Lesson 3

The atmosphere

The atmosphere on Earth

Dry Air (= Atmospheric air − H_2O − contaminants (dust, pollen, ⋯))

	Mole Fraction	Molecular Weight $\frac{kg}{kmol} \sim \frac{lbm}{lbmol}$	
Nitrogen	78.08%	28.02	$N \equiv N$ strong triple bond!
Oxygen	20.95%	32.0	$O=O$ double bond
Argon	0.93%	39.94	Greek αργον "inactive" Ar no bond
Carbon Dioxide	0.03%	44.01	$O=C=O$ linear shape double bond
Other	0.01%		
	100%		

Apparent Molecular Weight $= \sum a_i M_i$

$$M \approx 28.02 \cdot 0.7808 + 32.0 \cdot 0.2095 + 39.94 \cdot 0.0093 + 44.01 \cdot 0.0003$$

$$\approx 28.97 \ \frac{lbm}{lbmol} = 28.97 \ \frac{kg}{mol}$$

Gas Constant for air

$$R = \frac{\bar{R}}{M} = \frac{1545.34 \text{ ft lbf}}{R \text{ lbmol}} \cdot \frac{\text{lbmol}}{28.97 \text{ lbm}} \cdot \frac{32.174 \text{ lbm}}{\text{slug}} = 1716.5 \ \frac{\text{ft lbf}}{R \text{ slug}}$$

$$= 53.35 \ \frac{\text{ft lbf}}{R \text{ lbm}}$$

Air density SSL (14.696 psi, 59°F)

$$\rho = \frac{PM}{\bar{R}T} = \frac{14.696 \text{ psi}}{} \cdot \frac{28.97 \text{ lbm}}{\text{lbmol}} \cdot \frac{R \text{ lbmol}}{1545.34 \text{ ft lbf}} \cdot \frac{}{518.67 R} \cdot \frac{\text{slug}}{32.174 \text{ lbm}} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \cdot \frac{\text{lbf}}{\text{psi in}^2}$$

$$= 0.00237 \ \frac{\text{slug}}{\text{ft}^3}$$

Wet air

The addition of water vapor changes the properties of "air":

$N \equiv N$ $\quad O=C=O$ $\quad N \equiv N$

Ar $\quad N \equiv N$

H_2O $\quad O=O$

Water vapor behaves as an ideal gas, thus we can model the mixture as an IG.

$$\rho = \frac{P_{dry}}{R_{dry}\, T_{dry}} + \frac{P_{vapor}}{R_{vapor}\, T_{vapor}}$$

where the partial pressures add to total pressure
$P = P_{dry} + P_{vapor}$

$$= \frac{P_{dry}\, M_{dry}}{R\, T_{dry}} + \frac{P_{vapor}\, M_{vapor}}{R\, T_{vapor}}$$

Temps are identical

$$= \frac{P_{dry}\, M_{dry} + P_{vapor}\, M_{vapor}}{\bar{R}T}$$

Partial pressure of vapor
$P_{vapor} = \phi P_{sat}$
ϕ = relative humidity

$$= \frac{(P - P_{vapor})M_{dry} + P_{vapor}\, M_{vapor}}{\bar{R}T}$$

$$= \frac{(P - \phi P_{sat})M_{dry} + \phi P_{sat}\, M_{vapor}}{\bar{R}T}$$

$$= \frac{P M_{dry} + \phi P_{sat}\left(M_{vapor} - M_{dry}\right)}{\bar{R}T}$$

$M_{dry} = 28.97$

$M_{vapor} = 18.0$

Thus, $M_{vapor} - M_{dry}$ is negative

Increasing the water vapor decreases air density

Saturated Water Vapor ⇒ Partial Pressure

Arden - Buck Equation (curve fit)

$$p_s^{[hPa]}\left(T^{[C]}\right) = 6.1121 \exp\left(\left(18.678 - \frac{T}{234.5}\right)\left(\frac{T}{257.14 + T}\right)\right)$$

$$p_s^{[psi]}\left(T^{[R]}\right) = 0.08865 \exp\left(\frac{-0.002369\,(T - 8375.65)(T - 491.67)}{T - 28.818}\right)$$

Ex:

what is the partial pressure of water vapor at 100°F?

$$p_s(100^F) = 0.08865 \exp\left(\frac{-0.002369\,(559.67 - 8375.65)(559.67 - 491.67)}{559.67 - 28.818}\right)$$

$$= 0.9502 \text{ psi}$$

$p_s(100^F)$ from my thermodynamics steam table is 0.9503 psi

Ex: What is $p_{s_{H_2O}}$ at 212^F? Hint: Boiling Water

You don't need the formula! $p_s = p_{SSL} = 14.7^{psi} = 1$ atm

Ex: At what altitude must you fly to boil water in your hand?

