

Aerodynamics I

AEM 313

# Lesson 1

Nomenclature  
and  
Reference frames

It's easy to explain how a rocket works,  
but explaining how a wing works takes a  
rocket scientist.

— Philippe Spalart

The answer!

$$2\pi$$

Corrected for aspect ratio

Corrected for Mach #

and Reynolds #

But only valid within a linear range of AOA

Except when decambered through ....

With an unsteady term associated with the instantaneous lift and ....

Local transonic regions change the fundamental solution type.

But only valid within a range of  $K_n$  sufficient for continuous approach.

Don't forget about turbulence and the Kolmogorov scale

Leading edge stall

Corrected for an equivalent roughness

Wing tip shape influences the tip role force

Except in the presence of vortex generators

Standard day vs Hot vs tropic vs ....

Upper surface suction

Flap gain schedules, trim ipsun

But air isn't truly an ideal gas.

Ice formation

But past the drag divergence H ....

Bombard layer separation

Hysteresis

Tunnel turbulence

Crit drag

Eiffel 10 (Wright)



Gottingen 346



CLARK Y



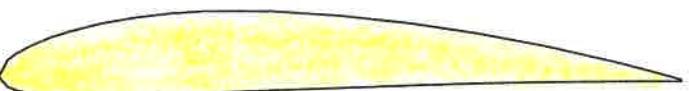
NACA Munk M-4 airfoil



NACA 0012



NACA 4412



See + understand

EPPLER 625



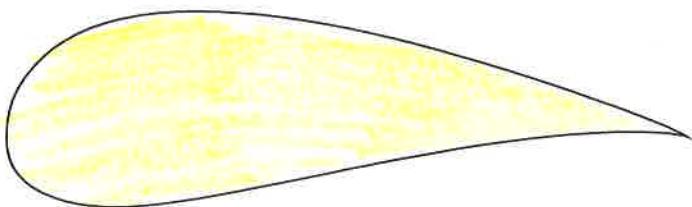
WORTMANN FX 2



AG455ct02r



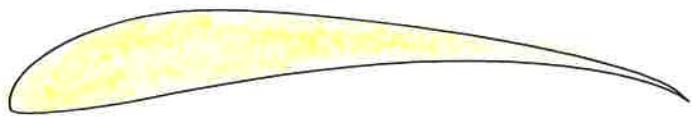
FX 69-PR-281



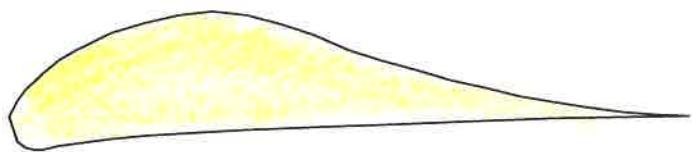
NASA SC(2) 0010



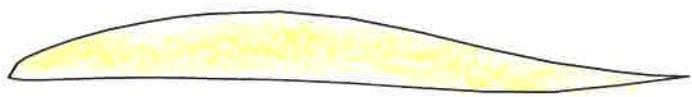
Selig 1223



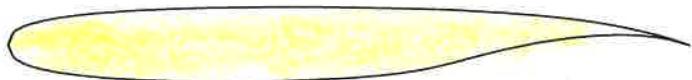
Liebeck L1003



NACA 6-H-10 Rotorcraft



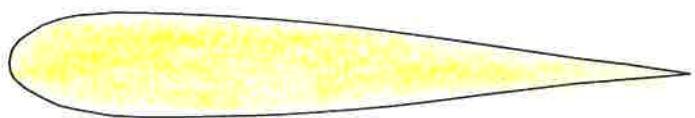
Whitcomb Supercritical



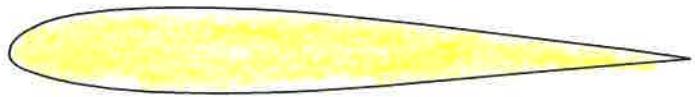
Roncz 1080 Voyager



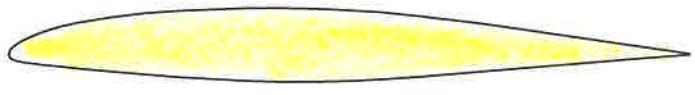
Boeing 737 Root



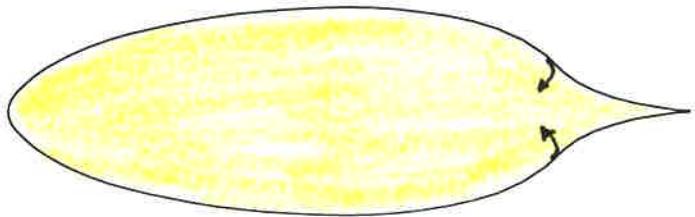
Boeing 737 Midspan



Boeing 737 Tip

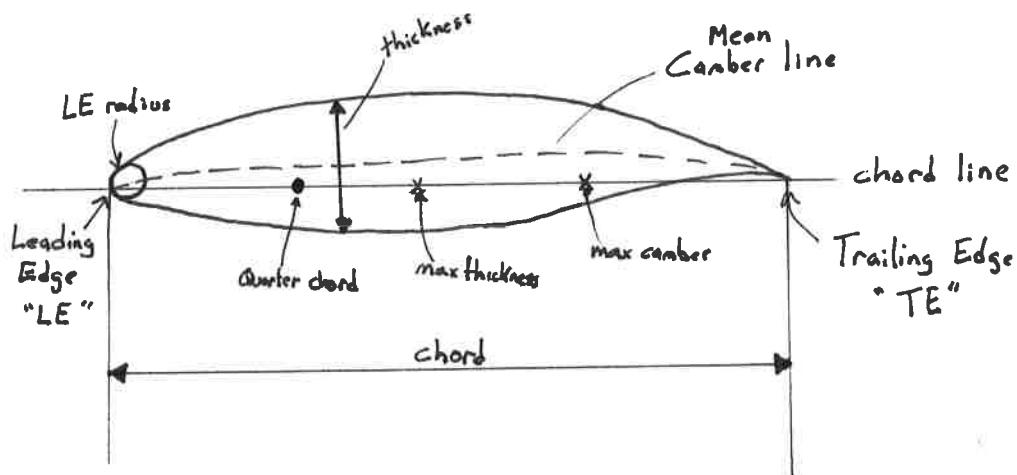


Griffith Suction Airfoil



## Airfoil:

- Two-dimensional closed curve
- Cross section of a wing
- Fluid boundary condition (usually air-solid or water-solid)



Thickness ratio:

$$T = \frac{\text{Max thickness}}{\text{chord}} \quad \text{usually given in percent (e.g. 12%)}$$

Above FX 2 is  $\approx 20\%$

LE radius:

Often, the leading edge is specified as a circular radius to prevent sharp edges in the curve's definition.



