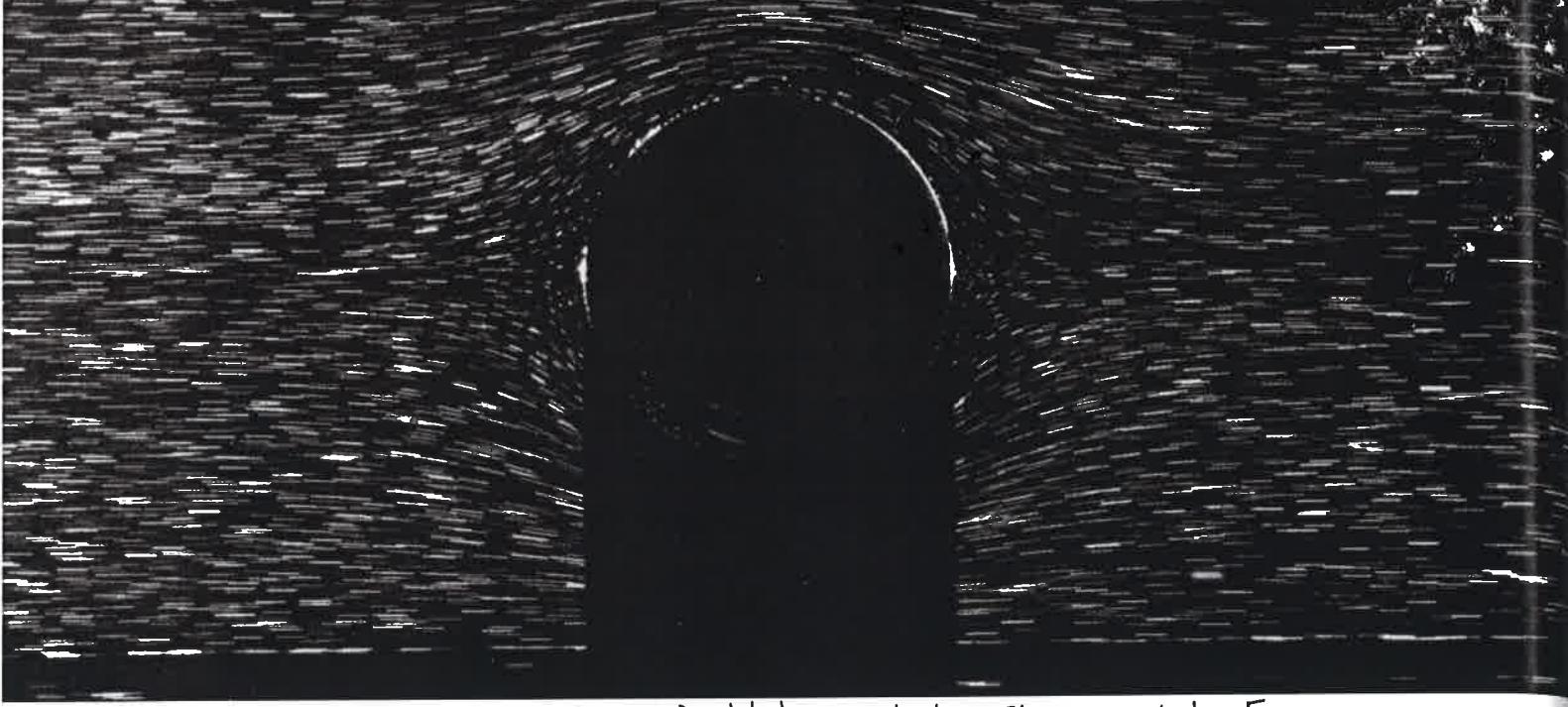


Lesson 10 part 3

Stream functions ψ

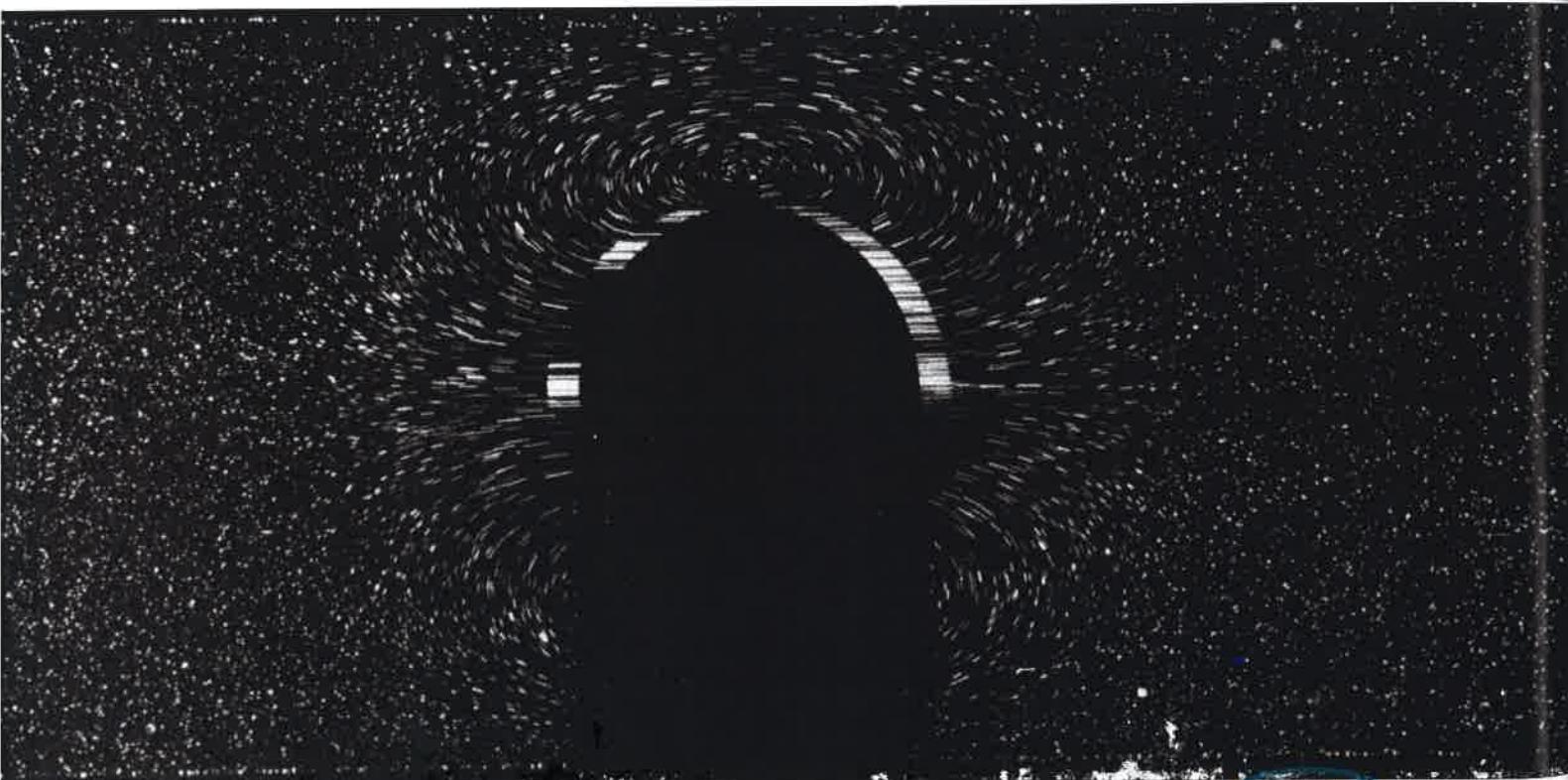
Cylinder pressure + Lift



$U_\infty + \text{Doublet} = \text{Cylinder Flow in body Frame}$

8. Sphere moving through a tube at $R=0.10$, relative motion. A free sphere is falling steadily down the axis of a tube of twice its diameter filled with glycerine. The camera is moved with the speed of the sphere to show the flow rel-

ative to it. The photograph has been rotated to show flow from left to right. Tiny magnesium cuttings are illuminated by a thin sheet of light, which casts a shadow of the sphere. Coutanceau 1968



