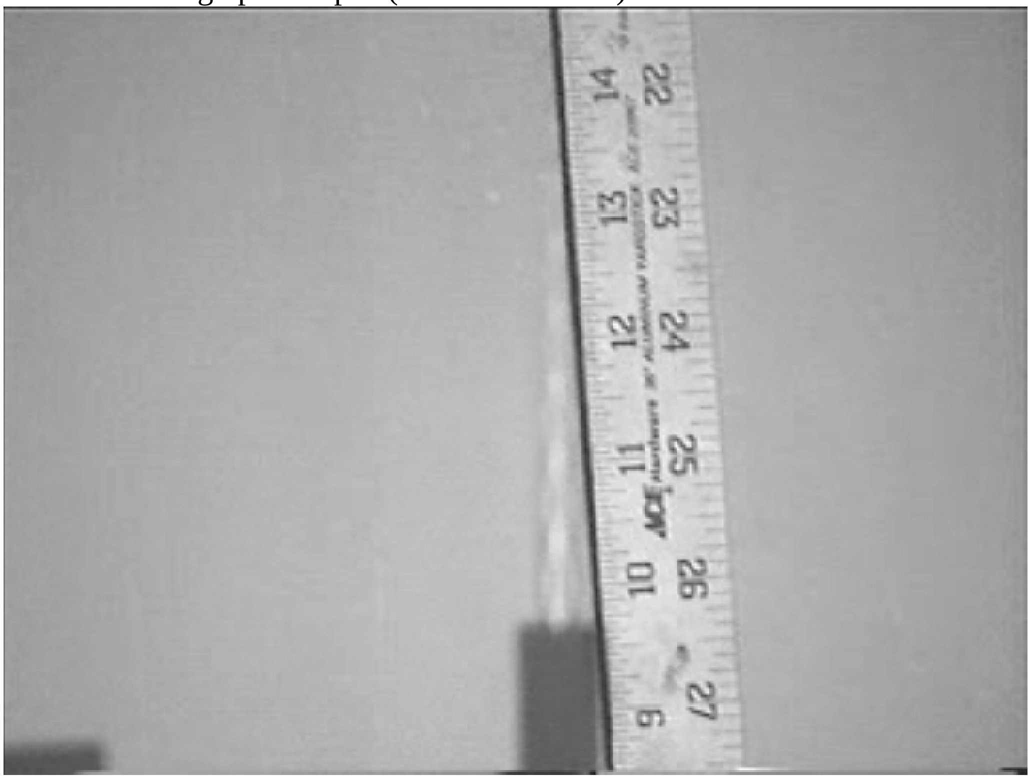


PHOTO 8. Shadowgraph 276.4 psi. (Design Mach Number)

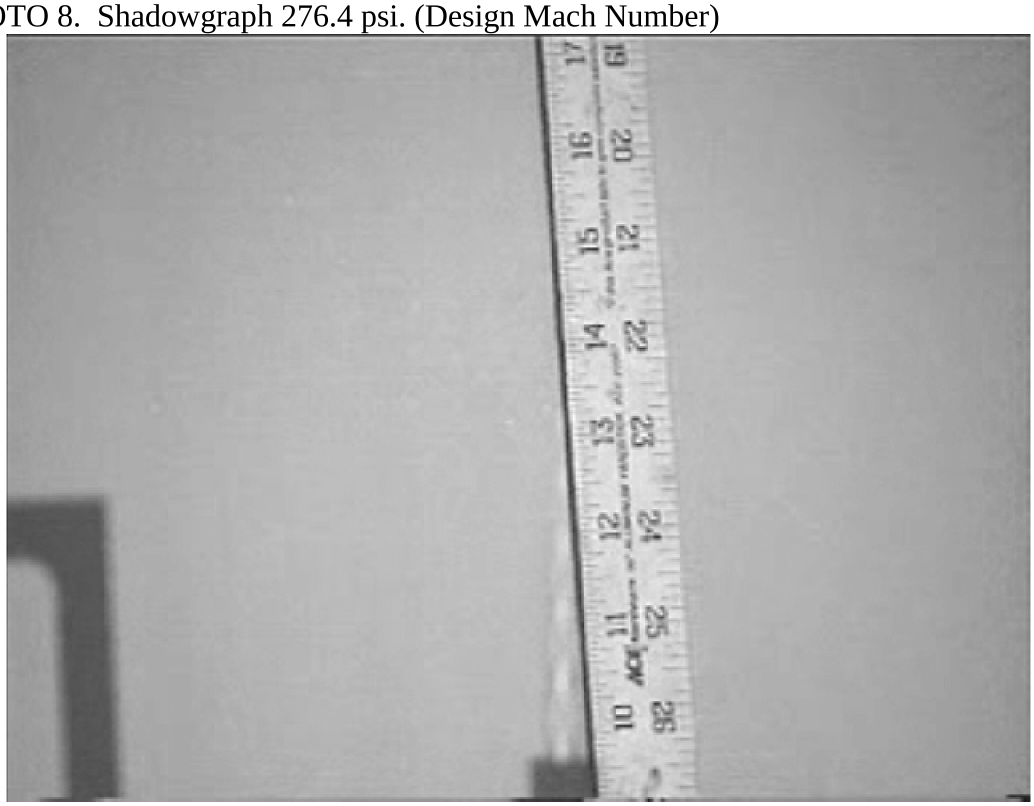


PHOTO 9. Shadowgraph 300 psi. (Above Design Mach Number)