Chord line:

The chord line is defined as the line connecting the LE and TE.

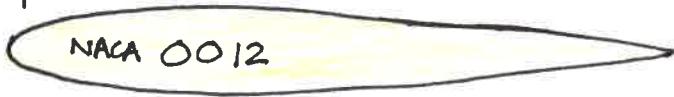
NACA "En - Ay - Cee - Ay"

4 digit airfoils

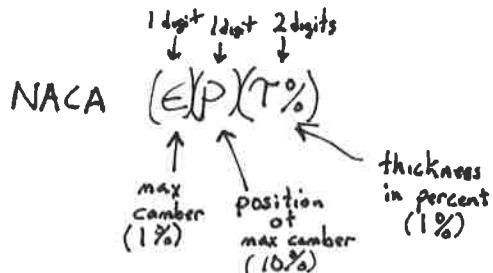
"When the NACA four digit wing sections were derived, it was found that the thickness distributions of efficient wing sections such as the Göttingen 398 and the Clark Y were nearly the same when their camber was removed ... and they were reduced to the same maximum thickness."

Theory of Wing Sections - Abbott and von Doenhoff

Example:



Code:



Thickness

$$T = 10T_c \left( 0.2969 \sqrt{\frac{x}{c}} - 0.12600 \frac{x}{c} - 0.35160 \left(\frac{x}{c}\right)^2 + 0.28430 \left(\frac{x}{c}\right)^3 - 0.10150 \left(\frac{x}{c}\right)^4 \right)$$

- This  $T$  gives a blunt TE. Some people replace the 2nd order term to give a coincident TE ( $-0.35160 \left(\frac{x}{c}\right)^2 \rightarrow -0.3537 \left(\frac{x}{c}\right)^2$ ), but technically the definition is correct. (cf. NACA TN 385).
- Max thickness at 30%.

Camber line

$$y_c = \frac{Ex}{p^2} \left( 2p - \frac{x}{c} \right) \quad \text{for} \quad 0 \leq \frac{x}{c} \leq p$$

$$y_c = \frac{E(c-x)}{(1-p)^2} \left( 1 + \frac{x}{c} - 2p \right) \quad \text{for} \quad p \leq \frac{x}{c} \leq 1$$

2 parabolas  
joined at  
max camber pt.

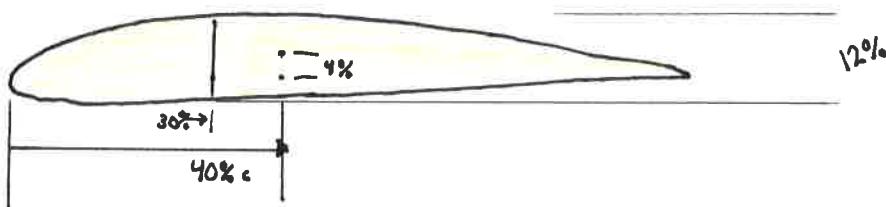
Curves

$$y^+ = y_c + \frac{1}{2}T$$

$$y^- = y_c - \frac{1}{2}T$$

Example:

NACA 4412: 4 digit NACA airfoil with 4% chord max camber at 40% chord with 12% thickness



Q: Can you have a non-canonical NACA 4 digit airfoil?

$$\epsilon = 4.5\%$$

$$\rho = 25\%$$

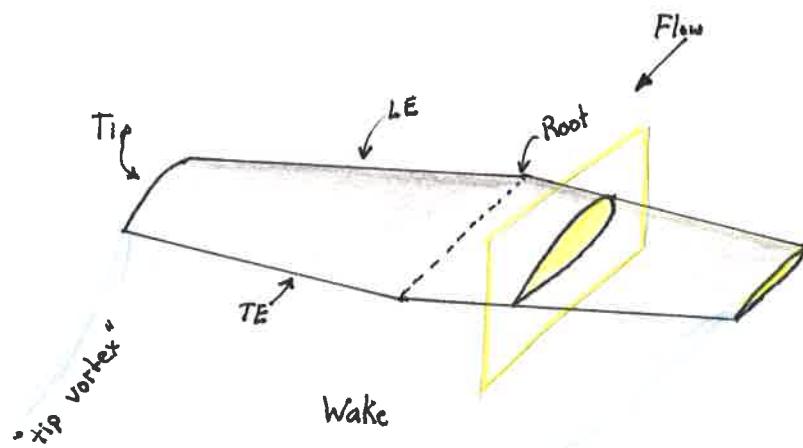
$$\tau = 11.25\%$$

A: Yes, but it won't be describable by the 4 digit code.

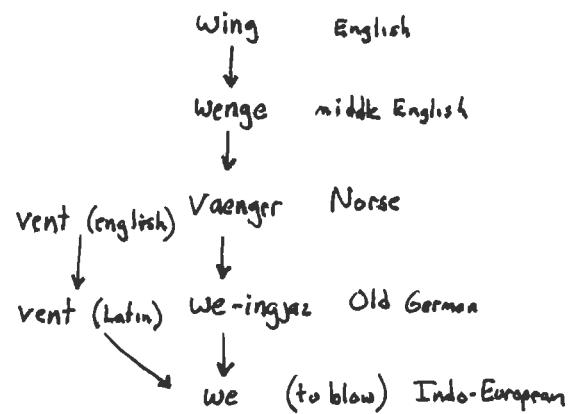
A2: Yes, the shapes are continuous. The code is not.