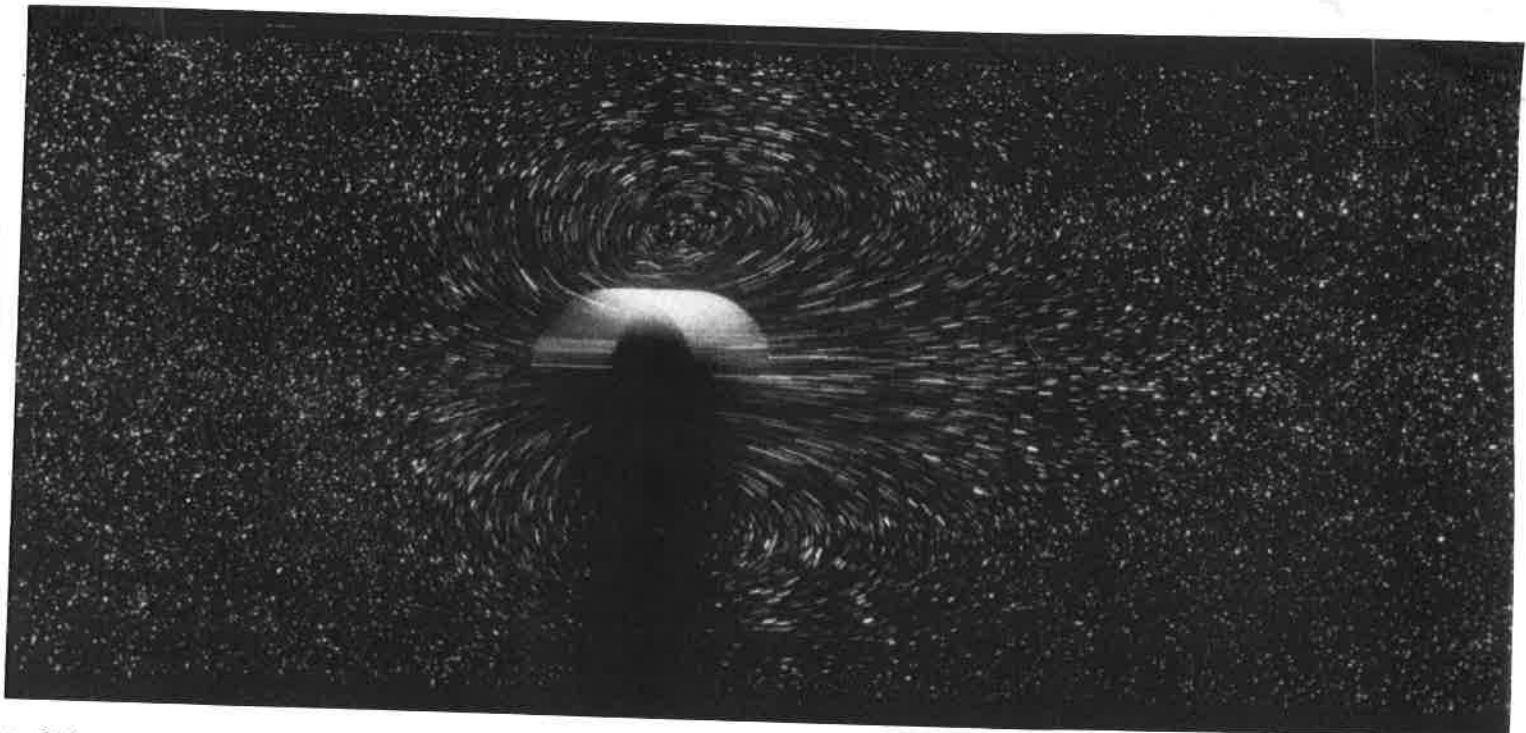
$\text{Cylinder Flow in } U_\infty \text{ Frame} = U_\infty + \text{Doublet} - U_\infty = \text{Doublet}$

