

Lesson 14

Lumped Vortex TAT

Panel methods

Lumped Vortex TAT

We derived TAT with source and vortex sheets ($\lambda(x)$ and $\gamma(x)$). Now, we will simplify the physics further by lumping vortex sheets into vortex lines.

$$\text{becomes} \quad \int_a^b \gamma(x) dx \quad \text{where} \quad \Gamma = \int_a^b \gamma(x) dx$$

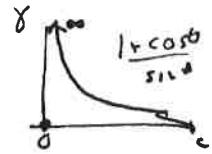
We ignore the source sheet. $\lambda(x) = 0$

You probably know that the vortex line is applied to the quarter chord. From TAT, we know that $C_m \gamma_q = 0$, thus the lift force would act through this γ_q point. However, we can use math to prove this is true.

$$X_{\text{action}} = \frac{\text{Moment}}{\text{Force}} \Rightarrow X_{CG} = \frac{\text{Moment}}{\text{Weight}} \dots \text{etc.}$$

We will do the same operation to $\gamma(x)$

For a flat panel, $\frac{\gamma(\theta)}{V_\infty} = 2A_0 \frac{1+\cos\theta}{\sin\theta}$



The integral of γ over the panel is:

$$\int_0^\pi 2 \underbrace{\frac{1+\cos\theta}{\sin\theta}}_{\gamma} \underbrace{\sin\theta d\theta}_{dx} = 2\pi$$

The moment integral of γ is

$$\int_0^\pi 2 \underbrace{\frac{1+\cos\theta}{\sin\theta}}_{\gamma} \underbrace{\left(\frac{C}{2}\right)(1-\cos\theta)}_{x} \underbrace{\sin\theta d\theta}_{dx} = \frac{\pi C}{2}$$

Thus, the vortex acts at:

$$\frac{\int_0^\pi 2 \frac{1+\cos\theta}{\sin\theta} \left(\frac{C}{2}\right)(1-\cos\theta) \sin\theta d\theta}{\int_0^\pi 2 \frac{1+\cos\theta}{\sin\theta} \sin\theta d\theta} = \frac{\frac{\pi C}{2}}{2\pi} = \frac{C}{4} \quad \text{The quarter chord.}$$

For a single panel, which collocation point gives $C_{e\alpha} = 2\pi$?



$$V_\theta = -\frac{\Gamma}{2\pi r} \quad \alpha \rightarrow \frac{V_\infty \sin \alpha}{U_\infty}$$

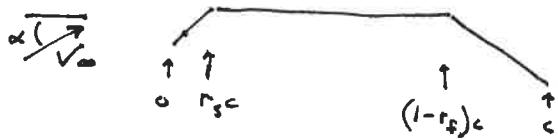
From flow tangency $\Gamma = V_\infty \sin \alpha / 2\pi r$ and $L = \rho V \Gamma$ $\Rightarrow L = \rho V_\infty^2 / 2\pi r \sin \alpha$

$$C_L = \frac{L}{\frac{1}{2} \rho V_c^2} = \frac{\rho V_\infty^2 / 2\pi r \sin \alpha}{\frac{1}{2} \rho V_c^2} = \frac{1}{2} \frac{\sin \alpha}{r} = \frac{1}{2} \frac{\sin \alpha}{c} \Rightarrow r = \frac{1}{2} c \text{ from } \frac{1}{4} c$$

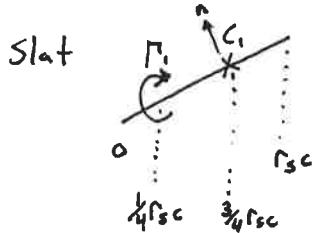
The equivalent collocation point is at $\frac{3}{4} c$.



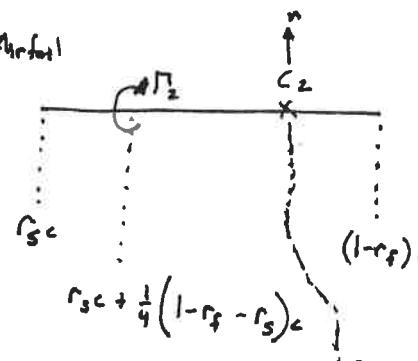
Ex: Slotted flap airfoil.