# Wing:

- Three dimensional closed surface
- Generates aerodynamic force
- Cross sections are airfoils

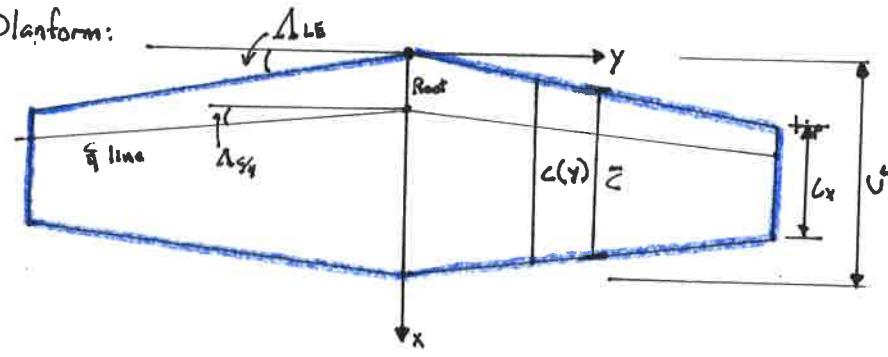


## Etymology:

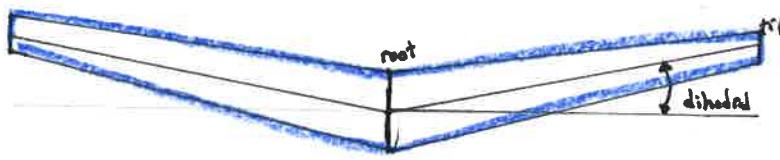


Interestingly, not from Indo-European word for "fly" which is "petr" from which we get "feather" and "pen".

## Planform:



Front view



$$S \equiv \text{Wing Area} \quad [L^2]$$

$$b \equiv \text{Span} \quad [L]$$

$$c \equiv \text{chord}$$

$$AR \equiv \text{Aspect Ratio} \equiv \frac{b^2}{S}$$

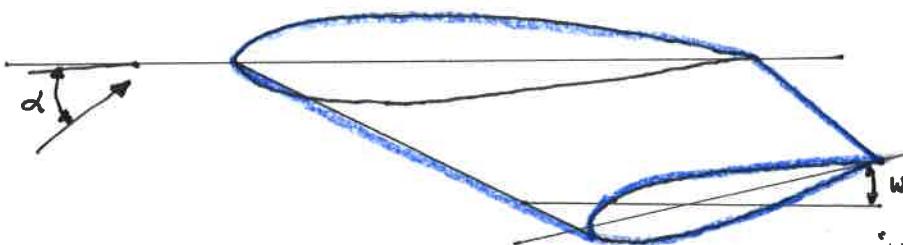
$$\lambda \equiv \text{taper ratio} = \frac{C_r + t_{\text{tip}}}{C_r}, \left( \frac{t_{\text{tip}}}{C_r} \right)$$