Human body ≈ 98°F

$p_s(98°F) = 0.89$ psi

Consult a standard atmosphere table

$h \approx 63000^{ft}$

please don't try at home!

Wet air (continued)

$$\rho = \frac{P M_{dry} + \phi \; 0.08865 \exp\left(-\frac{0.002369(T - 8375.65)(T - 491.67)}{T - 28.818}\right)(M_{vapor} - M_{dry})}{\bar{R} T}$$

function of p, ϕ, T

Ex: $p = 14.696$ psi, $90°F$, 90% rh

$\rho = \cdots\cdots = 0.00220 \frac{slug}{ft^3}$

Ex: " " 0% rh

$\rho = 0.002243 \frac{slug}{ft^3}$

Impact of Wet Air.

					% SSL
• Standard Sea Level (SSL) Std-Day	0^{ft} MSL	59°F	0% rh		
				$\rho = 0.00237 \frac{slug}{ft^3}$	100
• Alabama Summer (hot + humid)	90°F	90% rh	$\approx 0^{ft}$ MSL	$\rho = 0.002206 \frac{slug}{ft^3}$	93%
• " " Dry	90°F	0% rh		$\rho = 0.00224 \frac{slug}{ft^3}$	94%
• Alabama Winter (wet)	40°F	90% rh		$\rho = 0.00246 \frac{slug}{ft^3}$	104%
• " " Dry	40°F	0% rh		$\rho = 0.00246 \frac{slug}{ft^3}$	104%
• Antarctica (cold + dry)	−126°F	0% rh		$\rho = 0.00369 \frac{slug}{ft^3}$	155%
• Denver, CO (std day)	40°F	0% rh	5000^{ft} MSL	$\rho \approx 0.00205 \frac{slug}{ft^3}$	86%

ASHRAE Chart Comparison

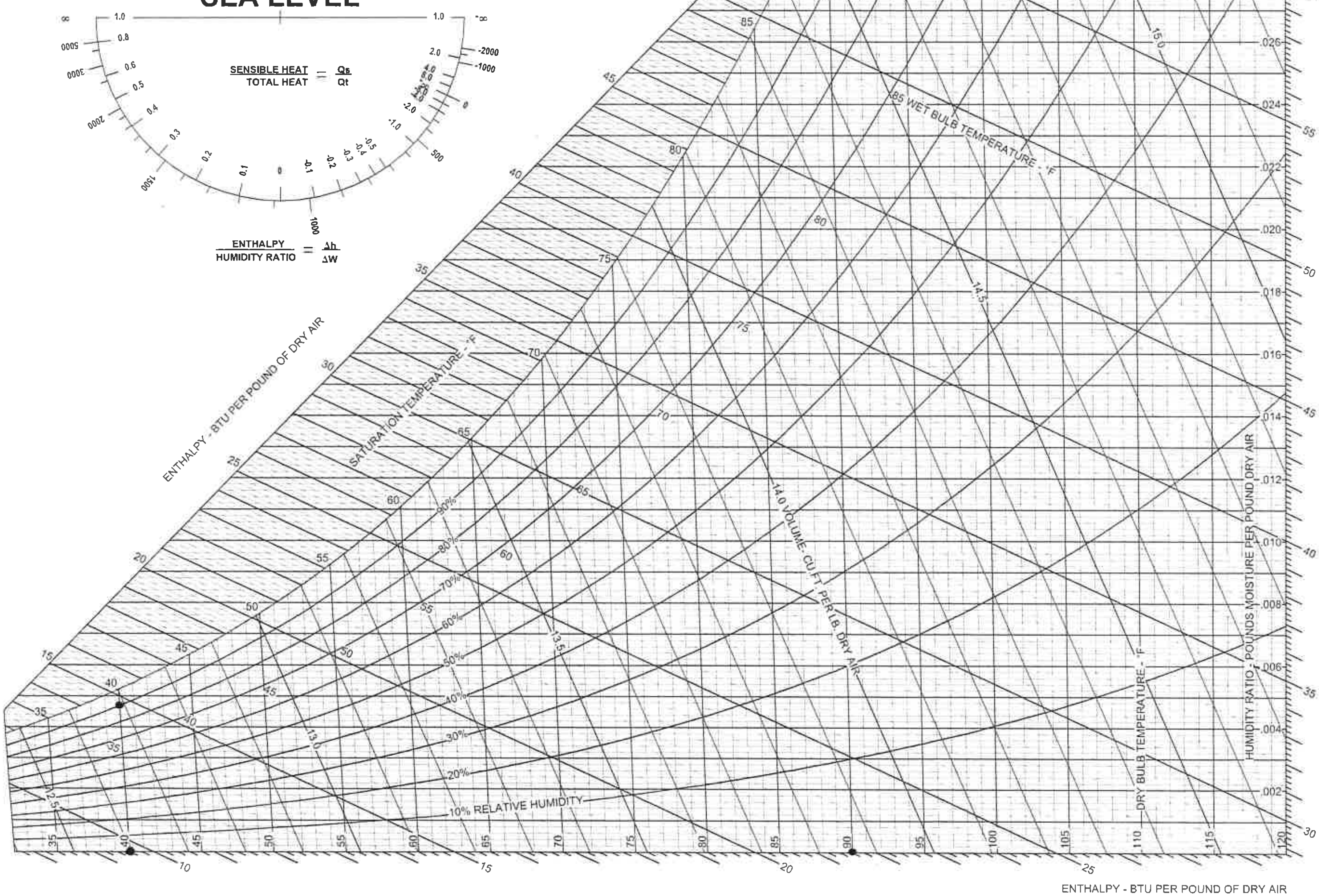
ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

