AIAA 2005-0230
Aircraft Flight Dynamics with a Non-Inertial CFD Code

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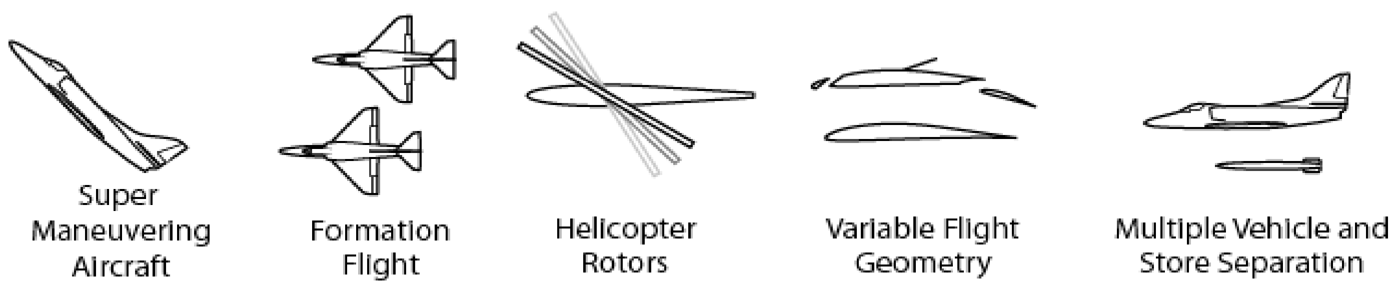

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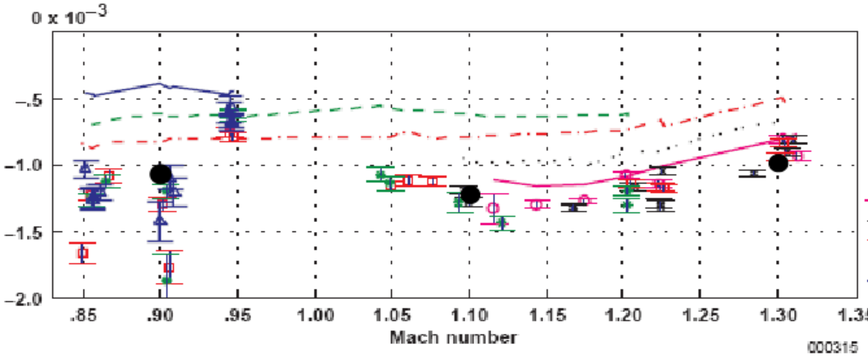
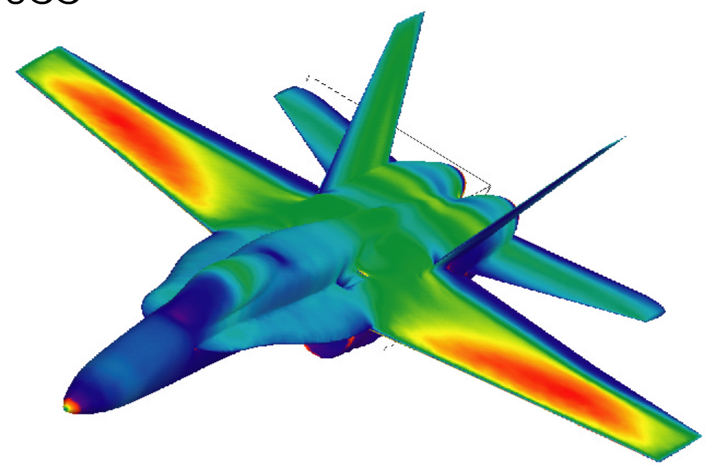
F-18 Stability Derivatives

