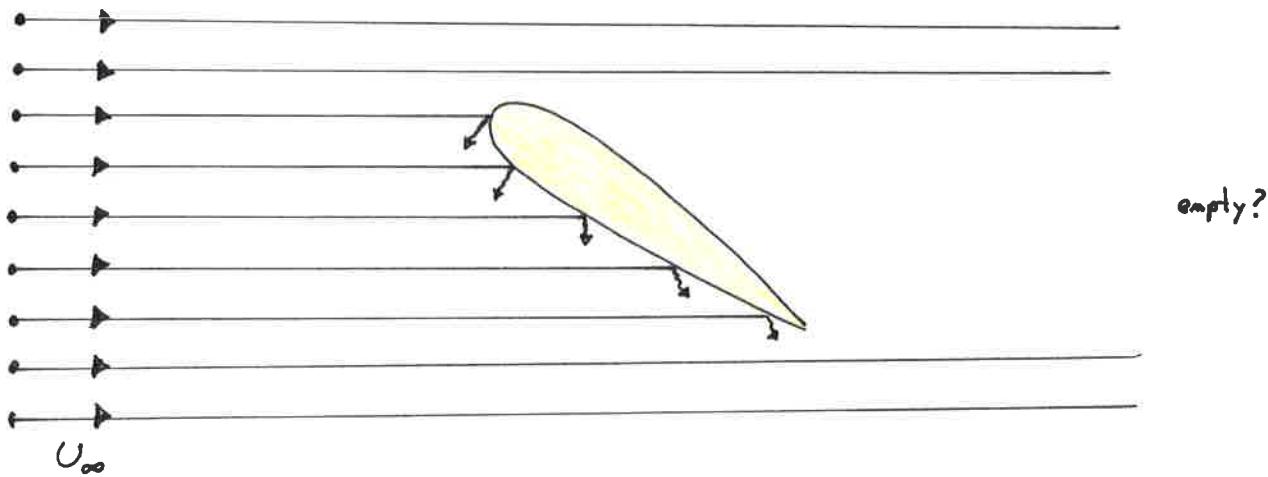


Lesson 2

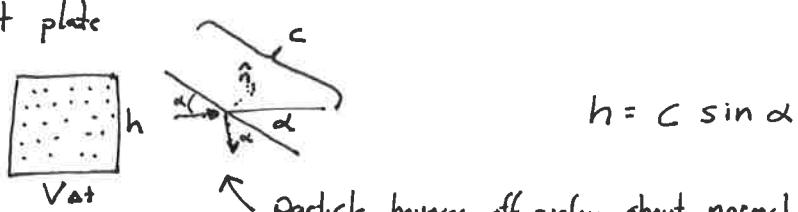
Ideal Gas

# Newton's Lift Model

Non-interacting kinetics



For a simple flat plate



particle bounces off surface about normal  $\hat{n}$  (exit angle is  $2\alpha$ )  
[Historical note, Newton actually used a momentum loss / sliding on surface!]

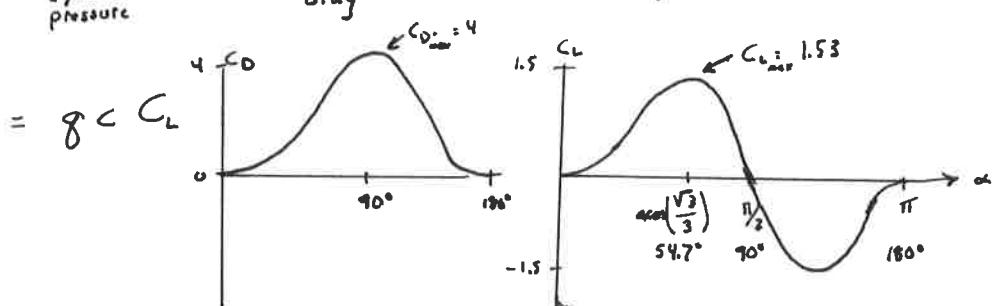
Impulse ( $J = \Delta p = \sigma(mv)$ )

$$J = \underbrace{\rho V \Delta t h}_{\text{mass}} \underbrace{V}_{\text{momentum}} \left[ (\cos 2\alpha - 1) \uparrow - \sin 2\alpha \uparrow \right]$$