9. Sphere moving through a tube at $R=0.10$, absolute motion. In contrast to the photograph above, here the camera remains fixed with respect to the distant fluid. During the exposure the sphere has moved from left to right

less than a tenth of a diameter, to show the absolute motion of the fluid. At this small Reynolds number the flow pattern, shown by magnesium cuttings in oil, looks completely symmetric fore-and-aft. Coutanceau 1968

Source : An Album of Fluid Motion

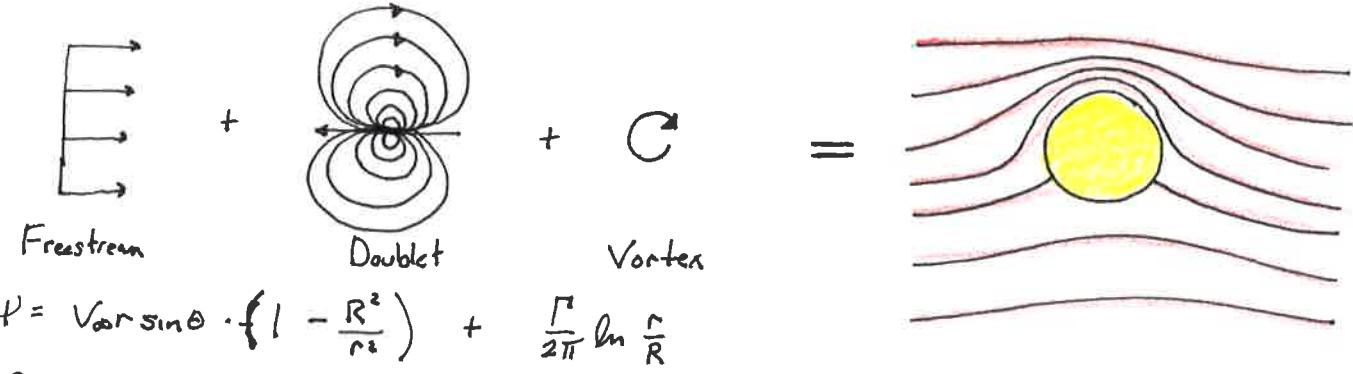
M. Van Dyke 1982



28. Sphere moving through a tube at $R=6.9$, absolute motion. The sphere is one-fourth the diameter of the tube. It has moved to the left through one radius. In contrast to the creeping motion of figure 9, there is at this modest Reynolds number the beginning of a wake: the dis-

turbances extend considerably farther behind the sphere than ahead. Magnesium cuttings are illuminated in silicone oil. Archives de l'Académie des Sciences de Paris. Cou-tanceau 1972

Spinning Cylinder



Remember that a vortex has only V_θ velocity component, so the circle's surface streamlines are preserved.

What is the velocity field?

$$V_r = \frac{1}{r} \frac{d\Psi}{d\theta} = \frac{1}{r} \frac{d}{d\theta} \left(V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi} \ln \frac{r}{R} \right) = \frac{1}{r} V_\infty r \cos\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$= V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$V_\theta = -\frac{d\Psi}{dr} = -\frac{d}{dr} \left(\underbrace{V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)}_{\text{previous part}} + \underbrace{\frac{\Gamma}{2\pi} \ln \frac{r}{R}}_{\frac{d \ln \frac{r}{R}}{dr} = \frac{1}{r}} \right)$$

$$= -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta - \frac{\Gamma}{2\pi r}$$