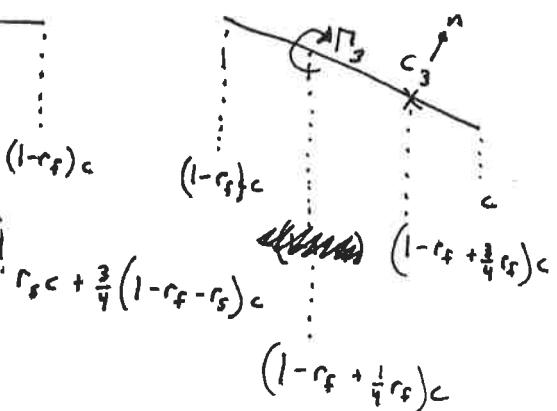
Assume small deflections such that $\Theta = -\alpha$



Airfoil



F_{L4}



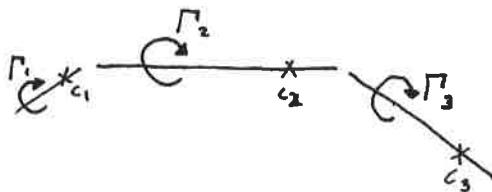
3 unknowns $\Gamma_1, \Gamma_2, \Gamma_3$

3 equations $V \cdot n = 0$ at c_1, c_2, c_3

Model an airfoil as a series of connected flat panels.



Place a vortex at the $\frac{1}{4}c$ of each panel.



A vortex has a potential of $\phi = -\frac{\Gamma}{2\pi} \theta = -\frac{\Gamma}{2\pi} \arctan\left(\frac{y}{x}\right)$

$$U = \frac{d\phi}{dx} = \frac{\Gamma y}{2\pi(x^2+y^2)} = \frac{\Gamma y}{2\pi r^2}$$

$$U_r = \frac{d\phi}{dr} = 0$$

$$V = \frac{d\phi}{dy} = -\frac{\Gamma x}{2\pi(x^2+y^2)} = -\frac{\Gamma x}{2\pi r^2}$$

$$U_\theta = \frac{1}{r} \frac{d\phi}{d\theta} = -\frac{\Gamma}{2\pi r}$$

A freestream U_∞ has the potential $\phi = V_\infty x$

$$U = \frac{d\phi}{dx} = V_\infty \quad \text{and} \quad V = \frac{d\phi}{dy} = 0$$

$$\text{or } \phi = V_\infty \cos\theta r$$

$$U_r = V_\infty \cos\theta \quad U_\theta = -\frac{1}{r} V_\infty \sin\theta r = -V_\infty \sin\theta$$

No flow across panel (at a particular location to be determined)

$$V \cdot n = 0 \text{ implies } U_\theta = 0$$

$$-\frac{\Gamma}{2\pi r} - V_\infty \sin\theta = 0 \Rightarrow \Gamma = -V_\infty \sin\theta \frac{2\pi r}{\Gamma}$$

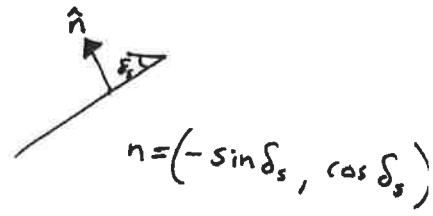
$$\text{with } \alpha = -\theta, \text{ we get } \Gamma = -V_\infty \sin(-\alpha) \frac{2\pi r}{\Gamma}$$

$$= V_\infty \sin\alpha \frac{2\pi r}{\Gamma}$$

At C_1 :

$$U_{C_1} = \frac{\Gamma_1 Y_{11}}{2\pi r_{11}^2} + \frac{\Gamma_2 Y_{21}}{2\pi r_{21}^2} + \frac{\Gamma_3 Y_{31}}{2\pi r_{31}^2} + V_\infty \cos \alpha$$

$$V_{C_1} = -\frac{\Gamma_1 X_{11}}{2\pi r_{11}^2} + -\frac{\Gamma_2 X_{21}}{2\pi r_{21}^2} + -\frac{\Gamma_3 X_{31}}{2\pi r_{31}^2} + V_\infty \sin \alpha$$



$$\mathbf{V} \cdot \mathbf{n} = 0$$

At C_2 :

$$V_{C_2} = -\frac{\Gamma_1 X_{12}}{2\pi r_{12}^2} + -\frac{\Gamma_2 X_{22}}{2\pi r_{22}^2} + -\frac{\Gamma_3 X_{32}}{2\pi r_{32}^2} + V_\infty \sin \alpha$$