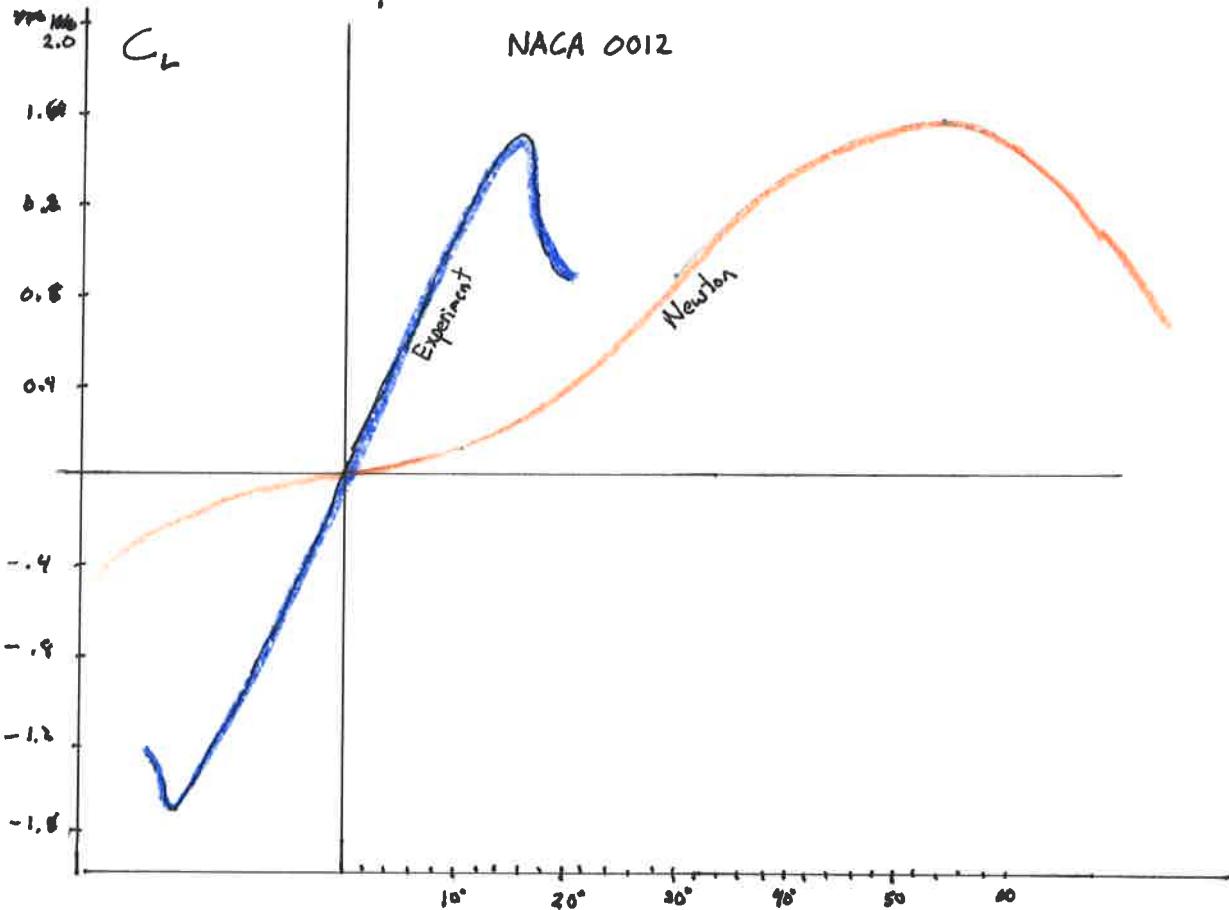
Average Force

$$F = -\frac{J}{\Delta t} = \rho V^2 C \sin \alpha \left[ (\cos 2\alpha - 1) \uparrow - (\sin 2\alpha) \uparrow \right]$$

$$= \frac{1}{2} \rho V^2 C \left[ \underbrace{(4 \sin^3 \alpha) \uparrow}_{\text{drag}} + \underbrace{(4 \sin^2 \alpha \cos \alpha) \uparrow}_{\text{lift}} \right]$$



Does this match experimental data?