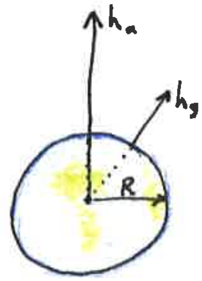
BAROMETRIC PRESSURE: 29.921 INCHES OF MERCURY

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.

SEA LEVEL



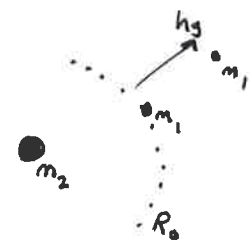
Atmosphere



Distinguish between absolute altitude from center of Earth (h_a) and altitude from the Earth's surface (h_g)

$$h_a = h_g + R$$

Gravity is not constant wrt altitude.



$$F = G\frac{m_1 m_2}{r^2}$$

How does gravity change with altitude?

$$F_{R_0} = \frac{G m_1 m_2}{R_0^2} \quad \text{force at } R_0$$

$$F_{R_0+h_g} = \frac{G m_1 m_2}{(R_0+h_g)^2} \quad \text{force at } R_0+h_g$$

$$\Rightarrow \frac{G m_2}{R_0^2} = g_0$$

$$g = \frac{G m_2}{(R_0+h_g)^2} = g_0 \frac{R_0^2}{(R_0+h_g)^2}$$

$$g = g_0 \frac{R_0^2}{(R_0+h_g)^2}$$

Define a new altitude called "geopotential altitude", h, where gravity is constant (g_0).

$$\underset{\text{geopotential}}{dh} = \frac{g}{g_0}\underset{\text{geometric}}{dh_G} \Rightarrow dh = \frac{R_0^2\, dh_G}{(R_0+h_g)^2} \Rightarrow h = \frac{R_0}{R_0+h_g}h_g$$

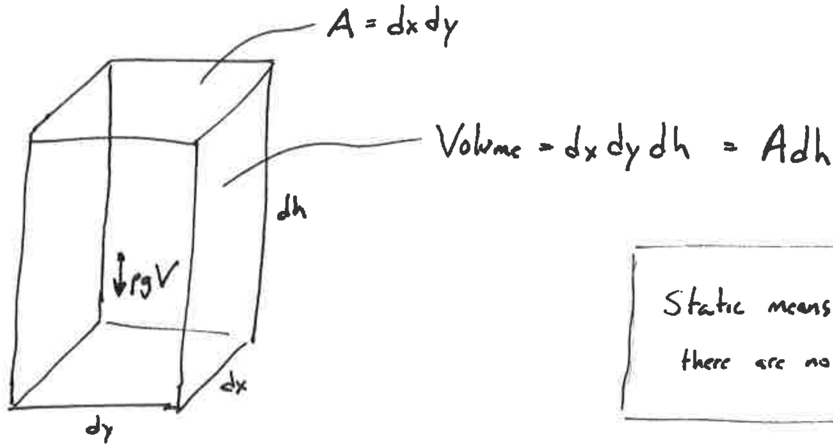
Negligible for most aircraft applications.