Experimental Data from
NASA TP-2000-209033

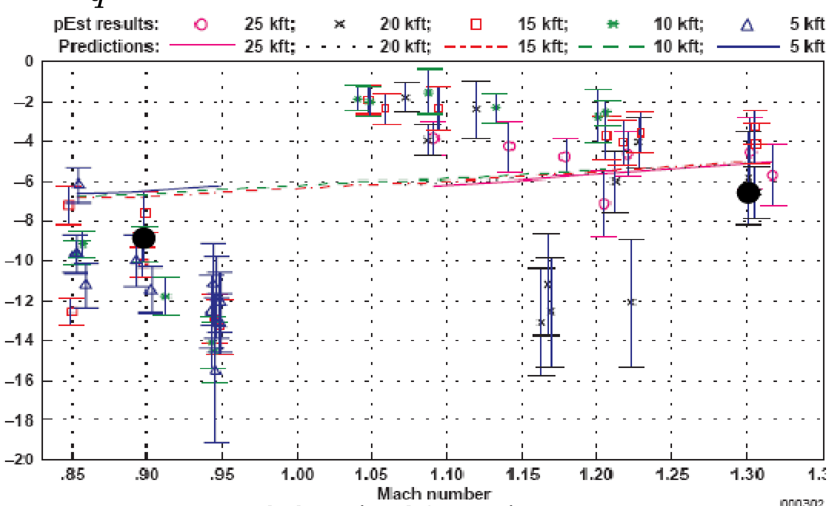
C_{L_β}



Mach Number

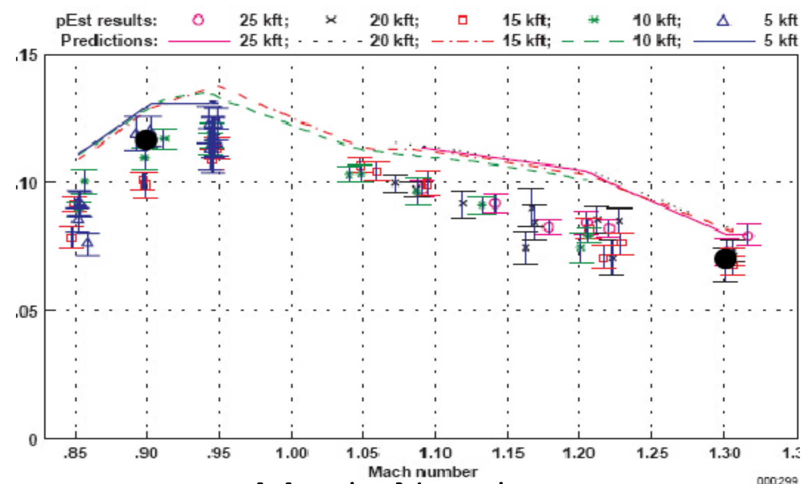


C_{M_q}



Mach Number

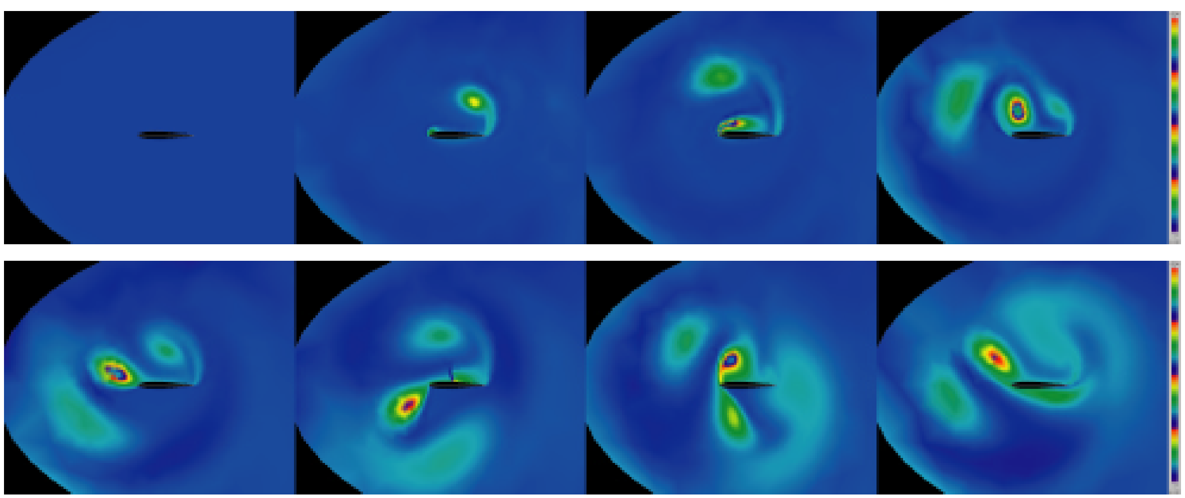
C_{N_α}



Mach Number

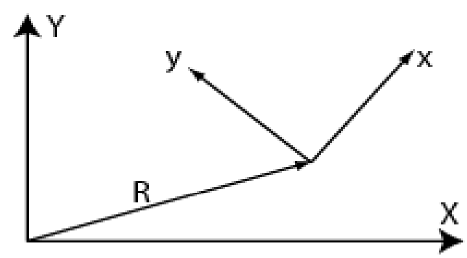
Non-Inertial Frame

- Arbitrary Motion without Remeshing
- CFD Computations in Non-Inertial (Body) Frame



Spinning Airfoil (Body Frame)

$$q_i = R_i + Br_b$$



Attitude Representation

- Euler Angles (Traditional)
- Quaternions
- Others…

Quaternions

Invented by Hamilton

Four parameter: scalar q_0 and vector q_1, q_2, q_3

Update Differential Equation

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 0 & p & q & r \\ -p & 0 & -r & q \\ -q & r & 0 & -p \\ -r & -q & p & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Constraint

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

Transition Matrix

$$B = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

Euler Angle Conversion

- Visualization usually requires Euler Angles.

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \mathsf{arctan}\left(2(q_0q_1 + q_2q_3)/q_0^2 - q_1^2 - q_2^2 + q_3^2\right) \\ \mathsf{arcsin}\left(2(q_0q_2 - q_1q_3)\right) \\ \mathsf{arctan}\left(2(q_0q_3 + q_1q_2)/q_0^2 + q_1^2 - q_2^2 - q_3^2\right) \end{bmatrix}$$

$$-\pi \leq \phi \leq \pi$$

$$-\pi/2 \leq \theta \leq \pi/2$$

$$-\pi \leq \psi \leq \pi$$

Body Frame Kinematics

Translation

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \frac{1}{m} \begin{bmatrix} X - mgS_\theta \\ Y + mgC_\theta S_\phi \\ Z + mgC_\theta C_\phi \end{bmatrix}$$

Rotation

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = I^{-1} \begin{bmatrix} L - (I_z - I_y)qr + I_{xz}pq \\ M - (I_x - I_z)rp - I_{xz}(p^2 - r^2) \\ N - (I_y - I_x)pq - I_{xz}qr \end{bmatrix}$$

6 DOF Equations of Motion

State Vector 13 states

$$Y = [x\ y\ z\ u\ v\ w\ p\ q\ r\ q_0\ q_1\ q_2\ q_3]^T$$

Update Differential Equation

$$\dot{Y} = \begin{bmatrix} 0 & B & 0 & 0 \\ 0 & -\Omega_B & 0 & 0 \\ 0 & 0 & -I^{-1}\Omega_B I & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\Omega_q \end{bmatrix} Y + \begin{pmatrix} 0 \\ m^{-1}F_B \\ J^{-1}T_B \\ 0 \end{pmatrix}$$

Numerical ODE Solution Method

$$y(t+1) = y(t) + \frac{dt}{24}\left(55\dot{y}(t) - 59\dot{y}(t-1) + 37\dot{y}(t-2) - 9\dot{y}(t-3)\right)$$