Wrong Physics

What does it get wrong?

- 
- 
- 
- 
- 

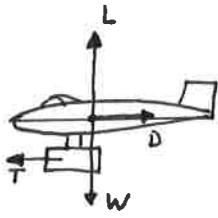
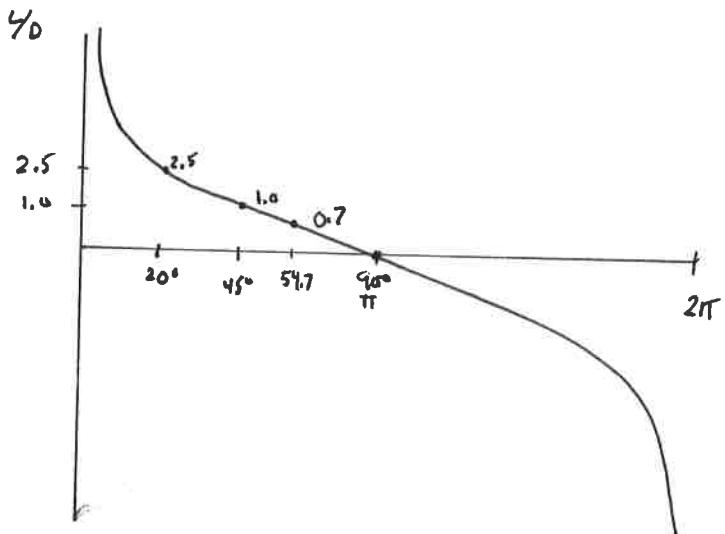
What does it get right?

- 
-

$$\frac{L}{D}$$

(i.e. how powerful of a propulsion system do we need?)  
ie. Thrust gain / amplifier.

$$\frac{L}{D} = \frac{4 \sin^2 \alpha \cos \alpha}{4 \sin^3 \alpha} = \frac{4 \cos \alpha}{4 \sin \alpha} = \frac{1}{\tan \alpha}$$



$$T = D$$

$$W = L$$

$$T = D : L \left( \frac{L}{D} \right)^{-1} = W \left( \frac{L}{D} \right)^{-1}$$

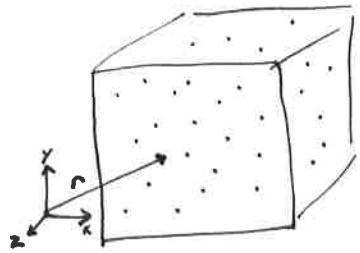
$$\therefore \frac{T}{W} = \left( \frac{L}{D} \right)^{-1}$$

$$\therefore \frac{W}{T} = \frac{L}{D}$$

In Newton's world, aircraft would fly fast at low  $\alpha$ . Maximum lift would be at  $\approx 54^\circ$  with an  $L/D$  of 0.7. Engines would need to be gigantic. Aero design is boring...

This world exists, but not at the bottom of our atmosphere.

# Molecular Dynamics



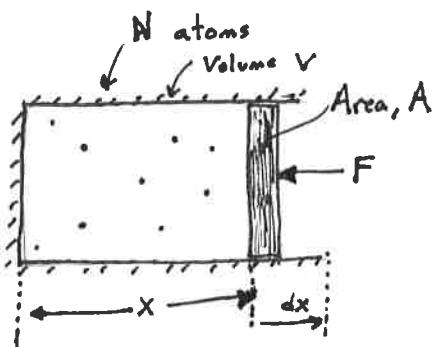
Molecules have a location and a velocity (6 states)  
"phase space"

$$V = (V_x, V_y, V_z)$$

$$X = (x, y, z)$$

Number density  $n(r) = \lim_{\Delta r} \frac{\Delta^3 N}{\Delta r^3} = \frac{\text{How many in a volume}}{\text{How big is a volume}}$

## Pressure



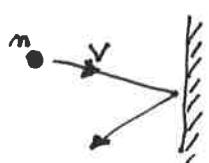
$$P = \frac{F}{A}$$

$$n = \frac{N}{V}$$

Work = Force · distance

$$dW = F(-dx) = -PA dx = -P dV$$

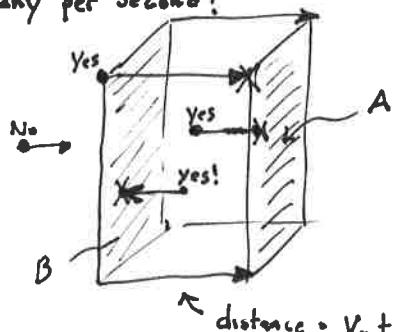
One atom



$$J = \Delta(mv)$$

$$= [m(-V_x)] - [mV_x] = -2mV_x$$

How many per second?