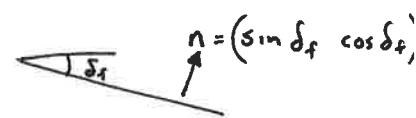
$$\mathbf{V} \cdot \mathbf{n} = V_{C_2} = 0$$



$$\text{At } C_3 : U_{C_3} = \frac{\Gamma_1 Y_{13}}{2\pi r_{13}^2} + \frac{\Gamma_2 Y_{23}}{2\pi r_{23}^2} + \frac{\Gamma_3 Y_{33}}{2\pi r_{33}^2} + V_\infty \cos \alpha$$

$$V_{C_3} = -\frac{\Gamma_1 X_{13}}{2\pi r_{13}^2} + -\frac{\Gamma_2 X_{23}}{2\pi r_{23}^2} + -\frac{\Gamma_3 X_{33}}{2\pi r_{33}^2} + V_\infty \sin \alpha$$

$$\mathbf{V} \cdot \mathbf{n} = 0$$



You notice that we create an influence matrix from the collocation equations

$$\begin{bmatrix} M & & \end{bmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \begin{pmatrix} F \end{pmatrix} V_\infty$$

For a complicated geometry, use a computer to populate and invert M .

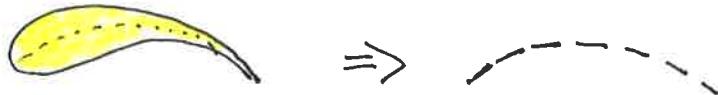
Solving for Γ_i requires inverting M (or an equivalent process).

What is the lift?

$$L = \rho V_\infty \sum \Gamma_i = \rho V_\infty M$$

Panel methods

The TAT lumped model used an open curve that represented the no-flow streamlines through/about the camber line.

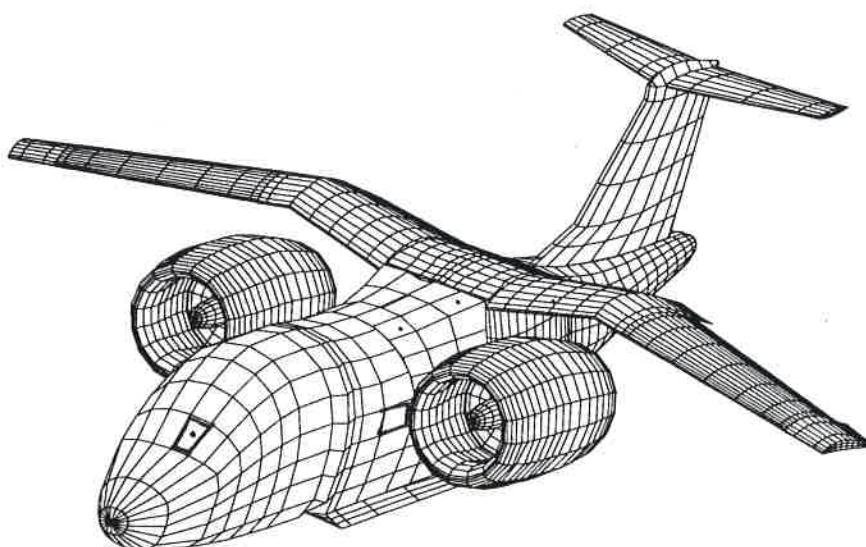


Panel methods extend the curve to a closed curve representing the surface.



Panel methods are powerful tools for conceptual and preliminary design.

The concept is not limited to 2D airfoils, 3D panel methods are also available
(often using doublet sheets)



Each panel has
an influence on
every other panel.

N panels $\Rightarrow N \times N$
matrix

$2546^2 = 6.5$ million
influence
coefficients

Figure 15.62 Detailed panel model of the tilt-nacelle airplane and its inlets, using 2546 panels per side. (Courtesy of S. Iguchi, M. Dudley, and D. Ashby, 1988, and NASA Ames Research Center.)

