

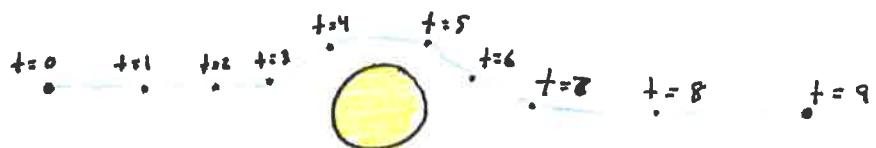
Lesson 9

Streamlines
Pathlines
Timelines
Streaklines

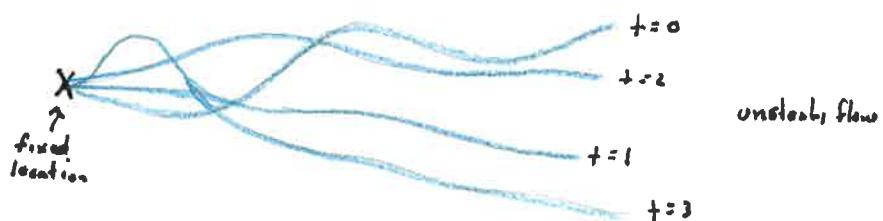
Flow Velocity Visualization

web.mit.edu/hai/ncfmf.html
"Flow Visualization"
00:00 - 13:00

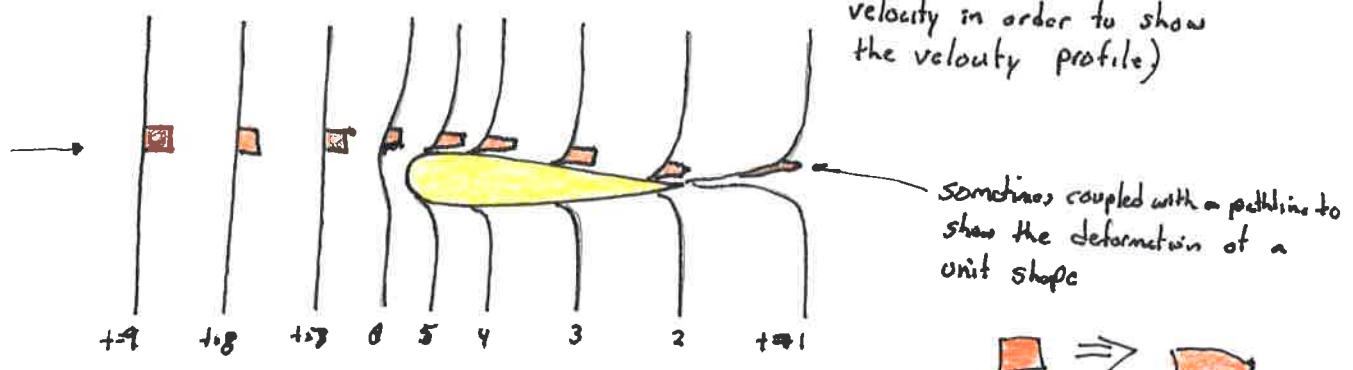
Pathline: Trajectory of a given fluid element



Streakline: Trace of all elements that flowed through a fixed location

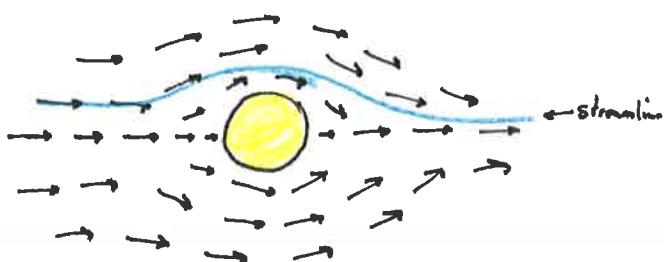


Timeline: Instantaneous location of a line of fluid particles. (Usually released normal to flow velocity in order to show the velocity profile)



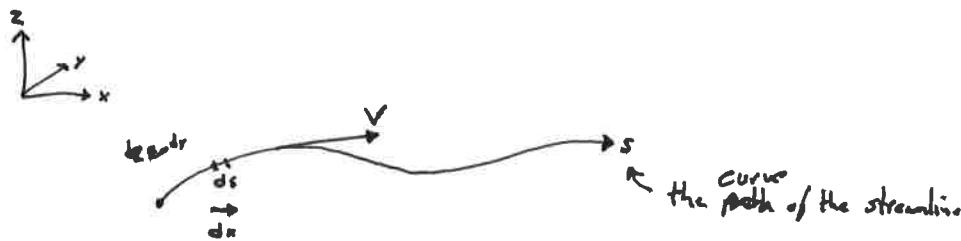
Streamline: A line tangent to the flow velocity vector

shows compressibility, viscosity, velocity, ...



