SUPERSONIC WIND TUNNEL
Project One

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ABSTRACT

Formulas for ratios of temperature, pressure and area of one dimensional isentropic nozzles and of normal shocks are reviewed. A computer program for determining the ratios of temperatures, pressures and areas of subsonic and supersonic nozzles was developed. Data is tabulated and plotted. A supersonic wind tunnel was analyzed using data calculated from the computer program.

INTRODUCTION

Converging-Diverging nozzles combined with the correct pressures and temperatures can accelerate a fluid to supersonic velocities. The magnitudes of velocities, pressures and temperatures cause precise modeling to become complicated quickly due to friction, boundary layers and heat transfer. Assumption of one dimensional isentropic conditions allows for simple relationships between the fluid properties of the nozzle.

The objective is to determine the fluid properties and geometry of a supersonic wind tunnel. A computer program will be developed and used to calculate the isentropic ratios of pressure, temperature and area. A normal shock in the diverging portion of the nozzle will also be analyzed.

THEORY

The flow of fluid in a supersonic wind tunnel can be modeled by assuming isentropic flow, ideal gas ($p = \rho RT$) and constant specific heat. The ratio of specific heats,γ, and the molecular mass specify fully the properties of the fluid due to these assumptions. With these assumptions, ratios of pressure and temperature can be related to the stagnation pressure and temperature at a given Mach number (John).

$$\frac{P}{P_t} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}} \qquad \frac{T}{T_t} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$

Also, the ratio of the current area over the throat area can be derived (John).

$$\frac{A}{A^*} = \frac{\sqrt{\gamma}\left(\frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2-2\gamma}}}{M\sqrt{\gamma}\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2-2\gamma}}}$$

At a shock, the properties of the fluid rapidly changes. The changes in Mach number and the ratio of stagnation pressures are given by (John),

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1} \qquad \frac{p_{t_2}}{p_{t_1}} = \left(\frac{\frac{\gamma+1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(\frac{1}{\frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}}\right)^{\frac{1}{\gamma-1}}$$

METHOD OF CALCULATION

A computer program written in FORTRAN 77 and given in Appendix A was developed to calculate $\frac{P}{P_t}$, $\frac{T}{T_t}$ and $\frac{A}{A^*}$ when given the Mach number. The program first initilizes real variables for the Mach number and gamma. Next, functions for $\frac{P}{P_t}$, $\frac{T}{T_t}$ and $\frac{A}{A^*}$ given M are defined. A value for gamma is set and a header line is written. A DO loop steps through the Mach numbers. At each step, the needed ratios are calculated and the results are written out. The program then exits.

RESULTS AND DISCUSSION

Calculations were performed as described above and are given in Appendix D. For all calculations, γ equals 1.605, the reservoir temperature and pressure are 1000 K and 1 MPa and the molecular weight is 30.25 kg per kg-mole. The wind tunnel is designed to operate at Mach 4.367 with a throat area of 10 cm^2. The computer program outputs an isentropic flow table between M=0 and M=5 for steps of M=0.1, which is given in Table 1 (Appendix B). A flow table for selected Mach numbers was also calculated and is given in Table 2.

The mass flow rate is calculated (Appendix D) as $1.368\frac{kg}{s}$. Because the nozzle is choked and the throat area is fixed, this mass flow rate depends only upon the pressure and temperature of the reservoir. Any increase in pressure will result in a linear increase in mass flow rate. However, any increase in temperature will cause the mass flow rate to decrease as $\dot{m} = \frac{Constant}{\sqrt{T}}$.

For the wind tunnel to operate at Mach 4.367, the test section is required to have an area of $79.57cm^2$. A static temperature of 147.7^K and a static pressure of 6.3^{Kpa} will occur in the test section. Any increase in test section area will result in an increase in Mach number and a decrease in static temperature and pressure.

Plots of A,M,V,p,T and ρ versus distance x are given in Figures 1 through 6 (Appendix C). Between Mach zero and Mach one, the cross sectional area equals the area of the throat divided by the distance x, $A(x) = \frac{A_{throat}}{x}$. This enhances the low Mach number, large area flow portion of the plot. Above Mach 1, the area increases linearly as a function of x,

$A(x) = 80x - 70$. In the plots, the distance is measured in meters; however, any unit of distance can be used. The plots assumes constant test section properties will continue past the last data point. This corresponds with a constant test section area. All figures show that the properties change rapidly in the vicinity of the throat with the maximum rate of change at Mach 1.

When a normal shock exists at A=$20cm^2$, the test section becomes subsonic. From the calculations in Appendix D, the test section will have a Mach number of 0.12 and a pressure of 609^{Kpa}. The temperature will be 995.7^K. With an increasingly lower back pressure, the shock will move towards the exit until the entire test section is forward of the shock.

CONCLUSION

Assumption of a one dimensional isentropic condition gives a method for determining velocities, pressures and temperatures for subsonic and supersonic nozzles that depends upon simple calculations. While this method does find approximate solutions, its ultimate precision is limited by the presence of a real fluid. The computer program allows for ease in calculating the fluid property ratios and allows fluids with varied molecular weights and ratios of specific heats to be used. The assumption of one dimensional isentropic flow yields results suitable for a first calculation of a nozzle's properties.

REFERENCES

John, James E. A. (1984)

Gas Dynamics, Prentice Hall.

APPENDIX A

Computer Program

APPENDIX B

Tables

APPENDIX C

Figures

APPENDIX D

Calculations

Mass Flow Rate:

From theory, the mass flow rate is $\dot{m} = \frac{p}{RT} MA\sqrt{\gamma RT}$ and $R = \frac{\bar{R}}{M} = .2749\frac{KJ}{KgK}$

The Mach number is 1 at the throat. From Table 1, $\frac{T}{T_t} = 0.7678$ and $\frac{P}{P_t} = 0.4960$

$$\dot{m} = \frac{(1000)(.4960)(10)}{(0.2749)(.7678)(1000)(100^2)}\sqrt{(1.605)(.2749)(1000)(1000)(.7678)} = 1.368\frac{kg}{s}$$

Test Section: Area, Pressure, Temperature

The wind tunnel is designed to operate at M=4.367 with a throat area of 10 cm^2. From Table 2 at M=4.37, $\frac{A}{A^*}$ equals 7.957. A^* equals 10 cm^2, so that the area of the test section is $(7.957)(10) = 79.57cm^2$. $\frac{P}{P_t}$ equals 0.0063 so the pressure of the test section is $(0.0063)(1000) = 6.3^{Kpa}$. $\frac{T}{T_t}$ equals 0.1477 so the test section temperature is $(0.1477)(1000) = 147.7^{K}$.

Normal Shock at $20cm^2$:

With an area ratio of $\frac{A}{A^*} = 20/10 = 2$, Table 2 gives Mach 2.35 just before the shock. Solving for the Mach number after the shock yields $M_2 = 0.5585$ and 0.6165 for the ratio of stagnation pressures. At M=.5585, $\frac{A}{A^*}$ equals 1.2308 and $\frac{A_{exit}}{A_{shock}}$ equals $\frac{79.57}{20} = 3.979$ so that $\frac{A_{exit}}{A^*}$ is 4.8974. From Table 2, this corresponds to M=0.12 at the exit. $\frac{P}{P_t}$ equals 0.9885 so the pressure of the test section is $(0.9885)(1000)(.6165) = 609^{Kpa}$. $\frac{T}{T_t}$ equals 0.9957 so the test section temperature is $(0.9957)(1000) = 995.7^{K}$.