Lesson 4

Non-dimensional Aerodynamics

From the Buckingham Π theorem (AEM 311 Fluid Mechanics), a set of non-dimensional numbers naturally result from the units of length, time, and mass.

$m \equiv$ dimension of mass
$l \equiv$ dimension of length
$t \equiv$ dimension of time

All values and variables that we use can be represented in terms of m, l, t.

- density $\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{l^3} = ml^{-3}$
- Velocity $V = \frac{\text{distance}}{\text{time}} = \frac{l}{t} = lt^{-1}$
- Force $R = \text{mass} \cdot \text{acceleration} = m \cdot \frac{l}{t^2} = mlt^{-2}$
- ... and so on.

Given that we expect aerodynamic forces R to depend on ρ, V, c, ν, a (airfoil/wing size; density; velocity; viscosity; speed of sound)

We expect $N - K$ nondimensial #s.

$N = 6 \Rightarrow (R, \rho, V, c, \nu, a)$
$K = 3 \Rightarrow (m, l, t)$

Applying BT,

$$\Pi_1 = \frac{R}{\rho V^2 c^2}$$

ρV^2: twice Dynamic pressure; c^2: Area

$$\Pi_2 = \frac{\rho V c}{\nu}$$

Reynolds #

$$\Pi_3 = \frac{V}{a}$$

Mach #

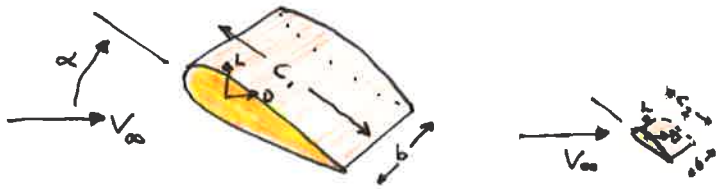
Matching this term matches the force magnitude.

$q = \frac{1}{2}\rho V^2$

Matching these 2 non-dim #s matches the flow behavior

"Similarity parameters"

Non-Dimensional Forces and Moments and Pressures



Lift is a force: Non-dimensionalize by dynamic pressure and area

$q = \frac{1}{2}\rho V^2$ Wing Area $= \bar{c} \cdot b = S$

$$C_L \equiv \frac{L}{\frac{1}{2}\rho V^2 S}$$

Units: $\frac{lbf}{\frac{slug}{ft^3}\left|\frac{s^2}{ft^2}\right|ft^2\left|\frac{slug\,ft}{lbf\,s^2}\right.} = 1$ ✓

Drag:

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 S}$$

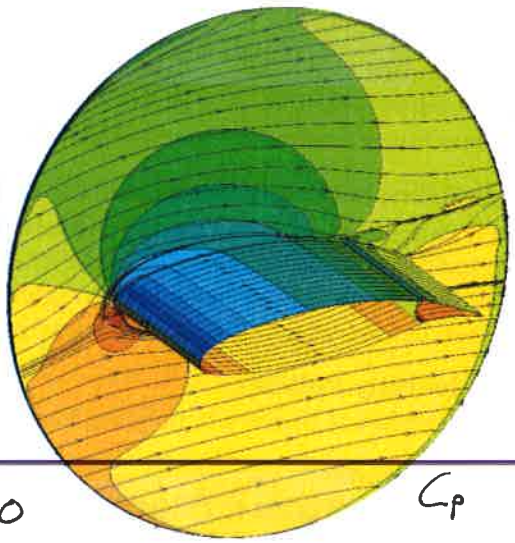
Moment is a force x distance: Non-dim by q, area, and length

$$C_M = \frac{M}{\frac{1}{2}\rho V^2 \cdot S \cdot c}$$

Units: $\frac{lbf\text{-}ft}{\frac{lbf}{ft^2}\left|ft^2\right|ft} = 1$ ✓

Pressure is a force per unit area: Non-dim by q

$$C_p = \frac{\Delta P}{\frac{1}{2}\rho V^2} = \frac{P - P_\infty}{q} = C_p$$



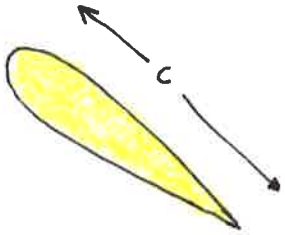
Ex: What is C_p in the freestream?

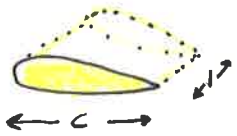
$P = P_\infty \Rightarrow C_p = 0$

Ex: What is C_p where $P = P_\infty + \frac{1}{2}\rho V^2$ (i.e. zero velocity)

$$C_p = \frac{P_\infty + \frac{1}{2}\rho V^2 - P_\infty}{\frac{1}{2}\rho V^2} = 1$$ "stagnation pt"

Airfoils (2D)





Forces, Moments are per unit span (think of an airfoil with exactly 1 unit span)

$$C_\ell = \frac{L'}{qc} = \frac{L}{qc \cdot b} \quad \text{since} \quad L' = \frac{L}{b} \quad \text{where } b = 1$$

notice the small case, this indicates 2D per-unit-span

$$C_d = \frac{D'}{qc}$$

$$C_m = \frac{M'}{qc \cdot c} = \frac{M'}{qc^2}$$

Ex: What is C_ℓ and C_d for an NACA 2412 at $Re \approx 9 \times 10^6$?