For steady flows, pathlines, streaklines and streamlines are identical.

Mathematics of streamlines



To maintain the streamline's curve s along the velocity vector,

$$ds \times V = 0$$

in other words, the component of velocity is always along the streamline.

A generic definition of ds and V are:

$$ds = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$V = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

Thus,

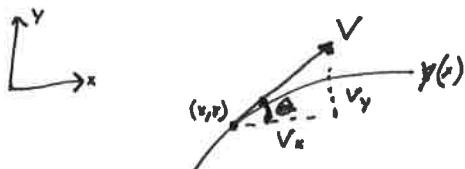
$$ds \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{dx}{ds} & \frac{dy}{ds} & \frac{dz}{ds} \\ V_x & V_y & V_z \end{vmatrix} = (V_z dy - V_y dz) \hat{i} - (V_z dx - V_x dz) \hat{j} + (V_y dx - V_x dy) \hat{k} = 0$$

Thus each part is zero

$$\begin{aligned} V_z dy - V_y dz &= 0 \\ -V_z dx + V_x dz &= 0 \\ V_y dx - V_x dy &= 0 \end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned} wdy - vdz &= 0 \\ -wdx + udz &= 0 \\ vdx - udy &= 0 \end{aligned}}$$

In 2D



Slope of $y(x)$ passing through the point (x, y) is:

$$\frac{dy}{dx} = \tan(\theta) = \frac{V_y}{V_x} = \frac{v}{u}$$

$$\boxed{\frac{dy}{dx} = \frac{v}{u}}$$

See above 3rd line (\hat{k} component)

$$vdx - udy = 0 \Rightarrow \frac{dy}{dx} = \frac{v}{u} \quad \checkmark$$

Ex:

Given a flow field: $U = y$ and $V = x$, what are the equations describing the streamlines?

Definition of Streamline.

$$\frac{dy}{dx} = \frac{V}{U}$$

Flow velocities subst'

$$\frac{dy}{dx} = \frac{x}{y}$$

Integrate,

$$y dy = x dx \Rightarrow \int y dy = \int x dx \Rightarrow \frac{y^2}{2} + C_1 = \frac{x^2}{2} + C_2$$

Rearrange

$$C_3 = C_2 - C_1 = \frac{y^2}{2} - \frac{x^2}{2}$$

We pick a constant C_3 and plot the resulting curve

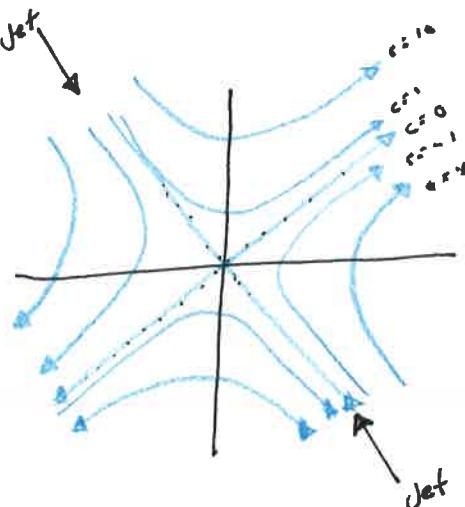
$$C = \frac{y^2}{2} - \frac{x^2}{2} \Rightarrow \frac{y^2}{2} = C + \frac{x^2}{2} \Rightarrow y^2 = 2C + x^2$$

$$y = \pm \sqrt{2C + x^2}$$

$$C = 0: \Rightarrow y = \sqrt{0 + x^2} = \pm x$$

$$C = 1: \Rightarrow y = \sqrt{2 + x^2} = \dots$$

plot



Similar to 2 jets merging pointed exactly towards each other

Dr. Ölmen's lab has done exactly this but at supersonic speeds.