Non-Inertial CFD Formulation

Euler Inviscid, Compressible Fluid Flow

$$\frac{\partial U}{\partial t} + \frac{\partial F_i}{\partial x_i} = S$$

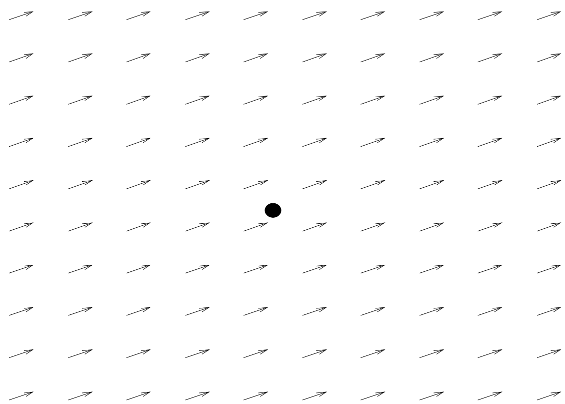
We Want Motion Description in the Body Frame

$$U = \begin{pmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho e_r \end{pmatrix} \quad F = \begin{pmatrix} \rho u_i \\ \rho u_i u_1 \\ \rho u_i u_2 \\ \rho u_i u_3 \\ \rho u_i e_r \end{pmatrix} + \begin{pmatrix} 0 \\ p\delta_{1i} \\ p\delta_{2i} \\ p\delta_{3i} \\ p u_i \end{pmatrix} \quad S = -\rho \begin{pmatrix} 0 \\ a'_t + \Omega V_r \\ a'_t \cdot (V'_t + V_r) \end{pmatrix}$$

$$V_t = B^{-1} V_0 + \Omega r_b, \qquad a'_t = B^{-1} a_0 + \Omega^2 r_b + \dot{\Omega} r_b + \Omega V_r \qquad \Omega_B = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

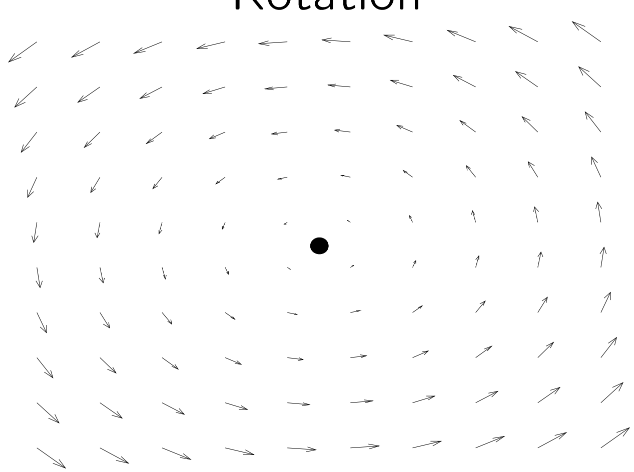
Non-Inertial Frame Motions

Translation

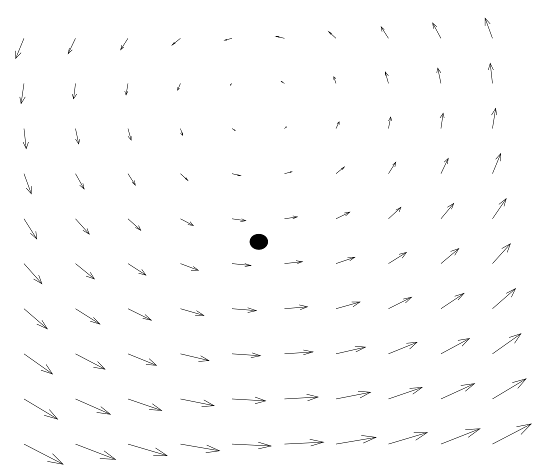


- Fixed Grid
- Fluid Motion referenced to Body Frame

Rotation

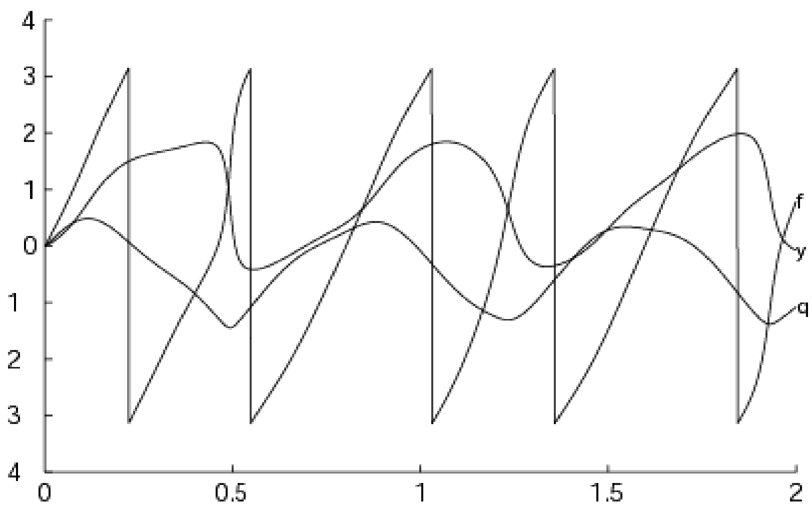


Translation + Rotation

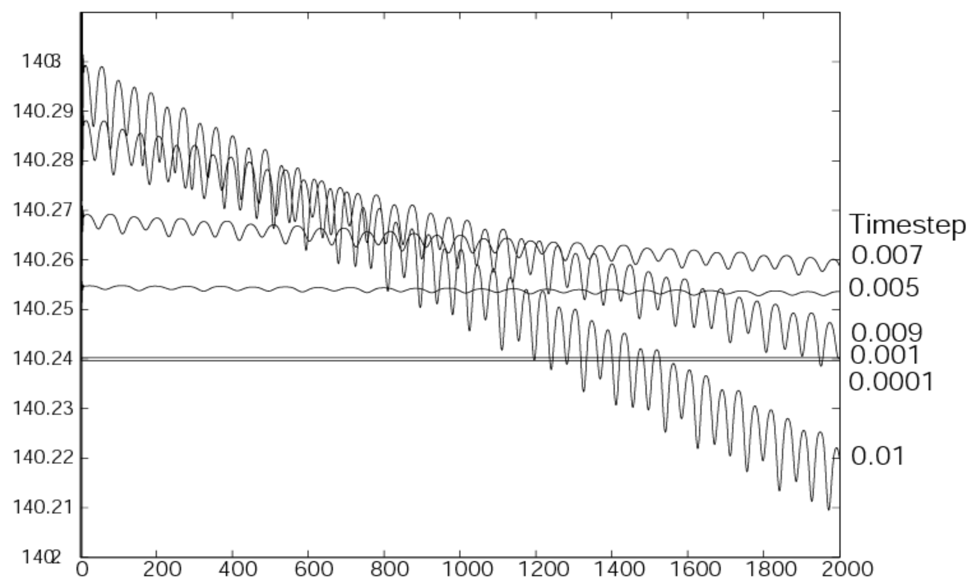


Verification

- Energy Conservation during 6 DOF translations and rotations.