"Volume" of atoms hitting a wall =  $V_x + A$

Number of atoms per second

$$N = nV_{\text{box}} = nV_x + A$$

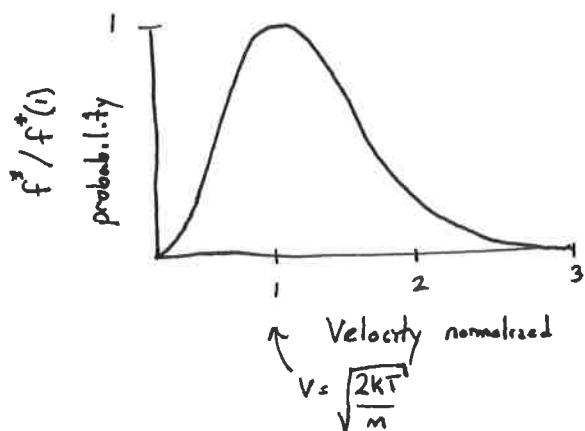
Total force on A and B

$$F = \frac{J \cdot N}{t} = -2mV_x \cdot nV_x A \Rightarrow P = \frac{F}{2A} = -\frac{2mnV_x^2}{2} = -mv_2 V_x^2$$

But not all atoms move at same speed!!

## Velocity Distribution

Maxwell Boltzmann distribution



$$f'(v) = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT}$$

For more information, refer to a gas kinetics source. These notes only mention this topic in passing. (Real or "Physical" Gas Dynamics)  
AEM 606

The most likely speed is not the average velocity.

Rather, take an average of velocity squared.

$$P = nM \langle V_x^2 \rangle \quad \text{since we only want atoms hitting one wall.}$$

If we make the assumption that the atomic velocities are directionally identical (ok for most cases)  
 ~~$V_x^2 + V_y^2 + V_z^2 = 3V_x^2$~~  (false in aerodynamics)  
 Not always.  
 Transients

$$P = \frac{2}{3} n \left\langle \frac{mv^2}{2} \right\rangle \quad \text{well, } \frac{mv^2}{2} \text{ is kinetic energy}$$

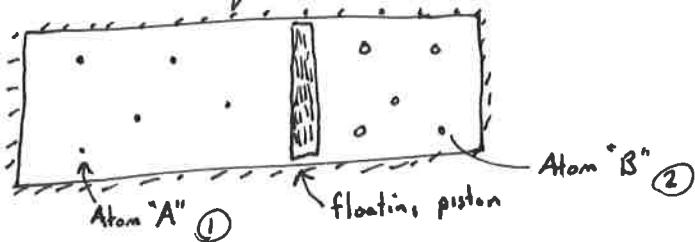
$$= \frac{2}{3} \frac{N}{V} \left\langle \frac{mv^2}{2} \right\rangle$$

$$\nwarrow \text{call this term } (\gamma-1) \frac{N}{V} \quad \gamma = \frac{5}{3} \text{ monatomic gas}$$

warning! physics definition of  $(\gamma-1)$

## Temperature

Q: What are the equilibrium conditions here?



A: Pressure on piston must be equal

$$P_1 = (\gamma - 1) \frac{N_1}{V_1} \left\langle \frac{m_1 v_1^2}{2} \right\rangle = P_2 = (\gamma - 1) \frac{N_2}{V_2} \left\langle \frac{m_2 v_2^2}{2} \right\rangle$$

$$n_1 \left\langle \frac{m_1 v_1^2}{2} \right\rangle = n_2 \left\langle \frac{m_2 v_2^2}{2} \right\rangle$$

A: But the piston reacts to the atomic impact and thus the KE must be equal.