Velocity squared V^2

$$V^2 = V_r^2 + V_\theta^2 = \underbrace{V_\infty^2 \cos^2\theta \left(1 - \frac{R^2}{r^2}\right)^2}_{\text{many terms cancel & simplify}} + \underbrace{\left(1 + \frac{R^2}{r^2}\right)^2 V_\infty^2 \sin^2\theta}_{\text{many terms cancel & simplify}} + \left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta \frac{\Gamma}{2\pi r} 2 + \frac{\Gamma^2}{4\pi^2 r^2}$$

$$= \left(1 + \frac{R^4}{r^4}\right) - 2 \frac{R^2}{r^2} \cos^2\theta V_\infty^2 + 2 \frac{R^2}{r^2} \sin^2\theta V_\infty^2 + \left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta \frac{\Gamma}{\pi r} + \frac{\Gamma^2}{4\pi^2 r^2}$$

$$= \left(1 + \frac{R^4}{r^4}\right) - 2 \frac{R^2}{r^2} V_\infty^2 (\cos^2\theta - \sin^2\theta) + \left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta \frac{\Gamma}{\pi r} + \frac{\Gamma^2}{4\pi^2 r^2}$$

Pressure Coefficient (incompressible)

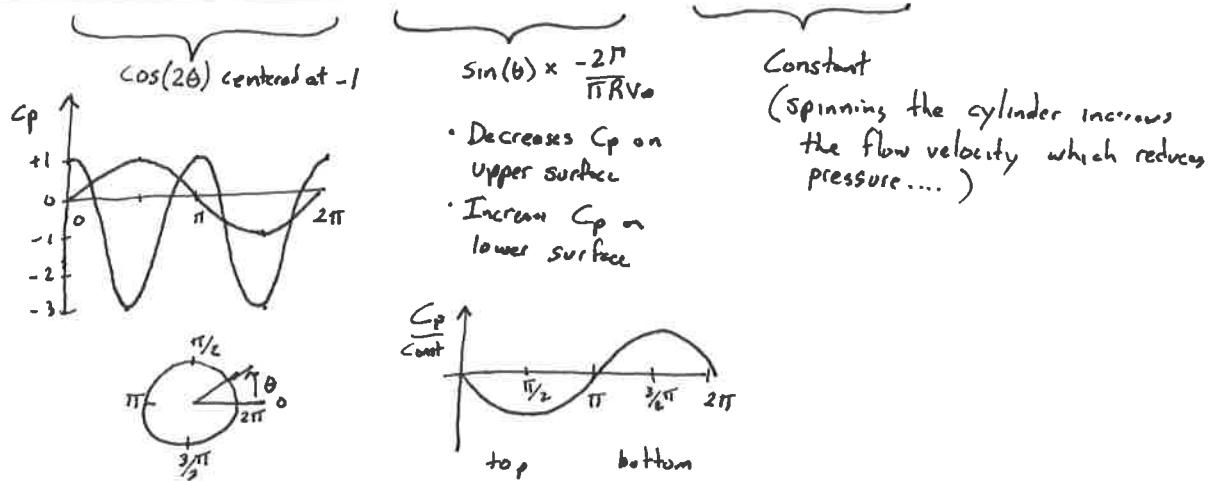
$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 + \frac{-1 - \frac{R^4}{r^4}}{2 \frac{R^2}{r^2} \cos(2\theta)} - \left(1 + \frac{R^2}{r^2}\right) \sin\theta \frac{\Gamma}{\pi r V_\infty} - \frac{\Gamma^2}{4\pi^2 r^2 V_\infty^2}$$

$$= -\frac{R^4}{r^4} + 2 \frac{R^2}{r^2} - 4 \frac{R^2}{r^2} \sin^2\theta - \left(1 + \frac{R^2}{r^2}\right) \sin\theta \frac{\Gamma}{\pi r V_\infty} - \frac{\Gamma^2}{4\pi^2 r^2 V_\infty^2}$$

At the surface of the cylinder, $r = R$

$$C_p = -1 + 2 - 4 \sin^2 \theta - \frac{2\Gamma}{\pi R V_\infty} \sin \theta - \frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2}$$

$$C_{p_{r=R}} = 1 - 4 \sin^2 \theta - \frac{2\Gamma}{\pi R V_\infty} \sin \theta - \frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2}$$