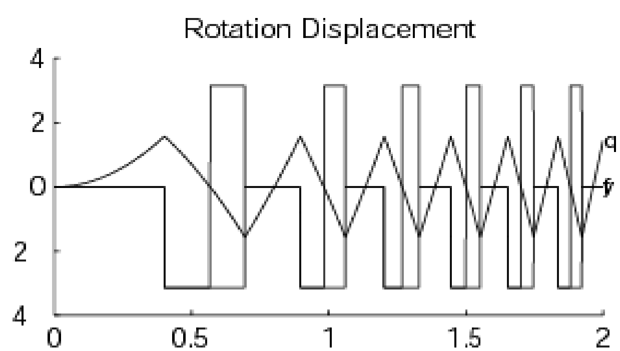
Motion vs. Time



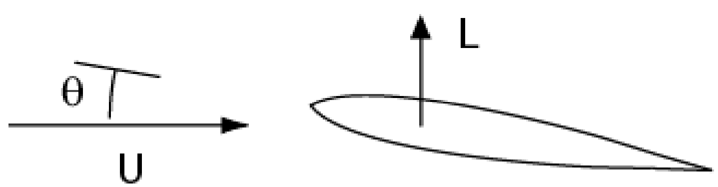
Energy vs. Time

Verification: Specified Pressure

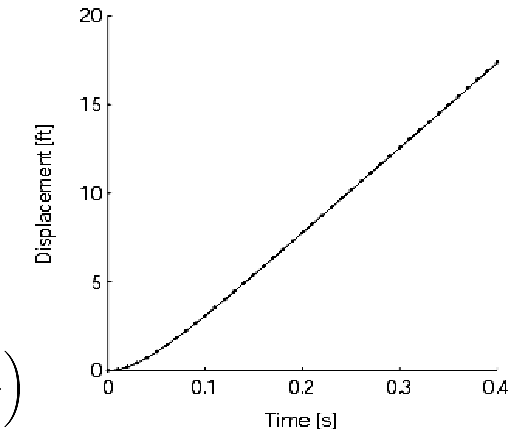
$$p^*(x,y,z,t) = \begin{cases} 1 & \text{if } z > \epsilon \\ -1 & \text{if } z < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

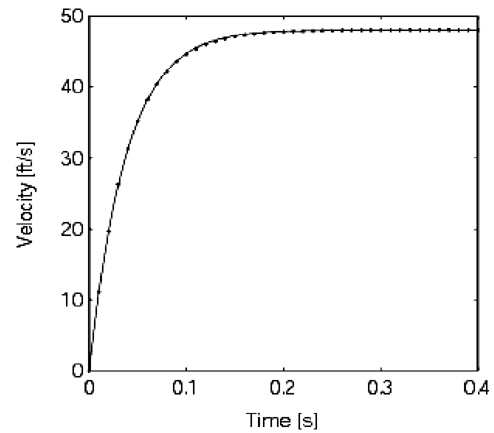


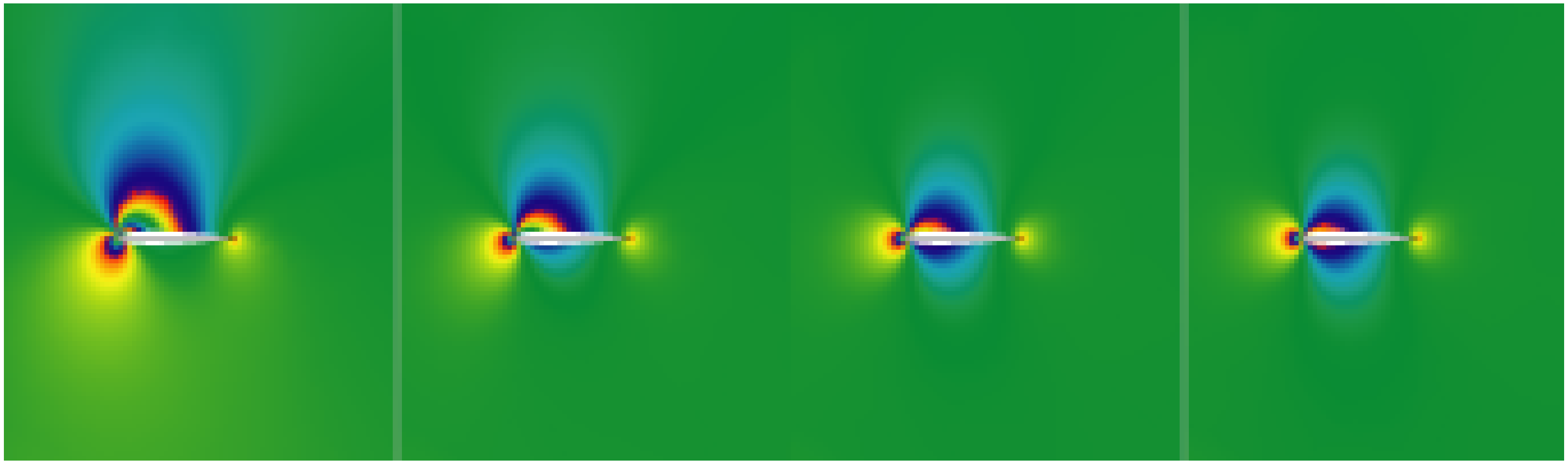
Validation Translation



$$x(t) = \alpha_0 V t - \frac{\alpha_0 V^2 m}{q C_{L\alpha} S} + \frac{\alpha_0 V^2 m}{q C_{L\alpha} S} \exp\left(-\frac{q C_{L\alpha} S t}{mV}\right)$$