$$n_1 \left\langle \frac{m_1 v_3^2}{2} \right\rangle = n_2 \left\langle \frac{m_2 v_3^2}{2} \right\rangle$$

where  $v_3^2$  is average of  $KE_1$  and  $KE_2$

Isothermal

Define temperature as a linear function of kinetic energy

$$T = C \cdot \left\langle \frac{m v^2}{2} \right\rangle$$

Rearrange to

$$\Delta U = C_v \Delta T$$

$\nwarrow$  Internal energy       $\uparrow$  Specific heat       $\uparrow$  Temp

for ideal gas where  $C_v$  can be a function of  $T$

## Ideal Gas

From before, we had the pressure from atomic impact as

$$P = C \cdot \frac{N}{V} \cdot \left\langle \frac{mv^2}{2} \right\rangle$$

Rearrange to

$$PV = N \cdot C \cdot T$$

Ideal Gas law

$$PV = N_m R T$$

also

$$P = \rho R T$$

$R$  depends on the gas

$$R = \frac{\bar{R}}{M} \text{ and } \bar{R} = 8.314 \times 10^3 \frac{\text{J}}{\text{mol K}}$$

$$= 1545.34 \frac{\text{ft lbf}}{\text{lbmol K}}$$

where  $\rho$  is density (aka ~~mass~~  $\frac{N}{V M}$ )

Example:

Compute the pressure of air with density  $1.2 \frac{\text{kg}}{\text{m}^3}$  at  $300^\circ\text{K}$

$$P = \frac{1.2 \frac{\text{kg}}{\text{m}^3} \cdot 8.314 \times 10^3 \frac{\text{J}}{\text{mol K}}}{28.97 \frac{\text{kg}}{\text{mol}}} \cdot 300 \text{ K} \cdot \frac{1 \text{ kPa}}{1000 \frac{\text{N}}{\text{m}^2}} \cdot \frac{\text{N}}{\text{J}} \cdot \frac{\text{m}^2 \text{ Pa}}{\text{N}}$$

$$P = 103 \text{ Pa}$$

Compute the pressure of air with density  $0.00237 \frac{\text{slug}}{\text{ft}^3}$  at  $500^\circ\text{R}$

$$P = \frac{0.00237 \frac{\text{slug}}{\text{ft}^3} \cdot 1716 \frac{\text{ft}^2}{\text{s}^2 \text{ R}} \cdot 500 \text{ R}}{5 \frac{\text{slug ft}}{\text{lbmol}}} \cdot \frac{1 \text{ lbft}}{144 \text{ in}^2} \cdot \frac{144 \text{ in}^2}{1 \text{ ft}^2}$$

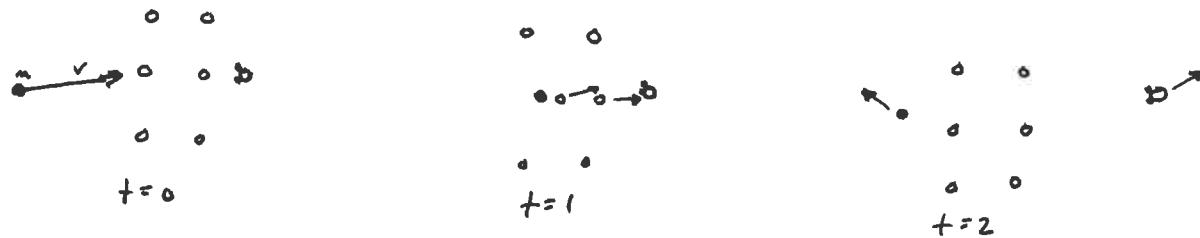
$$P = 14 \text{ psi}$$

# Reality

- Not all gases are monatomic.

Air is a mixture.

- Walls are composed of atoms



Heat (aka velocity distributions) "leak"

- Tracking individual atoms is tedious

Too small of scale

$$N \equiv N$$

$\xrightarrow{109.7 \text{ pm}} = 109700 \text{ nm}$

How small? The Knudsen # gives an indication of the ratio of the mean free path  $\lambda$  to the geometry scale.

$$\lambda_{SSL} \approx 65 \text{ nm}$$

- Ideal Gasses are common in aerodynamics at moderate temperatures

$$P = \rho R T$$

$$e = C_v T$$

$$h = C_p T$$

$$\gamma = \frac{C_p}{C_v}$$