Worth the effort? Useful?

Yes. Valuable for initial sizing, S+C, etc. Any aircraft with poor aerodynamics at low Mach and no viscosity will be worse at high Mach and lower Re.

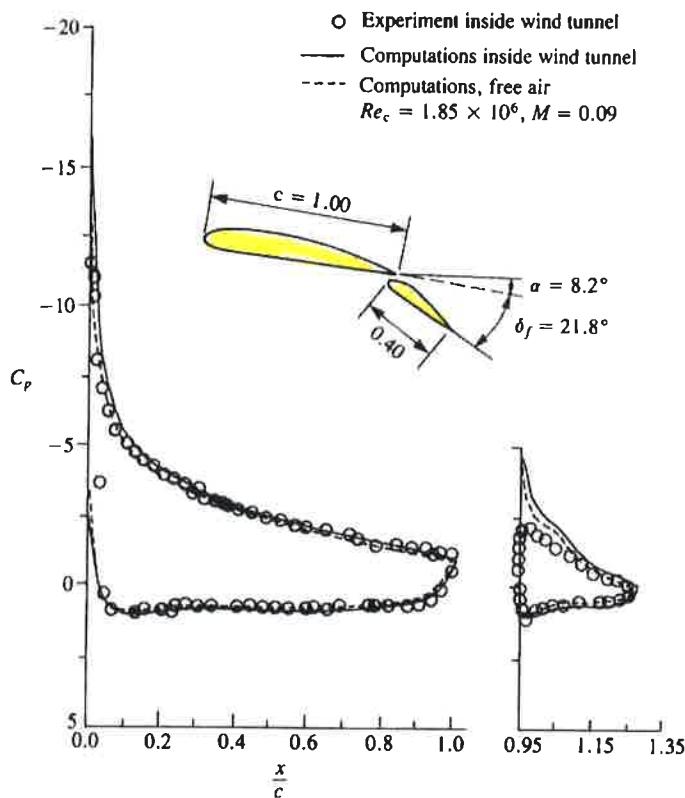


Figure 11.41 Two-dimensional experimental and computed (constant-source/doublet, with Dirichlet B.C.) chordwise pressure distribution on a NACA 4412 wing and a NACA 4415 flap (flap chord is 40% of wing chord). Experiments from Adair, D., and Horne, W. C., "Turbulent Separated Flow in the Vicinity of a Single-Slotted Airfoil Flap," *AIAA Paper 88-0613*, Jan. 1988.

Why might the flap C_p magnitude be lower in experiments when compared to inviscid?

Later, we will even study a method for predicting stall and $C_{p,\text{max}}$ from an inviscid C_p field. Foreseeing that lesson, $C_{p,\text{crit}} \approx -13$ and strongly dependent on Re.