$$MAC \equiv \text{Mean Aerodynamic Chord}$$

$$\frac{1}{S} \int_{-b/2}^{b/2} c(y)^2 dy$$

$$\bar{c} \equiv \text{Average Chord}$$

Side view

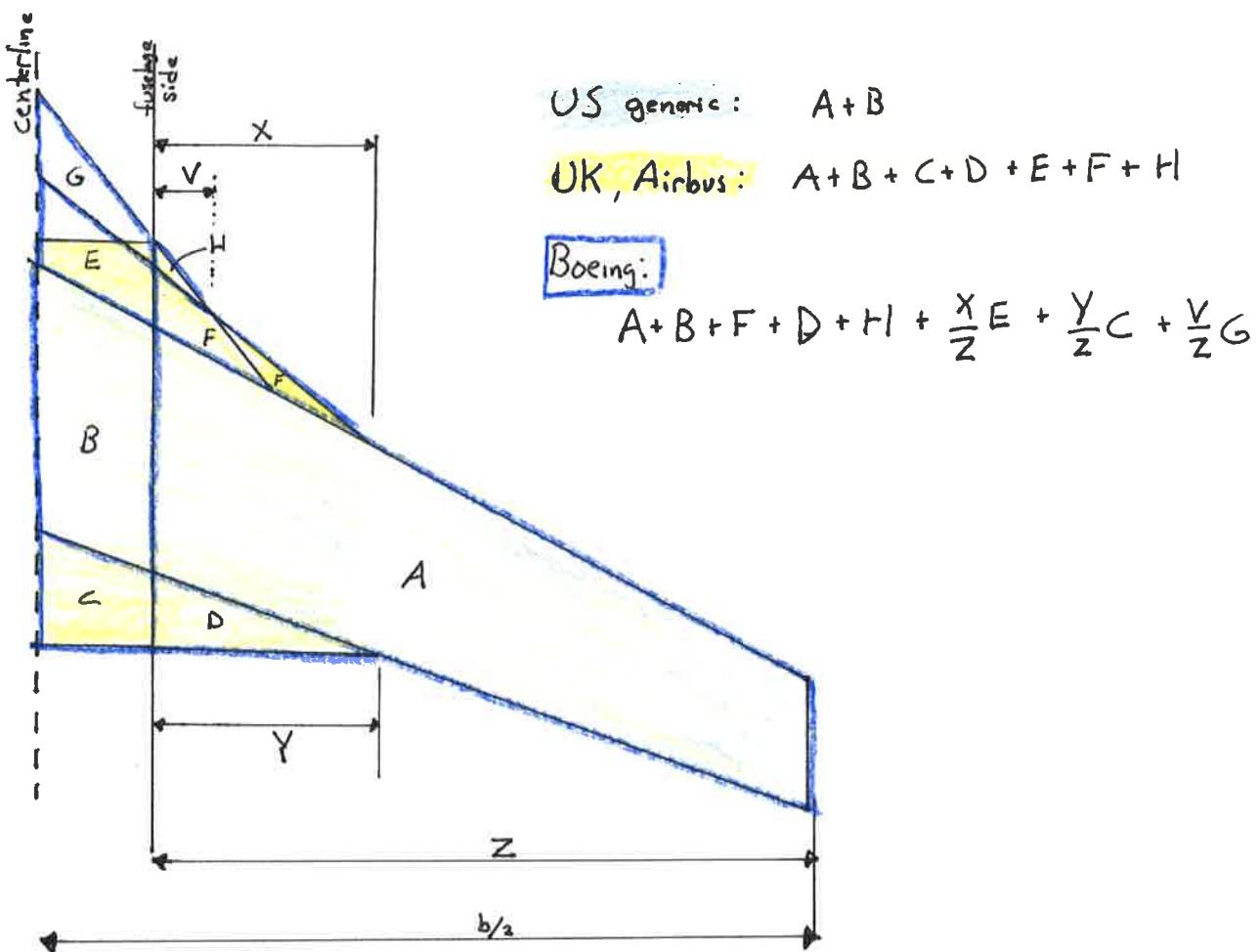


"wash out" = decrease in incidence angle at tip

"wash in" is a positive increase ... usually a bad idea

# Wing Reference Area

ADTA p 269



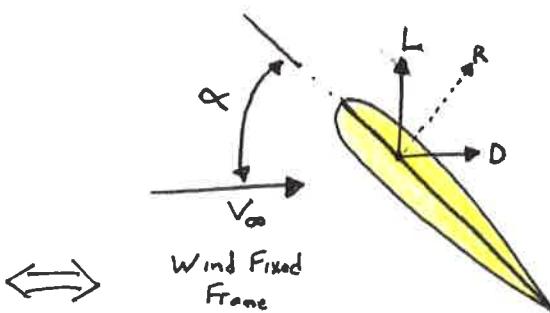
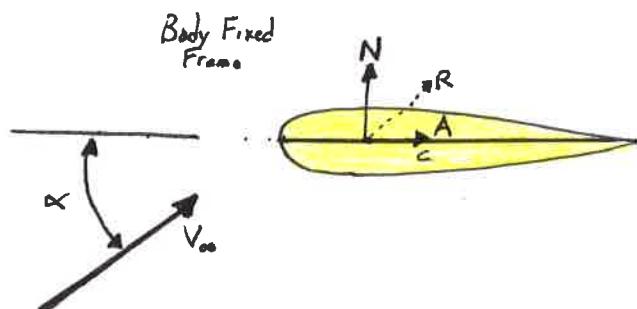
Or,  $S$  could be a planform projection from a CAD model.

Or,  $S$  could be the reference area of a previous version (i.e. historical).

Why is this important?

- Single most important ref #
- Compare performance

# Aerodynamic Forces and Moments



$\alpha \equiv$  angle of attack = "alpha"

$V_\infty \equiv$  Freestream Velocity = "Vee Infinity"

$N \equiv$  Normal force in body fixed frame perpendicular to  $c$

$A \equiv$  Axial force in body fixed frame parallel to  $c$

$L \equiv$  Lift force perpendicular to  $V_\infty$

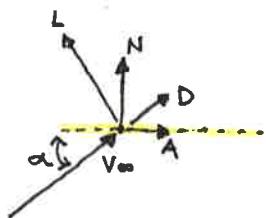
$D \equiv$  Drag force parallel to  $V_\infty$

} Body frame

} Wind frame

## Frame conversion

$$R^2 = N^2 + A^2 = L^2 + D^2$$



$$L = N \cos \alpha - A \sin \alpha$$

$$D = N \sin \alpha + A \cos \alpha$$

$$= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{pmatrix} N \\ A \end{pmatrix} = \begin{pmatrix} L \\ D \end{pmatrix}$$

well known 2D  
coordinate transform  
in matrix notation

Q: Transform from  $L$  and  $D$  to  $N$  and  $A$ ?

$$\begin{pmatrix} L \\ D \end{pmatrix} = [T] \begin{pmatrix} N \\ A \end{pmatrix} \Rightarrow T^{-1} \begin{pmatrix} L \\ D \end{pmatrix} = \begin{pmatrix} N \\ A \end{pmatrix}$$

The inverse of a  $2 \times 2$  is trivial

- 1) Swap diagonals
- 2) Negative of off diagonals
- 3) Divide by determinant

$$\Rightarrow T^{-1} = \frac{\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}}{\cos^2 \alpha + \sin^2 \alpha}$$

Q: In a wind tunnel, a wing is mounted at  $30^\circ$  angle of attack.

The normal force is 50 lbs. The axial force is -10 lbs.

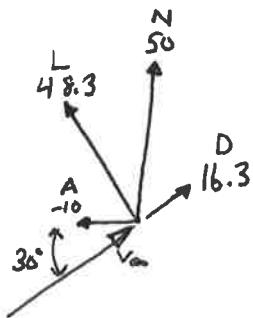
What is the lift to drag ratio?

$$L = N \cos \alpha - A \sin \alpha = 50 \cos(30^\circ) - (-10) \sin(30^\circ)$$
$$= 48.3 \text{ lbs}$$

$$D = N \sin \alpha + A \cos \alpha = 50 \sin(30^\circ) + (-10) \cos(30^\circ)$$
$$= 16.3 \text{ lbs}$$

$$\frac{L}{D} = \frac{48.3}{16.3} = 2.95$$

$$\boxed{\frac{L}{D} = 2.95}$$



One sentence commentary ....

# Units

The U.S. aerospace industry uses English units. You need to work in both SI and English comfortably. I can evaluate aero engineers by their capability of working in English units!

Ex:

$$1 \text{ Newton} \equiv 1 \text{ Kg} \cdot 1 \frac{\text{m}}{\text{s}^2}$$

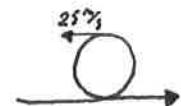
mass      acceleration

Ex: Your professor has a mass of 60 kg.

1) What is his weight on Earth?

$$W = m \cdot a = \frac{60 \text{ kg}}{\text{s}^2} \cdot \frac{\text{N}}{\text{kg} \cdot \text{m}} = 588 \text{ N}$$

2) What is his weight flying his C-172 at 2000 ft/s 25 m/s at the top of a 500 m loop?



$$a = \frac{V^2}{R} - g_0 \Rightarrow W = m \left( \frac{V^2}{R} - g_0 \right)$$

Ex:

$$1 \text{ lbf} \equiv 1 \text{ slug} \cdot 1 \frac{\text{ft}}{\text{s}^2}$$

↑ pronounced "pound"  
lb comes from Latin "libra"  
meaning scales. Thus the  
pound is by etymology a weight

$$1 \text{ lbf} \equiv 1 \text{ slinch} \cdot 1 \frac{\text{in}}{\text{s}^2}$$

Ex: Your professor weighs 132 lbf. What is his mass in slugs?

$$M = \frac{W}{g} = \frac{132 \text{ lbf}}{32.174 \text{ ft}} = \frac{\text{lbf}}{\frac{\text{ft}}{\text{s}^2}} = \frac{\text{slug ft}}{\text{lbf s}^2} = 4.1 \text{ slugs}$$

Ex: How many slinches?

$$M = \frac{W}{g} = \frac{132 \text{ lbf}}{32.174 \text{ ft}} = \frac{\text{lbf}}{\frac{\text{ft}}{\text{s}^2}} = \frac{\text{slug in}}{\text{lbf s}^2} = 0.34 \text{ slinches}$$

Q:

Why on Earth would anyone use slinches?

A: Aircraft are designed in the U.S. with the inch as the reference length.