Depends on:

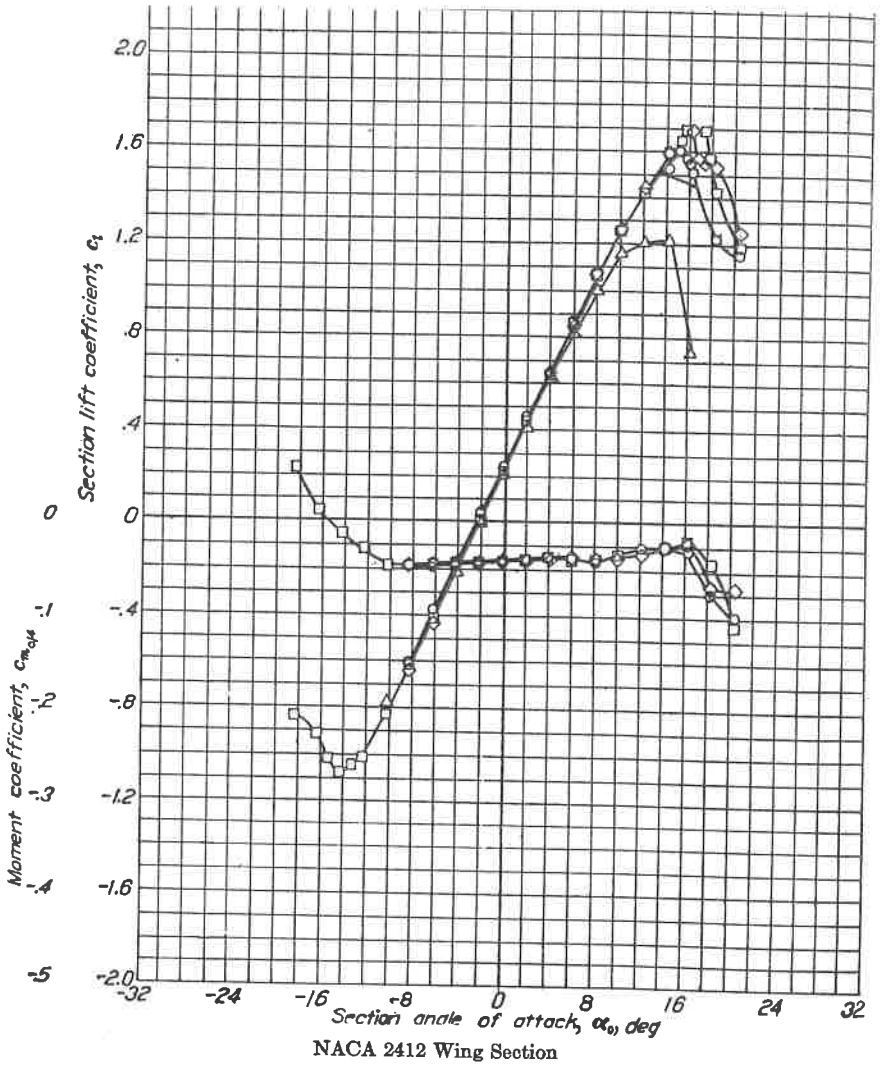
α, Re, M

Lookup in Theory of Wing Sections

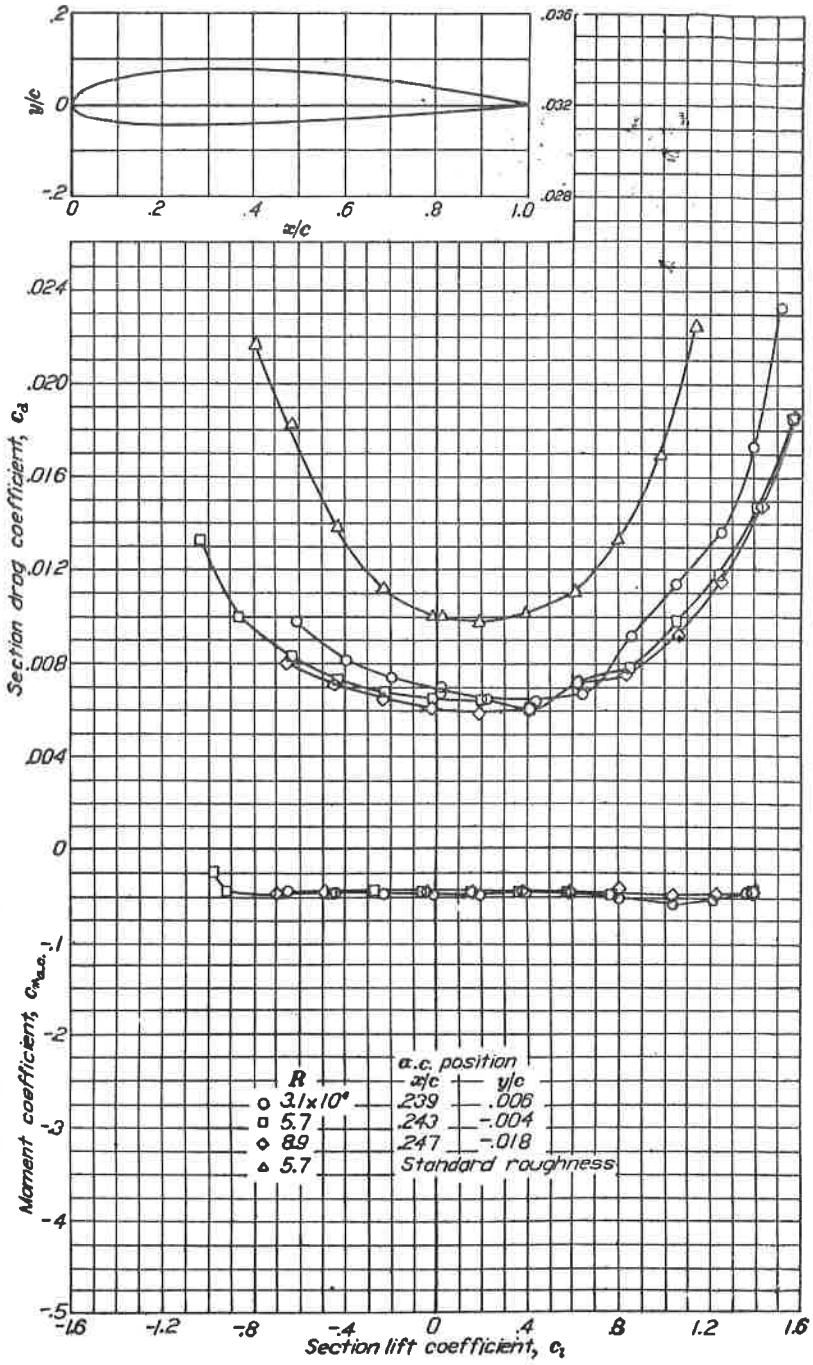
- $\alpha = 0°$

  $C_\ell \approx 0.25$ $C_d \approx 0.0060$ (60 counts) $C_{m_{1/4c}} = -0.048$

- $\alpha = 8°$ $C_\ell \approx 1.05$ $C_d \approx 0.0090$ (90 counts) $C_{m_{1/4c}} = -0.048$



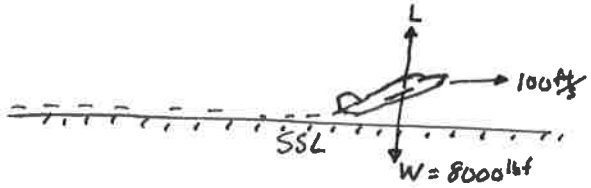
NACA 2412 Wing Section



NACA 2412 Wing Section (*Continued*)

Ex:

Consider a small jet which is known to takeoff at $100\ ft/s$ at SSL at $8000\ lbf$.



Q1: What is the takeoff/liftoff velocity at $12000\ lbf$?

Lift Coefficient:

$$L = W \quad \text{and} \quad L = \underbrace{\tfrac{1}{2}\rho V^2}_{q} S C_L \Rightarrow W_0 = \left(\tfrac{1}{2}\rho_0 V_0^2\right)(S_0)(C_{L_0})$$

These are not likely to change much (geometry + aero).

Also, $W_N = \frac{1}{2}\rho_N V_N^2$