A primary source for implementing panel methods is

Low Speed Aerodynamics

by

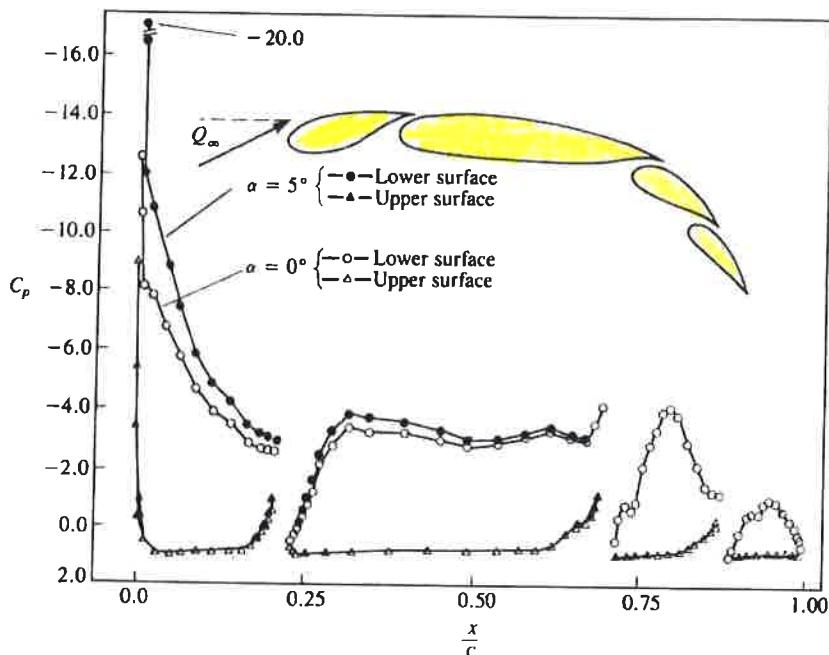
Katz and Plotkin

ISBN 0-521-66219-2

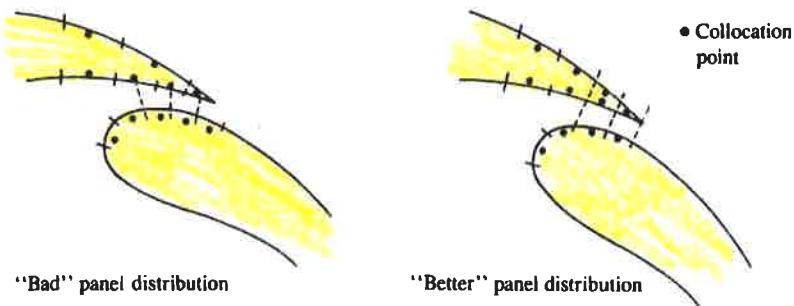
This book includes

- 2D and 3D panel methods.
- Closed form integrals and perturbation velocities for different sheet and panel distributions.
- Implementation hints
- Unsteady panel methods.

Panel methods are suited to multi-element experiments and intuition increasing configuration design



However, there are some issues to avoid.



Compared to full Computation Fluid Dynamics grids, these panel method restrictions and distributions are simple.

How sensitive are panel methods to missing, bad, or moved collocation boundary conditions?

The TE is where most problems appear.

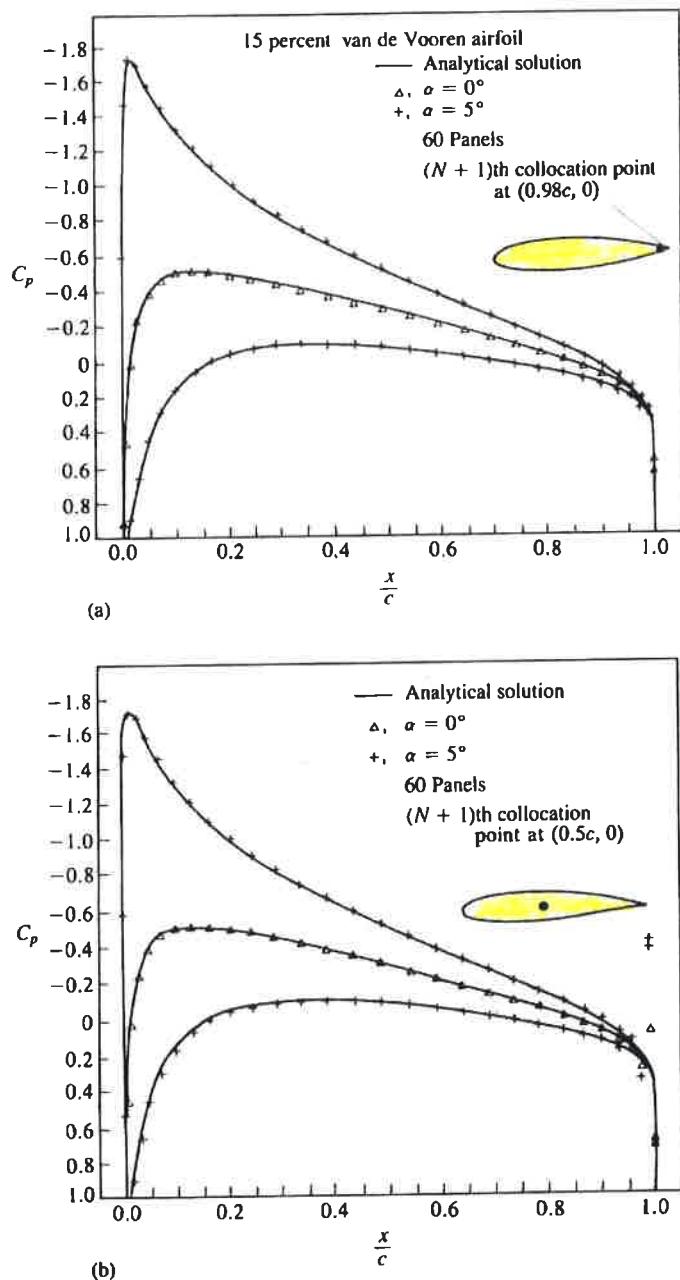
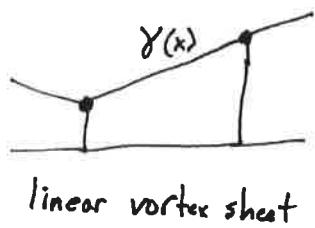
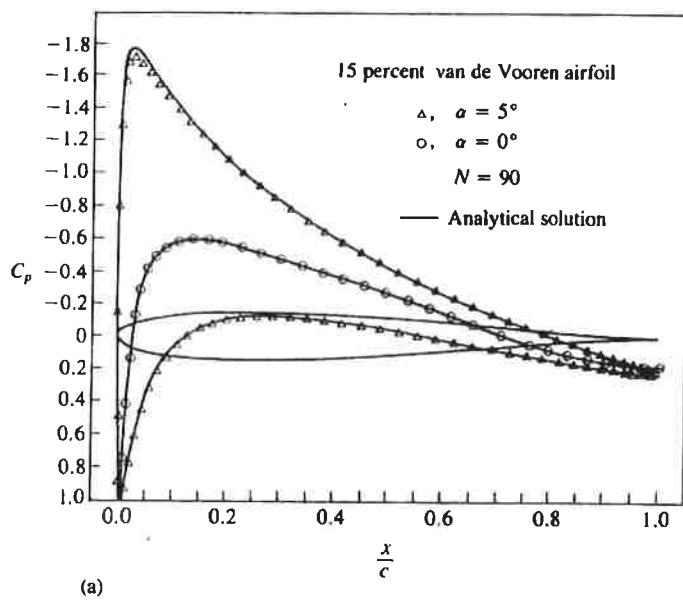