Q: What about the pound mass used in thermodynamics?

$$F = m \frac{a}{g_c} \Rightarrow 1 \text{ lbf} = \frac{1 \text{ lbm}}{32.174 \text{ ft}} \cdot \frac{\text{s}^2 \text{lbf}}{\text{lbf} \cdot \text{ft}} = \frac{1}{g_c}$$

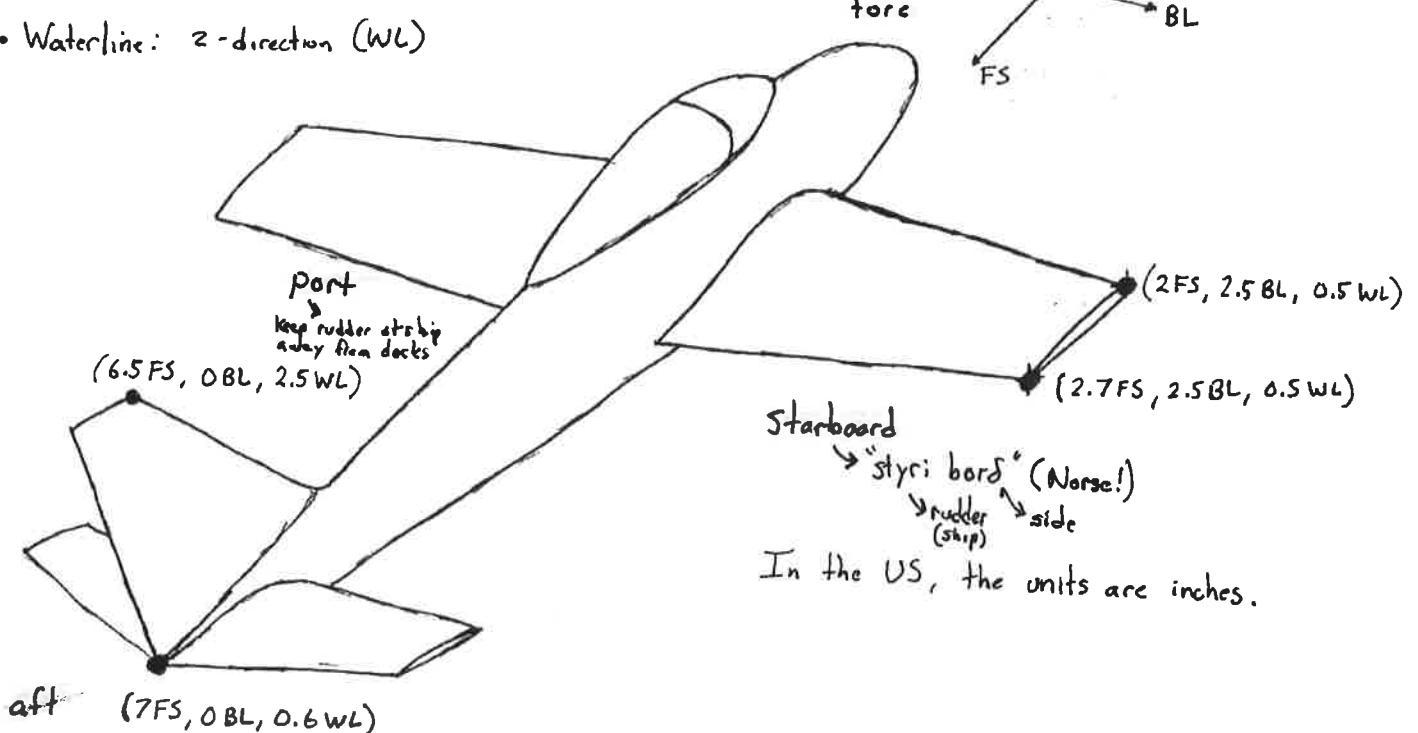
We will avoid lbm.

SI has a similar monstrosity called the kg force.

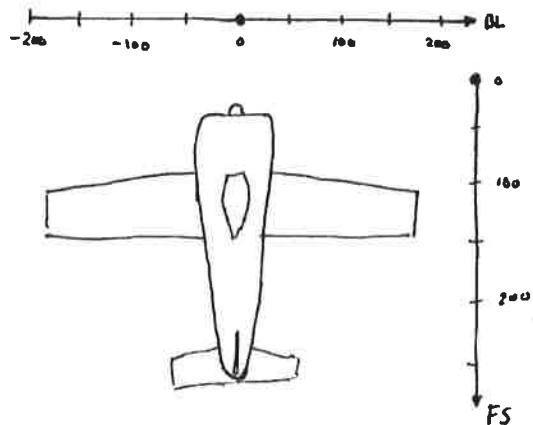
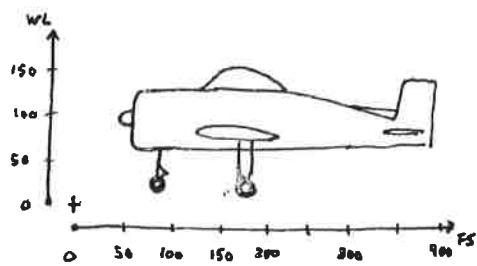
~~NaCl~~

## Locations on an Aircraft (Aircraft Station Coordinates)

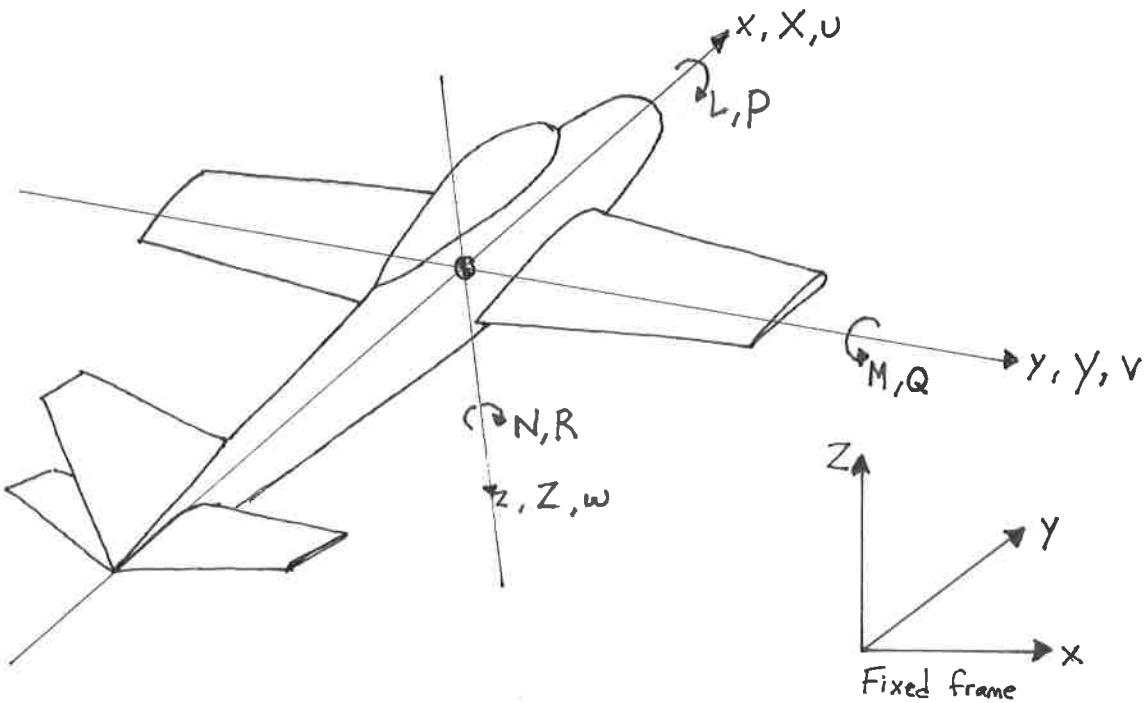
- Fuselage Station: along <sup>x-direction</sup> fuselage (FS)
- Butt line: <sup>y-direction</sup> (BL)
- Waterline: <sup>z-direction</sup> (WL)