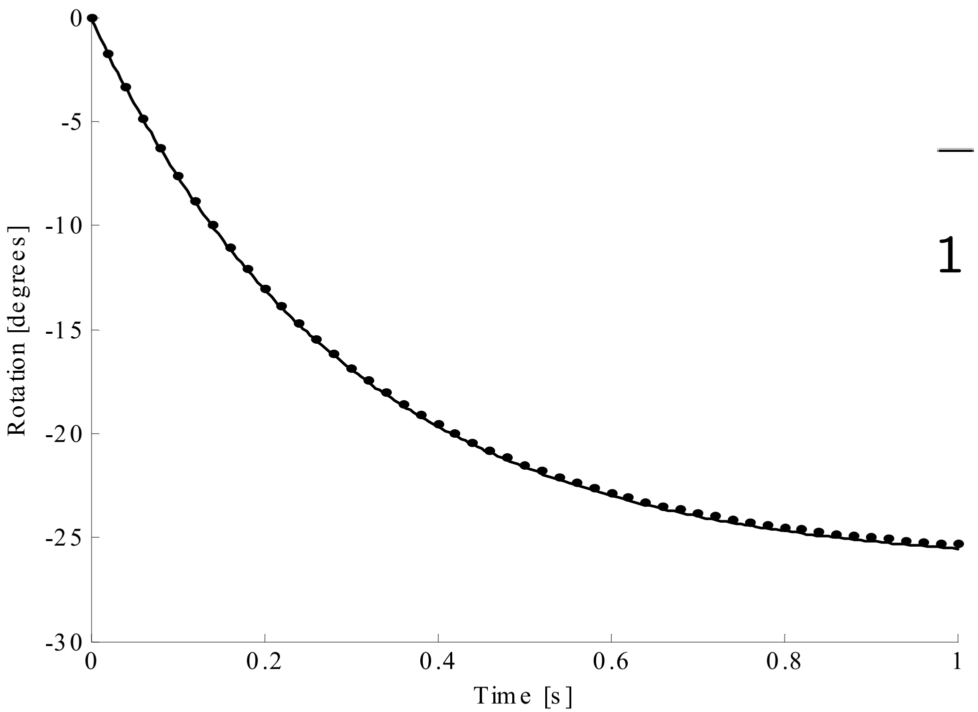




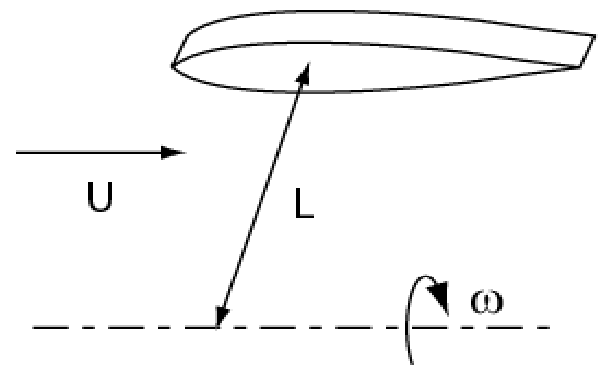


Validation Rotation

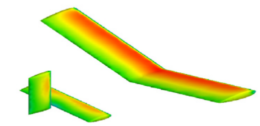
- Compared to quasi-steady aerodynamics



ϕ Rotation Angle vs. Time



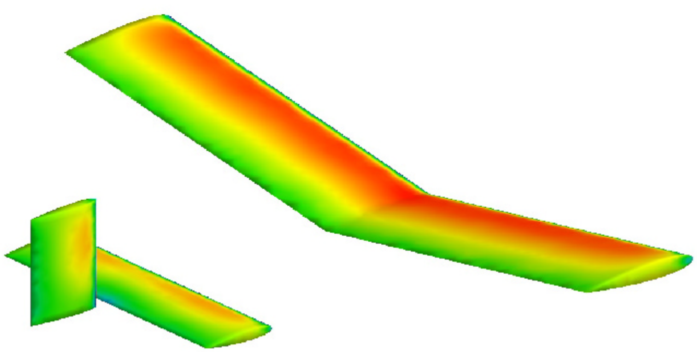
1 DOF Free to Rotate

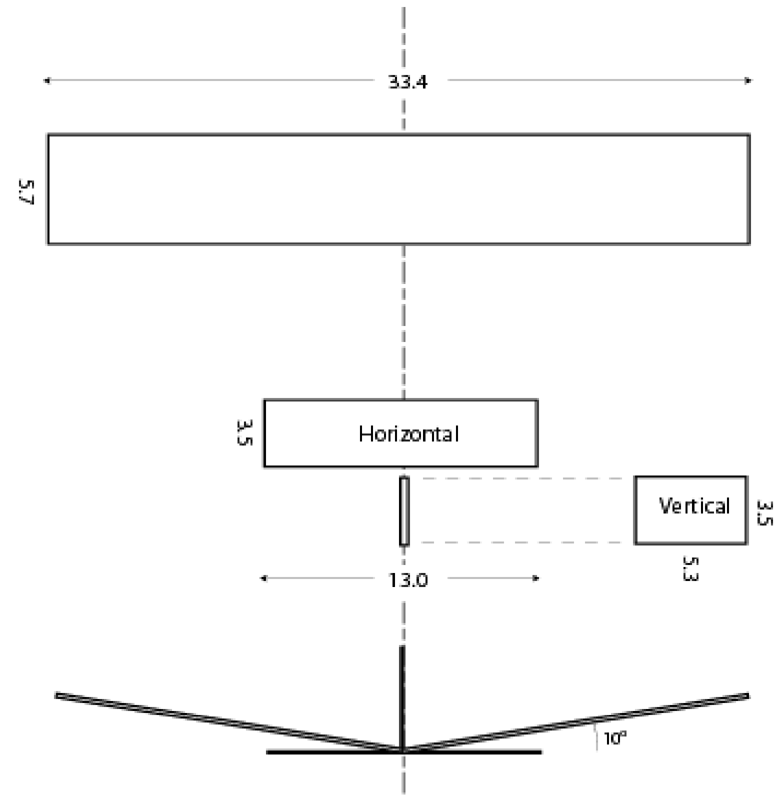


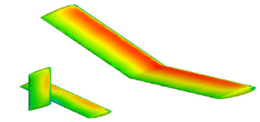
Proof of Concept with a Simplified Navion