Figure 11.38 Effect of placing the $(N + 1)$ th collocation point inside the 15% thick van de Vooren airfoil (using a quadratic doublet method with the Dirichlet B.C.).

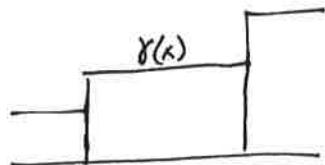
Cusped TEs have a history of causing pathological behavior with some lower order methods.



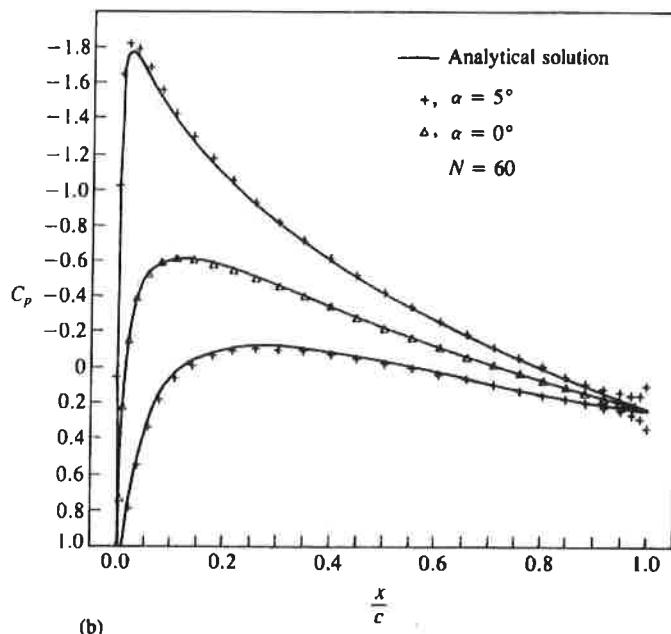
linear vortex sheet



(a)