$\approx$ 0.1% error at 21 000 ft

1% error at 200 000 ft

Nevertheless, we will use h (geopotential) altitude.

Static Column of Fluid



Static means no velocity, so no $\frac{du_i}{dx_j}$, thus there are no viscous forces.

Summation of forces in h direction

$$P_{bottom} A = P_{top} A + \rho g V$$

Taylor Series expansion for $P_{top} = P_{bottom} + \frac{dP}{dh} dh$

$$PA = \left(P + \frac{dP}{dh} dh\right) A + \rho g A dh$$

Reduce (divide by A, cancel PA terms)

$$\frac{dP}{dh} dh + \rho g dh = 0$$

Divide by dh

$$\frac{dP}{dh} + \rho g = 0$$

Gov Equ

$$\boxed{dp = -\rho g_o dh}$$

Atmosphere (continued)

$$dp = -\rho g_0 dh$$

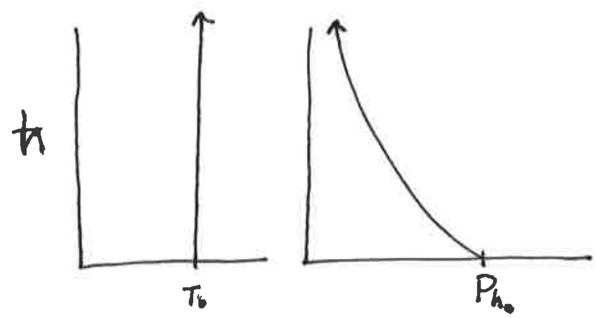
Ideal Gas is $p = \rho R T$, substitute for ρ

$$dp = -\frac{p}{RT} g_0 dh \quad \Rightarrow \quad \frac{dp}{p} = -\frac{g_0 \, dh}{RT}$$

