

Lesson 10 part 2

Streamfunctions (Doublets)

ψ

Aerodynamic Modeling

↖ Not an exact solution, but sufficient for understanding.

- Uniform Flow



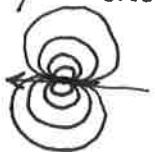
Models a freestream (i.e. Motion!)

- Vortex



Models lift generation

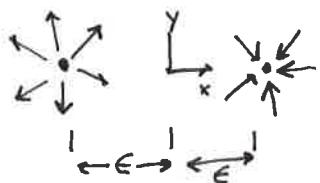
- Doublet / Source + Sink



Models thickness and volume

These are conceptual tools for thinking about aerodynamics

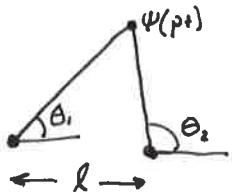
Source + Sink (proto - doublet)



$$\Psi = \frac{\Delta}{2\pi} \theta$$

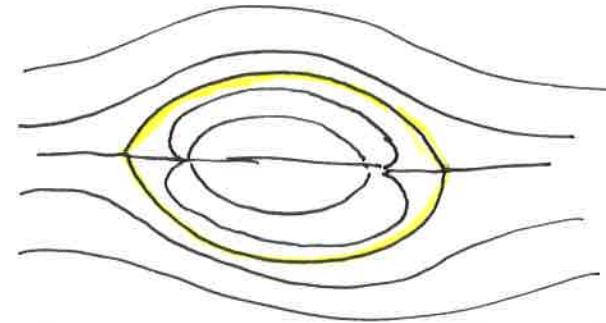
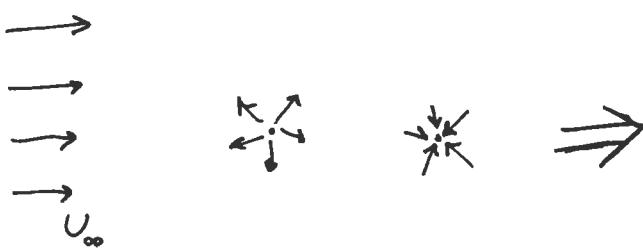
↑ Notice that the stream function depends only on the angle from the source or sink

Thus, the Ψ for a source/sink pair of equal magnitude is visually



$$\Psi = \frac{\Delta}{2\pi} \theta_1 - \frac{\Delta}{2\pi} \theta_2$$

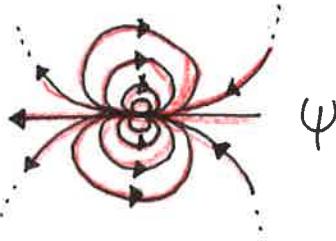
When combined with a freestream, this gives a **Rankine oval**.



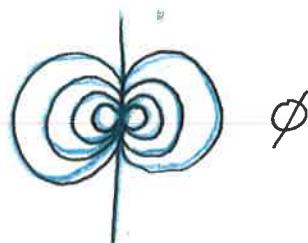
$$\Psi = V_\infty r \sin \theta + \frac{\Delta}{2\pi} \theta_1 - \frac{\Delta}{2\pi} \theta_2$$

In the limit, as the distance between the source and sink is reduced to zero (in the limit) the doublet emerges.

$$\boxed{\Psi_{\text{doublet}} = -\frac{K}{2\pi} \frac{\sin \theta}{r}}$$



$$\boxed{\phi_{\text{doublet}} = \frac{K}{2\pi} \frac{\cos \theta}{r}}$$



Doublet

The beauty of the doublet is that when combined with a freestream, a circular streamline is formed



$$\Psi = V_\infty r \sin\theta + \frac{-K}{2\pi} \frac{\sin\theta}{r}$$

What value of K ? Pick a streamline of radius R for the circle.