Constant strength sheet

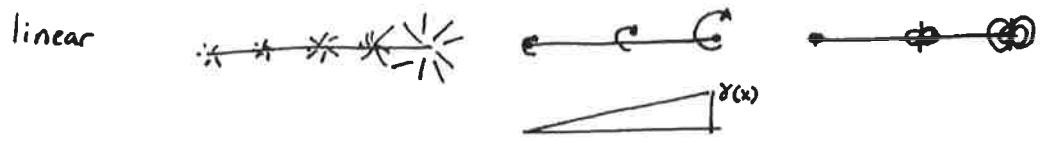
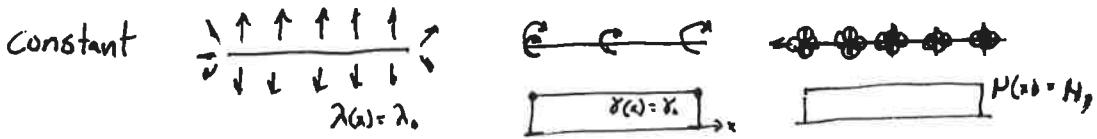
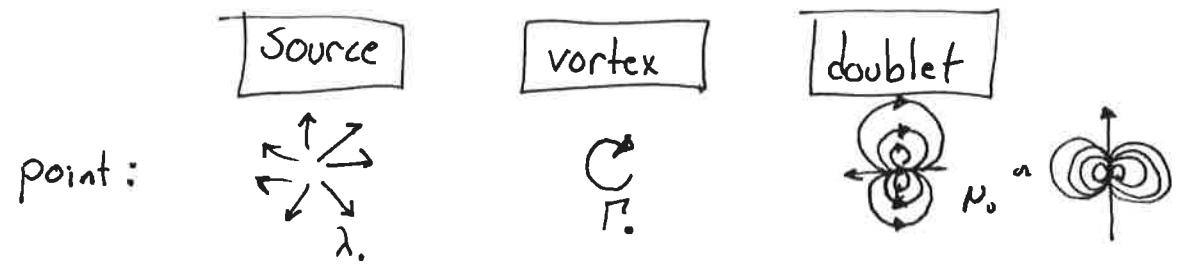


(b)

Figure 11.39 Pressure distribution on a cusped trailing edge 15%-thick van de Vooren airfoil using:
(a) linear vortex method with Neumann B.C., and (b) constant-strength source/doublet method with the Dirichlet B.C.

For a cusped trailing edge, use a linear strength vortex sheet and watch for poor behavior.

Classify panel methods



The lower the order, the more singularities and non-natural perturbations show up.
Luckily, the far field flow is sufficient to determine what engineers typically need.

C_a, C_D, etc

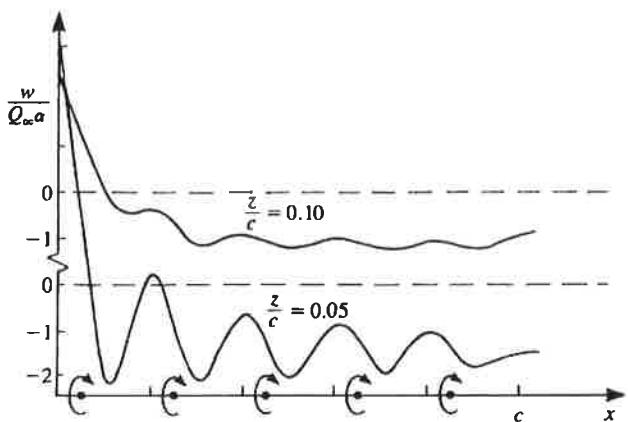
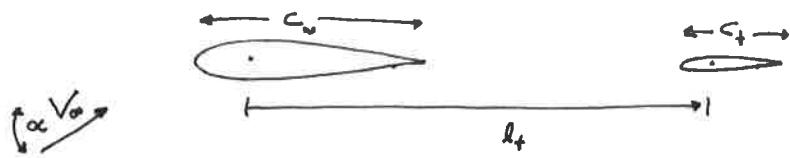


Figure 11.11 Survey of induced normal velocity above a thin airfoil (as shown in Fig. 11.4) modeled by discrete vortices.