North American Navion

V_0=174 ft/s

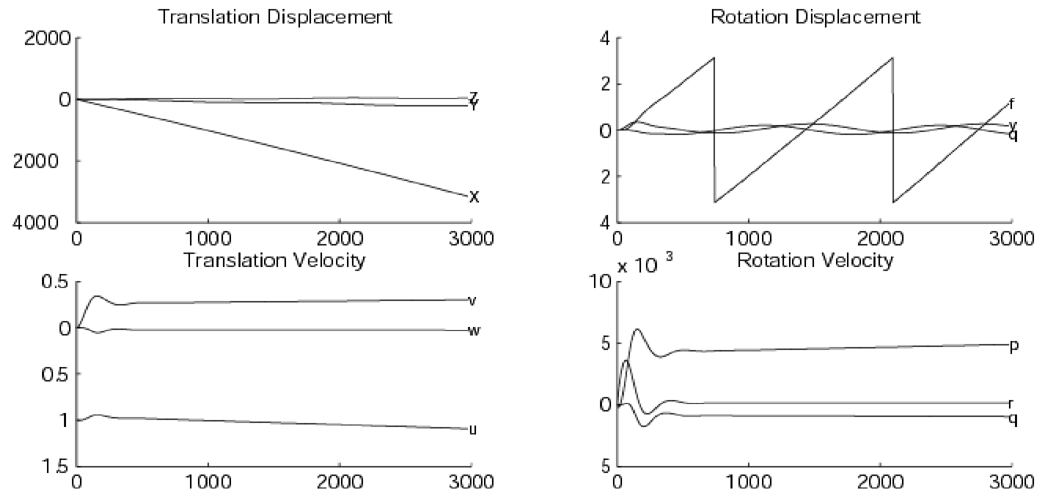


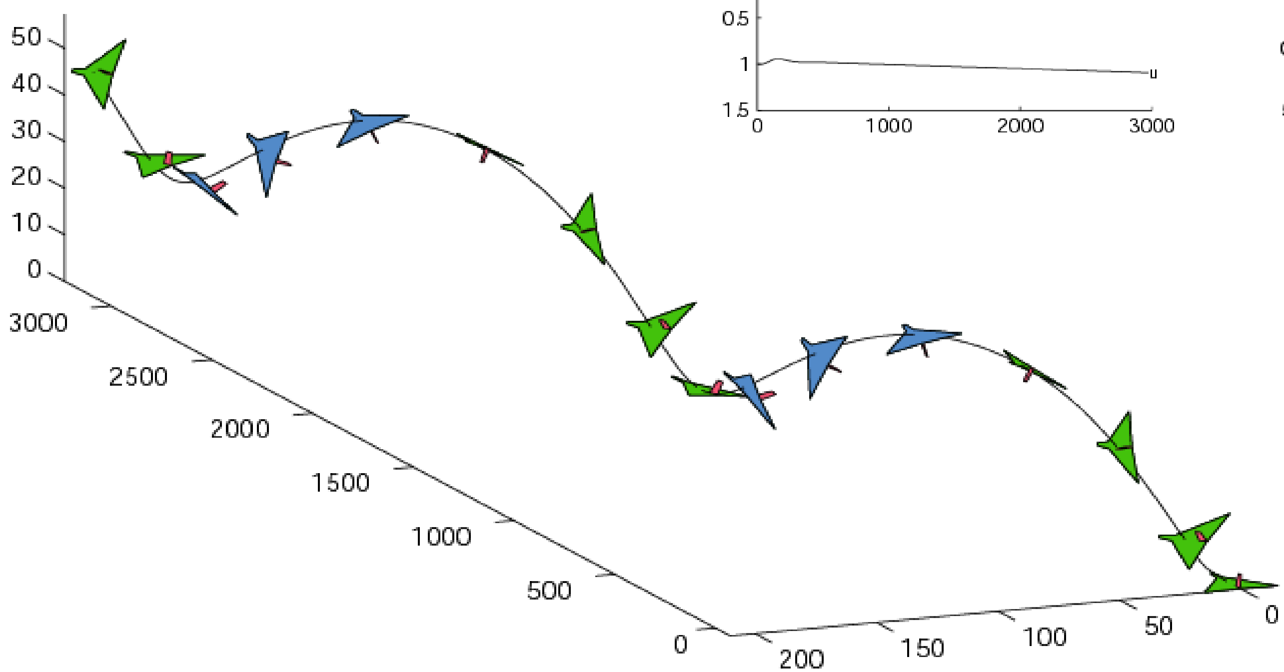


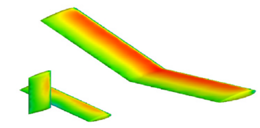


Rudder-Dihedral Induced Roll

$$V_0 = 174 ft/s$$
$$\delta_r = -20°$$



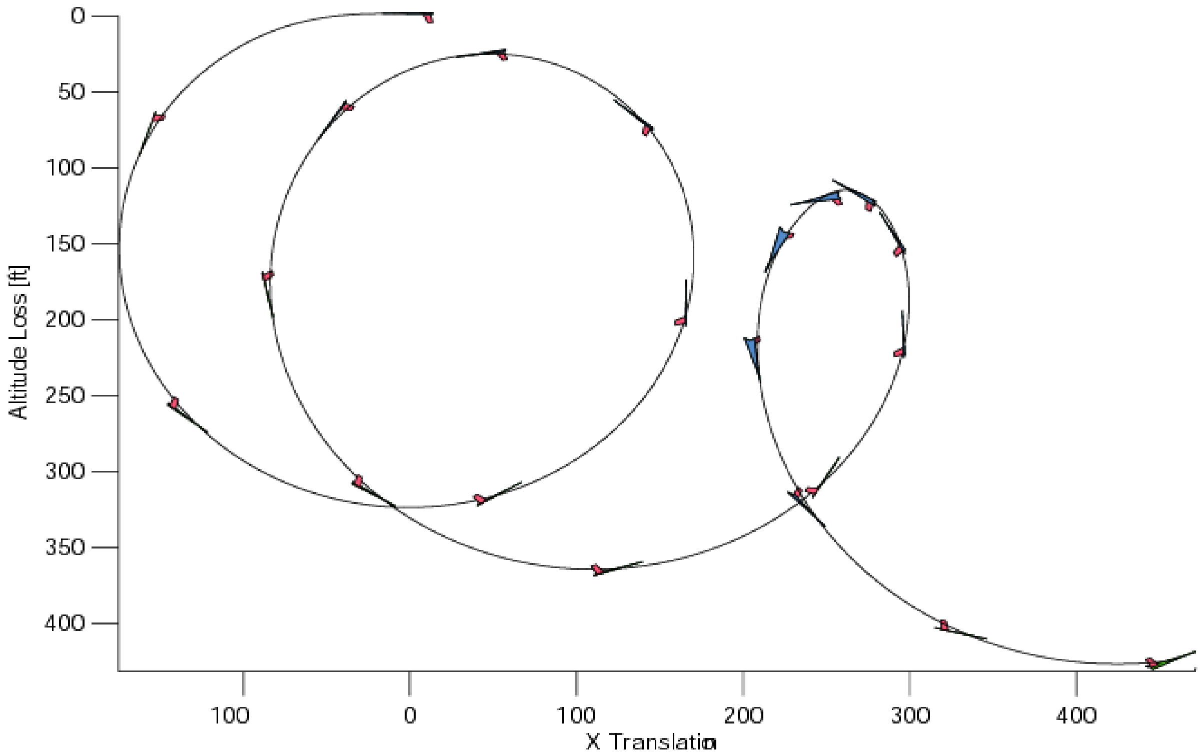




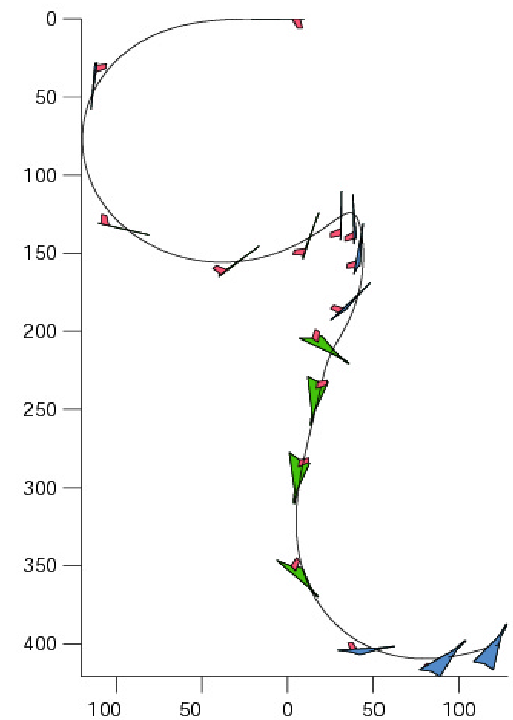
Inverted
$V_0 = 174 ft/s$
$\delta_e = 20°$