Spinning Cylinder Drag and Lift (?)

$$C_d = - \int_0^{2\pi} P \cos \theta R d\theta \cdot \frac{1}{\frac{1}{2} \rho V_\infty^2 (2R)} \quad C_P = \frac{P}{\rho}$$

$$= -\frac{1}{2} \int_0^{2\pi} C_p \cos \theta d\theta$$

$$= -\frac{1}{2} \int_0^{2\pi} \left(1 - 4 \sin^2 \theta - \frac{2\Gamma}{\pi R V_\infty} \sin \theta - \frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2} \right) \cos \theta d\theta = \boxed{\textcirclearrowleft} = C_d$$

$\int_0^{2\pi} \cos \theta d\theta = 0$

$$C_d = -\frac{1}{2} \int_0^{2\pi} C_p \sin \theta d\theta$$

$$= -\frac{1}{2} \int_0^{2\pi} \sin \theta d\theta + \frac{4}{2} \int_0^{2\pi} \sin^3 \theta d\theta - \frac{1}{2} \int_0^{2\pi} \frac{-2\Gamma}{\pi R V_\infty} \sin^2 \theta d\theta - \frac{1}{2} \int_0^{2\pi} -\frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2} \sin \theta d\theta$$

$\int_0^{2\pi} \sin \theta d\theta = 0$

$\int_0^{2\pi} \sin^3 \theta d\theta = \pi$

$\int_0^{2\pi} \sin^2 \theta d\theta = \pi$

$\int_0^{2\pi} -\frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2} \sin \theta d\theta = \frac{\Gamma^2}{4\pi^2 R^2 V_\infty^2}$

$$\boxed{C_d = + \frac{\Gamma}{R V_\infty}}$$

or

$$L' = C_d \frac{1}{2} \rho V_\infty^2 (2R)$$

$$L' = \frac{\Gamma}{R V_\infty} \frac{1}{2} \rho V_\infty^2 (2R)$$

$$\boxed{L' = \rho V_\infty \Gamma}$$

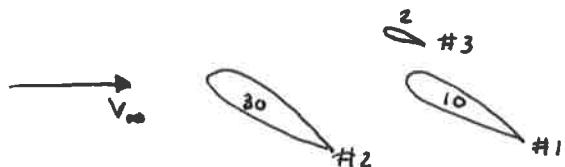
Kutta - Joukowski

Ex: At SSL and V_∞ of 100 ft/s , an airfoil is generating $10 \frac{\text{lbf}}{\text{ft}}$ of lift. What is the circulation about the wing?

$$L' = \rho V_\infty \Gamma \Rightarrow \Gamma = \frac{L'}{\rho V_\infty}$$

$$\Gamma = \frac{10 \frac{\text{lbf}}{\text{ft}}}{\frac{0.00237 \text{ slugs}}{\text{ft}^3}} = \frac{10 \frac{\text{lbf}}{\text{ft}}}{\frac{0.00237 \text{ slugs}}{\text{ft}^3}} \cdot \frac{\text{ft}^3}{\text{lbf}} = \boxed{42 \frac{\text{ft}^2}{\text{s}}}$$

Ex: 3 airfoils in a wind tunnel are at 100 ft/s and SSL are creating circulations of $10 \frac{\text{ft}^2}{\text{s}}$, $30 \frac{\text{ft}^2}{\text{s}}$, and $2 \frac{\text{ft}^2}{\text{s}}$. Visually, the wings airfoils are:



- What is the total lift?

$$L' = \rho V \Gamma = \frac{0.00237 \text{ slugs}}{\text{ft}^3} \cdot \frac{100 \text{ ft}}{\text{s}} \cdot \frac{(10 + 30 + 2) \frac{\text{ft}^2}{\text{s}}}{\text{s}} = \boxed{10 \frac{\text{lbf}}{\text{ft}}}$$

- Does airfoil #2's lift generation impact the flowfield at airfoil #3?

Q: What is the most pressing issue now?

What value for Γ ?