- Zero FS is usually not the most forward location of the aircraft. Rather the origin is placed arbitrarily forward such that negative FS does not occur.
- Zero BL is usually along the centerline
- Zero WL is usually placed such that all values are positive



# Aircraft Coordinate System



$X, Y, Z$  Aircraft "stability" frame location

$X, Y, Z$  Forces in stability frame

$u, v, w$  Velocity in stability frame

$L, M, N$  moment in stability frame

$P, Q, R$  Angular velocities (roll, pitch, yaw)

$\phi, \theta, \psi$  Euler angles (orientation)

Warning:

Outside of aerodynamics, the x axis is usually down the length of the a/c.

$$X_{\text{lift}} = -X_{\text{aero stability}}$$

$$Y_{\text{lift}} = Y_{\text{aero stability}}$$

$$Z_{\text{lift}} = -Z_{\text{aero stability}}$$

# Euler Angles $\phi, \theta, \psi$ (order dependent!)

~~global to local~~  
~~local to global~~: yaw( $\psi$ ), pitch( $\theta$ ), roll( $\phi$ )  
~~local to global~~  
~~global to local~~: roll( $\phi$ ), pitch( $\theta$ ), yaw( $\psi$ )

Yaw

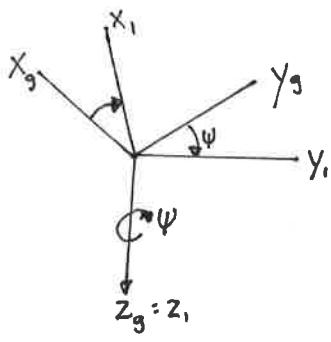


Diagram showing a coordinate system transformation from global to local (yaw). A vertical z-axis  $z_g = z_1$  is rotated by  $\psi$  around its own axis. The resulting local axes are  $x_1$ ,  $y_1$ , and  $y_g$ . The angle  $\psi$  is indicated at the origin.

$$\bar{X}_1 = \underbrace{\begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_\psi^{1g}} \bar{X}_g$$

Pitch

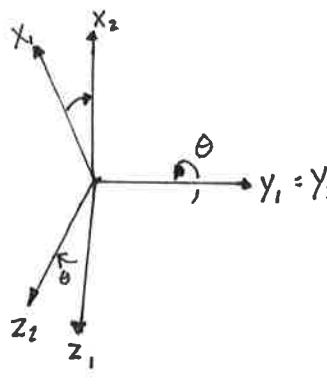


Diagram showing a coordinate system transformation from local to global (pitch). A horizontal y-axis  $y_1 = y_2$  is rotated by  $\theta$  around its own axis. The resulting global axes are  $x_2$ ,  $y_2$ , and  $z_1$ . The angle  $\theta$  is indicated at the origin.

$$\bar{X}_2 = \underbrace{\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}}_{T_\theta^{1g}} \bar{X}_1$$

Roll

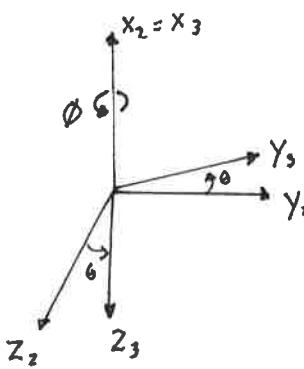


Diagram showing a coordinate system transformation from global to local (roll). A vertical x-axis  $x_2 = x_3$  is rotated by  $\phi$  around its own axis. The resulting local axes are  $x_2$ ,  $y_3$ , and  $z_2$ . The angle  $\phi$  is indicated at the origin.

$$\bar{X}_3 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}}_{T_\phi^{1g}} \bar{X}_2$$

Global

$$\bar{X}_g = \underbrace{T_\psi^{1g} T_\theta^{1g} T_\phi^{1g}}_{T^{1g}} \bar{X}_l$$

$$T^{lg} = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix}$$

A beautiful property of transformation matrices is

$$T^{gl} = (T^{lg})^{-1} = T^{lg T}$$

The inverse is the transpose!