- Isothermal
  Zero lapse rate $(\Delta T \neq f(h))$ $(T = T_0)$

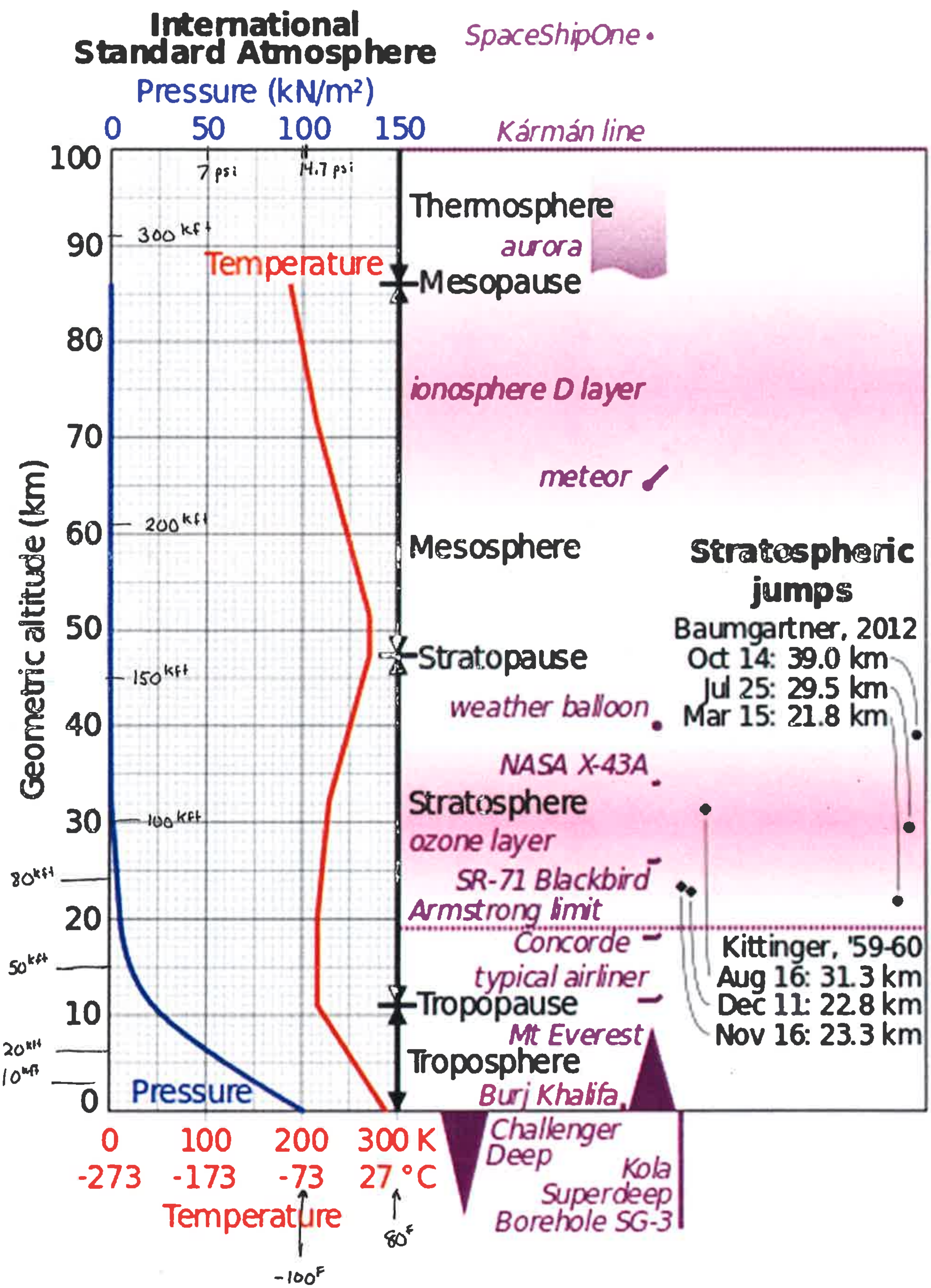
$$\frac{dp}{p} = \frac{-g_0}{R T_0} dh \qquad \text{Integrate} \qquad \ln p \Big|_{p_{h_0}}^{p_{h_1}} = \frac{-g_0}{RT_0} h \Big|_{h_0}^{h_1}$$

$$\underbrace{\ln p_{h_1} - \ln p_{h_0}}_{\ln \frac{p_{h_1}}{p_{h_0}}} = \frac{-g_0}{RT_0}(h_1 - h_0) \quad \Rightarrow \quad p_{h_1} = p_{h_0} e^{\frac{-g_0 (h_1 - h_0)}{RT_0}}$$



- Linear lapse rate $(T = T_0 + \lambda(h_1 - h_0))$

$$\frac{dp}{p} = \frac{-g_0}{R(T_0 + \lambda(h_1 - h_0))} dh \qquad \underset{\text{(slightly involved)}}{\text{Integrate}} \qquad \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{-g_0}{R\lambda}}$$


International
Standard Atmosphere
Pressure (kN/m²)
0
50
100
150
7 psi
14.7 psi
SpaceShipOne
Kármán line
100
90
80
70
60
50
40
30
20
10
0
Geometric altitude (km)
300 kft
200 kft
150 kft
100 kft
80kft
50kft
20kft
10kft
Temperature
Thermosphere
aurora
Mesopause
ionosphere D layer
meteor
Mesosphere
Stratopause
weather balloon
NASA X-43A
Stratosphere
ozone layer
SR-71 Blackbird
Armstrong limit
Concorde
typical airliner
Tropopause
Mt Everest
Troposphere
Burj Khalifa
Pressure
Stratospheric
jumps
Baumgartner, 2012
Oct 14: 39.0 km
Jul 25: 29.5 km
Mar 15: 21.8 km
Kittinger, '59-60
Aug 16: 31.3 km
Dec 11: 22.8 km
Nov 16: 23.3 km
0
100
200
300 K
-273
-173
-73
27 °C
Temperature
-100F
80F
Challenger
Deep
Kola
Superdeep
Borehole SG-3

Standard Atmosphere

h ft	λ R/ft
0	-0.003566
36089	0
65617	+0.000549
104987	+0.001536

$P(0^{ft}) = 14.696$ psi

$T(0^{ft}) = 59°F$

For more information, refer to the 1976 standard atmosphere.

Reality:

The atmosphere is not, has not, and will never be "standard" or static.

meteogram.

For engineering purposes, additional standard atmosphere models exist:

Hot Day

Cold Day

Tropics

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