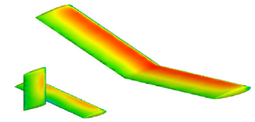
Loops

CG at 30% MAC



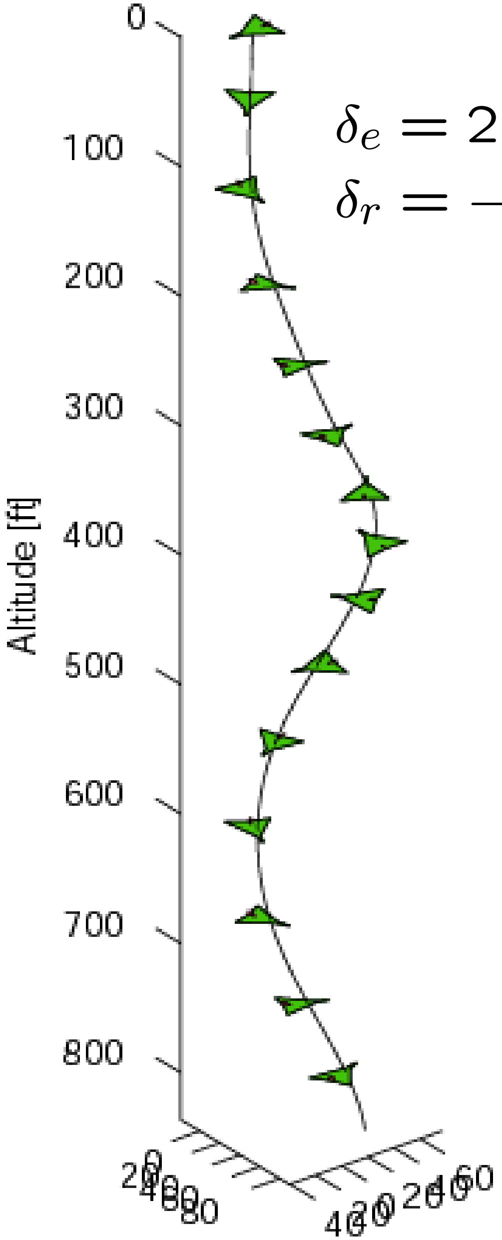
CG at 88% MAC

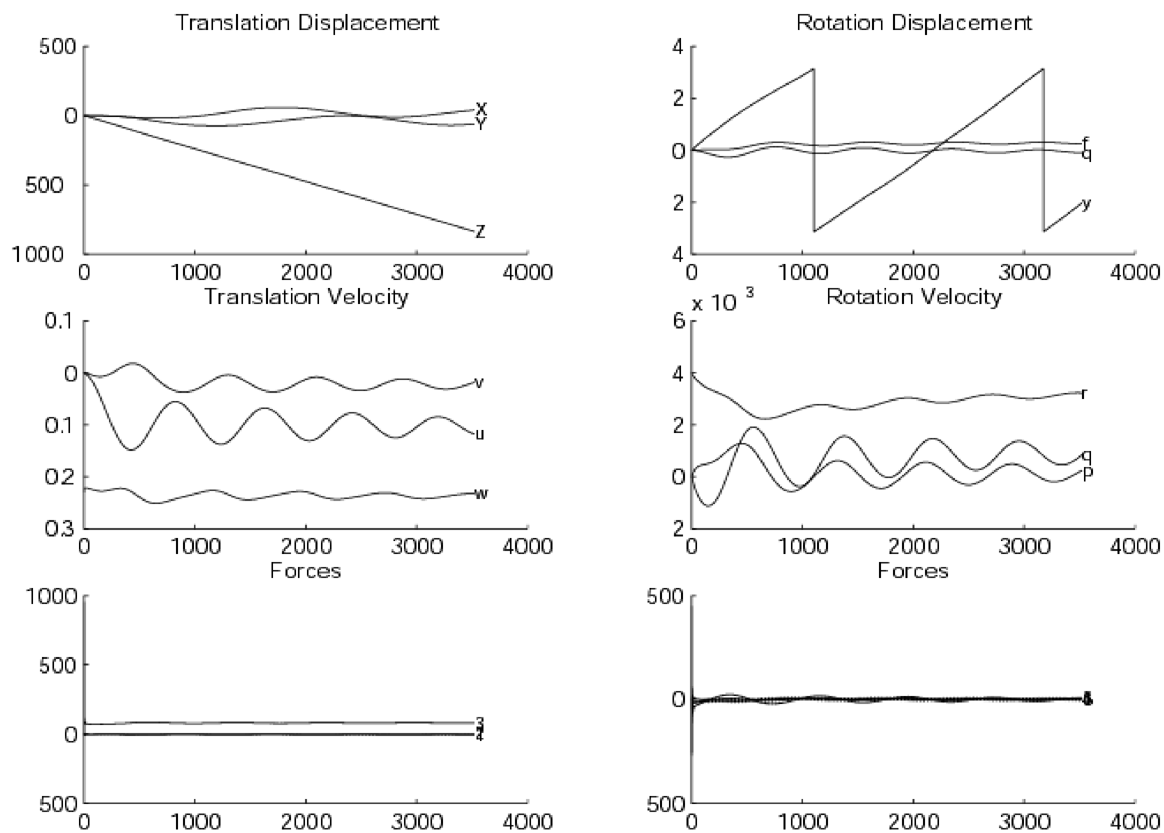




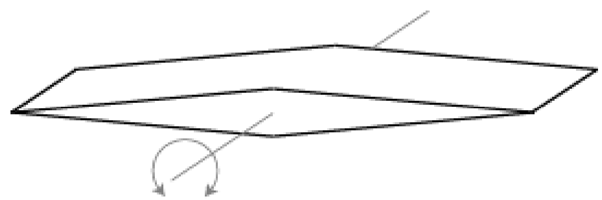
Spin

$$\delta_e = 20^\circ$$
$$\delta_r = -20^\circ$$

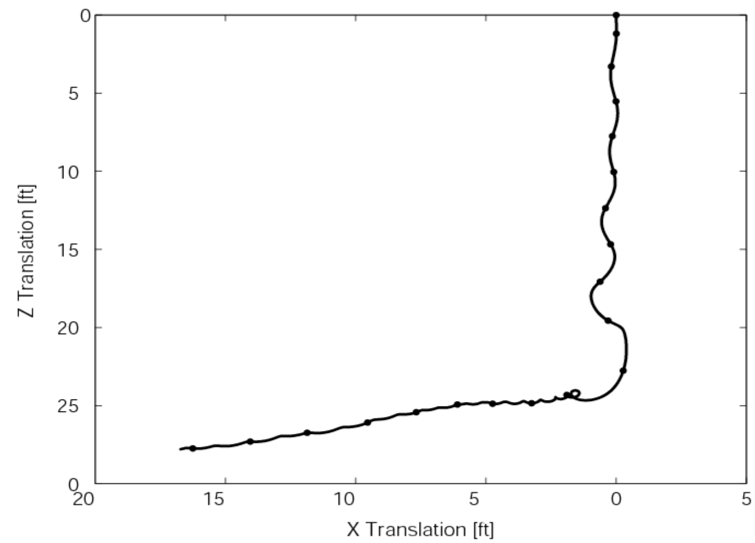


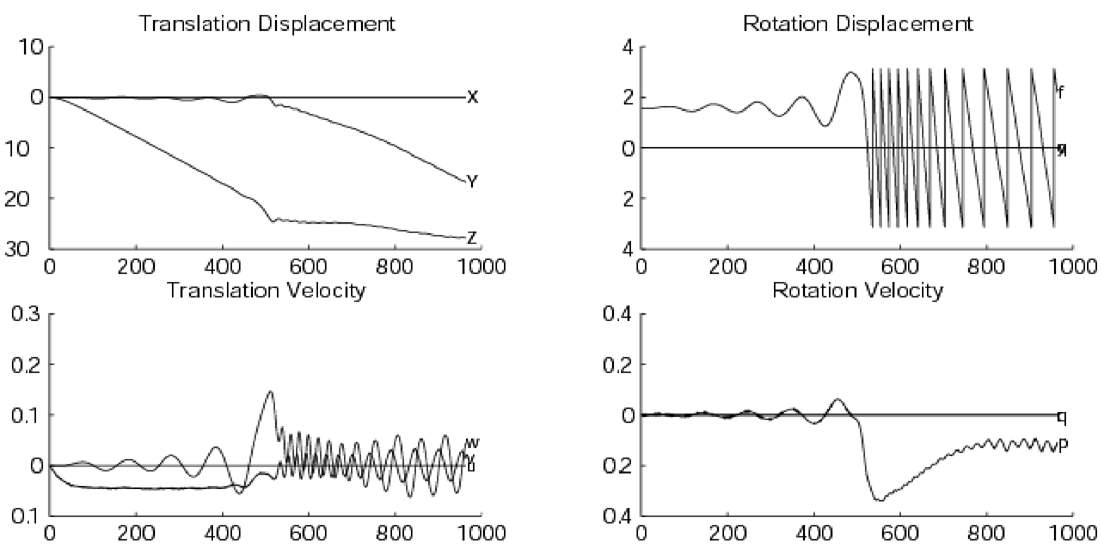


Tumbling Wedge



1-DOF rotate
2-DOF translate





Appears to be generating lift at an L/D of about 4.

Conclusions

- Rigid body solver successfully implemented
- Orientation uses quaternions
- Non-inertial CFD formulation offers advantages for rigid body motions
- Simulations gave qualitatively correct results regardless of the inviscid Euler CFD solver.
- Interesting rigid body dynamics appear correlated with viscous fluid flow.

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Acknowledgments:

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