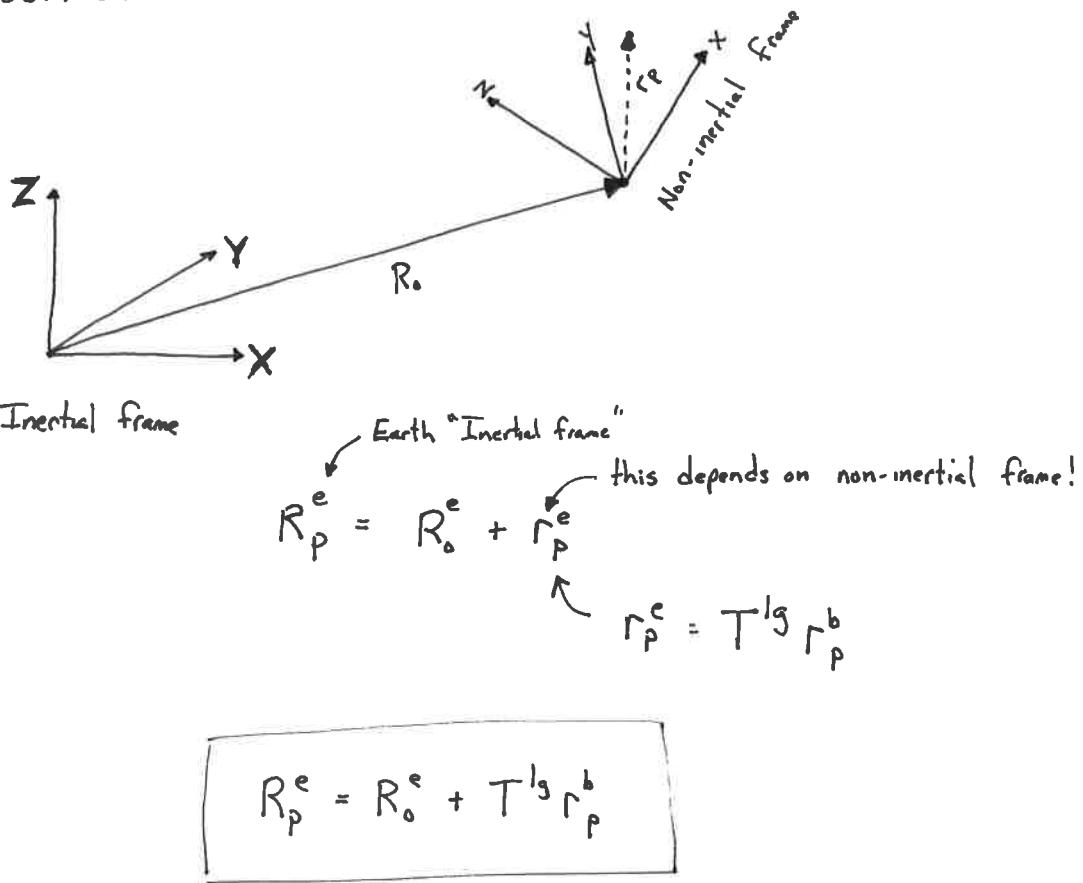
Ex:

Given a global frame velocity of  $\bar{V} = (1, 0, 0)$  and the orientation Euler angles of  $(\phi, \theta, \psi) = (0, 10^\circ, 10^\circ)$ , what is the body frame velocity vector  $(u, v, w)$ ?

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = T^{gl} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = T^{lg T} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0.9698 \\ -0.1413 \\ 0.1986 \end{pmatrix}$$

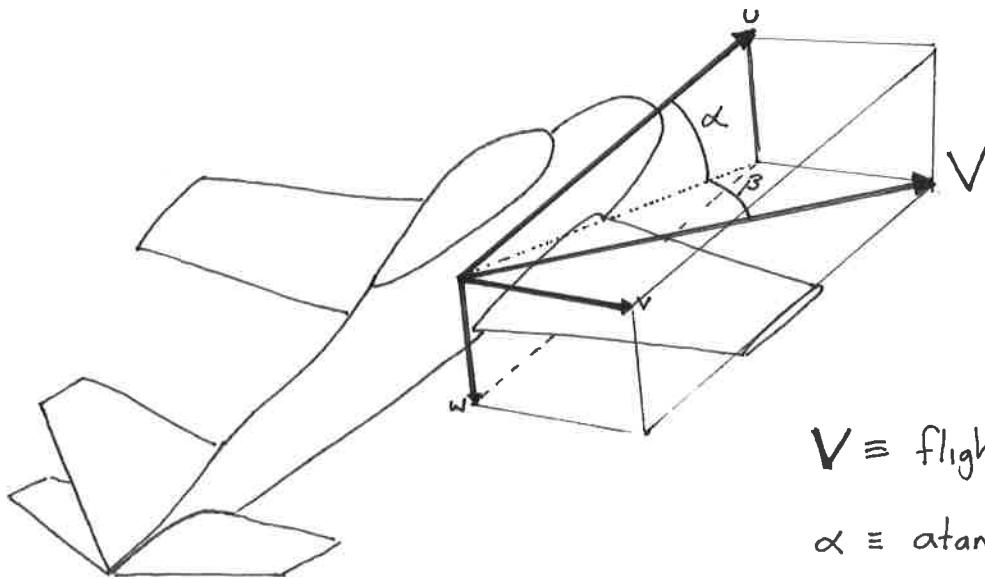
# Body Position



# Velocities

$$\begin{aligned} \frac{dR_p^e}{dt} &= V_p^e = \frac{dR_o^e}{dt} + \frac{dT^{1g}}{dt} r_p^b + T^{1g} \frac{dr_p^b}{dt} \\ &= V_o^e + \dots \\ &= T^{1g} V^b + \dots \\ &= T^{1g} V^b + \underbrace{\frac{dT^{1g}}{dt} r_p^b + T^{1g} \frac{dr_p^b}{dt}}_{\text{All terms in body frame!}} \end{aligned}$$

# Angle of Attack and Sideslip



$V \equiv$  flight velocity vector

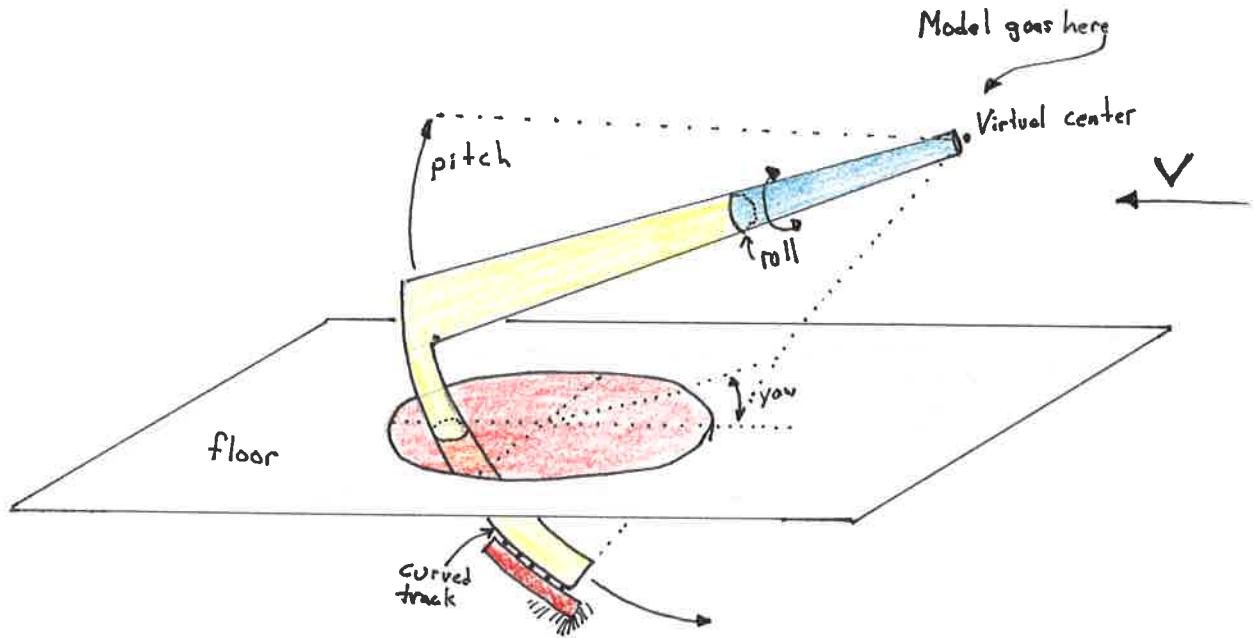
$\alpha \equiv \tan^{-1} \left( \frac{w}{u} \right)$  Angle of Attack