- Stagnation pt: $U_r = 0 \quad U_\theta = 0$

$$\begin{aligned} U_r &= \frac{1}{r} \frac{d\Psi}{d\theta} = \frac{1}{r} \frac{d}{d\theta} \left((V_\infty r \sin\theta) - \frac{K}{2\pi} \frac{\sin\theta}{r} \right) = \frac{1}{r} \left(V_\infty r \cos\theta - \frac{K}{2\pi r} \cos\theta \right) \\ &= V_\infty \cos\theta - \frac{K}{2\pi} \frac{1}{r^2} \cos\theta = 0 \text{ at } r=R \Rightarrow V_\infty = \frac{K}{2\pi R^2} \end{aligned}$$

$$K = 2\pi V_\infty R^2$$

Streamfunction

$$\Psi = V_\infty r \sin\theta - \frac{2\pi V_\infty R^2}{2\pi} \frac{\sin\theta}{r} = V_\infty \sin\theta \left(r - \frac{R^2 \sin\theta}{r} \right)$$

Mult/div by r and pull out r term

$$\boxed{\Psi = V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2} \right)}$$

What does this represent?

- Inviscid
- Incompressible
- Body of radius R in a flow of V_∞
-

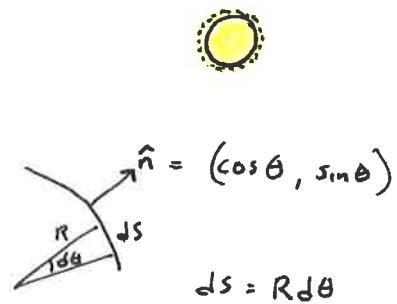
D'Alembert's Paradox.

We know that the drag on a ~~body~~ body is non-zero in flight. (See lesson 6 notes)

Compute the drag of the flow over a cylinder. C.V. fits exactly over cylinder,

$$\text{Drag} = - \iint_S \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) V_x dS - \iint_S \rho \hat{n}_x dS$$

No flow through
 dS , since flow
is tangent to circle!



$$= - \int_0^{2\pi} \rho \cos \theta R d\theta$$

What is the pressure on the surface? Assume Incompressible.. (B' eff)

$$P + \frac{1}{2} \rho V^2 = P_\infty + \frac{1}{2} \rho_\infty V_\infty^2 \Rightarrow P = P_\infty + \frac{1}{2} \rho_\infty (V_\infty^2 - V^2)$$

What is the velocity? (on the surface)

Body requires $U_r = 0$, otherwise not actually a body!

$$U_\theta = - \frac{d\psi}{dr} = - \frac{d}{dr} \left(V_\infty r \sin \theta \left(1 - \frac{R^2}{r^2} \right) \right)$$

$$= - V_\infty \sin \theta \left(1 - \frac{R^2}{r^2} \right) - V_\infty r \sin \theta \left(- \frac{2R^2}{r^3} \right)$$

$$= - V_\infty \sin \theta \left(1 - \frac{R^2}{r^2} \right) - V_\infty r \sin \theta \left(\frac{2R^2}{r^3} \right)$$

$$= - V_\infty \sin \theta \left(1 - \frac{R^2}{r^2} + r \frac{2R^2}{r^3} \right)$$

$$U_\theta = - V_\infty \sin \theta \left(1 + \frac{R^2}{r^2} \right)$$

$$V^2 = U_\theta^2 = V_\infty^2 \sin^2 \theta \left(1 + \frac{R^2}{r^2} \right)^2$$

$$\begin{aligned} \text{Drag} &= - \int_0^{2\pi} \left[P_\infty + \frac{1}{2} \rho_\infty V_\infty^2 - \frac{1}{2} \rho_\infty \left(V_\infty^2 \sin^2 \theta \left(1 + \frac{R^2}{r^2} \right)^2 \right) \right] \cos \theta R d\theta \\ &\quad \text{Sym} \quad \text{Sym} \\ &\quad + \text{const} \quad + \text{const} \\ &= + \int_0^{2\pi} \frac{1}{2} \rho_\infty V_\infty^2 \left(1 + \frac{R^2}{r^2} \right)^2 \underbrace{\sin^2 \theta \cos \theta R d\theta}_{\text{Integral at this is zero!}} \end{aligned}$$

$\boxed{\text{Drag} = 0}$

Why? This is what D'Alembert found in 1752.

Solved in 1904 by Prandtl