Ratio:

$$\frac{W_N}{W_0} = \frac{\frac{1}{2}\rho_N V_N^2 \, S_N \, C_{L_N}}{\frac{1}{2}\rho_0 V_0^2 \, S \, C_{L_0}} = \frac{\rho_N V_N^2}{\rho_0 V_0^2}$$

Solve for V_n:

$$V_N^2 = \frac{W_N}{W_0}\frac{\rho_0}{\rho_N} V_0^2 \Rightarrow V_N = \sqrt{\frac{W_N}{W_0}\frac{\rho_0}{\rho_N}}\, V_0$$

$$V_n = \sqrt{\frac{12000\ lbf}{8000\ lbf}\,\Big|\,\frac{1}{1}}\ 100\ ft/s$$

$$\boxed{V_N = 122\ ft/s}$$

Q2: What is the takeoff velocity at Denver on a std day?

$$V_n = \sqrt{\frac{W_n}{W_0}\frac{\rho_0}{\rho_N}}\, V_0$$

From prev notes, $\frac{\rho_N}{\rho_0} \approx 0.86$ for Denver

$$V_N = \sqrt{\frac{12000}{8000}\,\Big|\,\frac{1}{0.86}}\ 100\ ft/s = \boxed{132\ ft/s = V_N}$$

Q3: Original Weight at Denver.

$$V_N = \sqrt{\frac{1}{0.86}}\ 100\ ft/s = \boxed{107\ ft/s = V_N}$$

Worse news, the engines won't produce full power with the reduced density.