$\beta \equiv \sin^{-1} \left( \frac{v}{V} \right)$  Sideslip

$\alpha$  is defined wrt the projection of  $V$  onto the body frame (i.e.  $u$ )

$\beta$  is defined wrt the  $v$  projection and  $V$ .

# Wind - Tunnel Mount



What order is the transformation?

Floor  $\rightarrow$  Yaw  $\rightarrow$  pitch  $\rightarrow$  Roll (i.e. Euler angles!)

Ex: What mount rotations are needed for  $\alpha = 30^\circ$ ,  $\beta = 10^\circ$ ?

$$\alpha = \text{atan}\left(\frac{\omega}{\dot{v}}\right) = 30^\circ$$

$$\beta = \text{asin}\left(\frac{v}{\bar{V}}\right) = 10^\circ$$

$$\text{so, } v = \bar{V} \sin \beta$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \bar{V} \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix}$$

and

$$u^2 + \bar{V}^2 \sin^2 \beta + v^2 \tan^2 \alpha = \bar{V}^2 \Rightarrow u = \bar{V} \sqrt{\frac{1 - \sin^2 \beta}{1 + \tan^2 \alpha}} \Rightarrow w = \bar{V} \sqrt{\frac{1 - \sin^2 \beta}{1 + \tan^2 \alpha}} \tan \alpha$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0.852869 \\ 0.173648 \\ 0.492404 \end{pmatrix}$$

if  $\phi = 0^\circ$  (no roll)

$$\begin{aligned} \theta &= \alpha = 30^\circ \\ \psi &= -\beta = -10^\circ \end{aligned}$$

$\phi$  is an independent variable

Verify  $\theta = \alpha$   $\psi = -\beta$  when  $\phi = 0^\circ$  no roll

$$v = V C_\theta C_\psi$$

$$v = V (\cancel{S_\phi}^{\circ} \dots - \cancel{C_\phi}^1 S_\psi) = -V S_\psi$$

$$\omega = V (S_\theta C_\psi + 0) = VS_\theta C_\psi$$

$$\alpha = \tan\left(\frac{\omega}{v}\right) = \tan\left(\frac{VS_\theta C_\psi}{VC_\theta C_\psi}\right) = \tan(\tan(\theta))$$

$$\boxed{\alpha = \theta}$$

$$\beta = \sin\left(\frac{v}{\bar{v}}\right) = \sin\left(-\frac{VS_\psi}{V}\right) = \sin(-S_\psi) = -\psi$$

$$\boxed{\beta = -\psi}$$

# A demonstration of Gimbal Lock.

The transformation matrix when pitch  $\theta = 90^\circ$  is

$$T^g = \begin{bmatrix} 0 & S_\phi C_\psi - C_\phi S_\psi & C_\phi C_\psi + S_\phi S_\psi \\ 0 & S_\phi S_\psi + C_\phi C_\psi & C_\phi S_\psi - S_\phi C_\psi \\ -1 & 0 & 0 \end{bmatrix}$$

Remember that

$$\cos(A \mp B) = \cos(A)\cos(B) \pm \sin(A)\sin(B)$$

and

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

Substitute

$$T^g = \begin{bmatrix} 0 & S(\phi - \psi) & C(\phi - \psi) \\ 0 & C(\phi - \psi) & -S(\phi - \psi) \\ -1 & 0 & 0 \end{bmatrix}$$

Uh-oh!  $T^g$  is only a function of  $\phi - \psi$

We can not distinguish between  $\phi$  and  $\psi$  when  $\theta = 90^\circ$ !

# Apollo 11 transcript

(0000 NLP 1)

Tape 67/11

Page 337

- 04 08 46 11 CC Hello, Tranquility Base. Houston. On our DPS venting and that fuel problem, our heat exchanger is cleared up. We heard that the ice is melted, and we are in good shape now. Out.
- 04 08 49 39 CDR (TRANQ) Houston. Tranquility is going to put the track modes in P00 now.
- 04 08 59 27 CC Columbia, Houston. Over.
- 04 08 59 34 CMP (COLUMBIA) Columbia. Go.
- 04 08 59 35 CC Columbia, Houston. We noticed you are maneuvering very close to gimbal lock. I suggest you move back away. Over.
- 04 08 59 43 CMT (COLUMBIA) Yes. I am going around it, doing this CMC AUTO maneuvers to the PAD values of roll 270, pitch 101, yaw 45.
- 04 08 59 52 CC Roger, Columbia.
- 04 09 00 30 CMP (COLUMBIA) How about sending me a fourth gimbal for Christmas.
- 04 09 00 40 CC Columbia, Houston. You were unreadable. Say again please.
- 04 09 00 46 CMP (COLUMBIA) Disregard.
- 04 09 01 21 CC Columbia, Houston. Several items for you. Over.
- 04 09 01 28 CMT (COLUMBIA) Ready to copy.
- 04 09 01 30 CC Columbia, Houston. First of all, we'd like a waste-water dump to 10 percent on the backside. Secondly, it does not look like we are going to need any plane change at this time, so we will not be uplinking a new REFSMMAT. Third item, I would like all of your CRYO heaters to AUTO, and we are ready for a battery charge, battery BRAVO; it will last about 7 hours. If you should go to sleep, we will be terminating that BATT charge, but at the moment, we can go ahead and start the BATT charge on BATT Bravo. And a final item, for your SM RCS configuration for your rest period, register 1 for the DAP is 11111; DAP register 2, 01100. And your AUTO RCS select
- ↑ ↑ ↑ ↑  
Days hours min seconds  
Since liftoff

CC = CAP COM

CDR = N. Armstrong

CMP = M. Collins