Ex: Aircraft Stability "Neutral Point"



Model as



$$V = -\frac{R}{2\pi r}$$

$$\text{Distance from } R_1 \text{ to } C_2 = l_+ + \frac{c_t}{2}$$

$$\text{Distance from } R_2 \text{ to } C_1 = l_+ - \frac{c_w}{2}$$

Freestream

$$V_{fs} = V_\infty (\cos \alpha, \sin \alpha)$$

$$\text{Fore Wing collocation pt } (C_1) \quad \hat{n} = (0, 1)$$

$$V \cdot \hat{n} = 0 \quad V_{R_1} = (0, -\frac{R_1}{2\pi c_w}) \quad V_{R_2} = (0, -\frac{R_2}{2\pi} \left(\frac{1}{l_+ - \frac{c_w}{2}} \right))$$

$$V = V_{fs} + V_{R_1} + V_{R_2}$$

$$= V_\infty (\cos \alpha, \sin \alpha) + \left(0, -\frac{R_1}{2\pi c_w} \right) + \left(0, -\frac{R_2}{2\pi} \left(\frac{1}{l_+ - \frac{c_w}{2}} \right) \right)$$

$$V \cdot \hat{n} = V_\infty \sin \alpha - \frac{R_1}{\pi c_w} - \frac{R_2}{2\pi} \left(\frac{1}{l_+ - \frac{c_w}{2}} \right) = 0$$

Aft wing collocation pt (C_2)

$$V = V_{fs} + V_{R_1} + V_{R_2}$$

$$= V_\infty (\cos \alpha, \sin \alpha) + \left(0, -\frac{R_1}{2\pi} \left(\frac{1}{l_+ + \frac{c_t}{2}} \right) \right) + \left(0, -\frac{R_2}{2\pi} \frac{2}{c_t} \right)$$

$$V \cdot n = V_\infty \sin \alpha - \frac{R_1}{2\pi} \frac{1}{l_+ + \frac{c_t}{2}} - \frac{R_2}{2\pi} \frac{2}{c_t} = 0$$

Rearrange to matrix and solve

[Each collocation pt is a row]

unknown vector of $\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix}$

$$\begin{aligned} C_1 \rightarrow & \begin{bmatrix} \left(\frac{2}{2\pi C_w}\right) & \left(\frac{1}{2\pi} \left(\ell_+ - \frac{C_w}{2}\right)\right) \\ \vdots & \vdots \end{bmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = \begin{pmatrix} V_\infty \sin \alpha \\ V_\infty \sin \alpha \end{pmatrix} \\ C_2 \rightarrow & \begin{bmatrix} \left(\frac{1}{2\pi} \left(\ell_+ + \frac{C_w}{2}\right)\right) & \left(\frac{1}{2\pi} \left(\frac{2}{C_w}\right)\right) \\ \vdots & \vdots \end{bmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = \begin{pmatrix} V_\infty \sin \alpha \\ V_\infty \sin \alpha \end{pmatrix} \end{aligned}$$

Notice, pull out
2π from
matrix!!

Fast inverse of 2x2: flip diagonals, sign of off diagonals, divide by det

$$\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = \frac{\begin{bmatrix} \left(\frac{2}{C_w}\right) & \left(-\frac{1}{\ell_+ - \frac{C_w}{2}}\right) \\ \left(-\frac{1}{\ell_+ + \frac{C_w}{2}}\right) & \left(\frac{2}{C_w}\right) \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} (2\pi V_\infty \sin \alpha)}{\frac{4}{C_w} - \left(\frac{1}{\ell_+ - \frac{C_w}{2}}\right) \left(\frac{1}{\ell_+ + \frac{C_w}{2}}\right)}$$

$$\rho V \Gamma_2 = L$$

Moment about ~~middle~~, $\frac{S}{4}$ of main wings.

$$M = -\ell_+ \rho V_\infty \Gamma_2' = -\ell_+ L_z$$

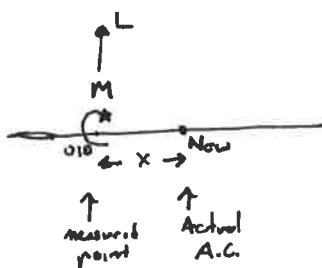
$$C_m = \frac{M}{\frac{1}{2} \rho V^2 c \cdot c} = -\frac{\ell_+ \rho V_\infty \Gamma_2}{\frac{1}{2} \rho V^2 c \cdot c}$$

Lift about $\frac{S}{4}$

$$L = \rho V_\infty (\Gamma_1 + \Gamma_2)$$

$$C_L = \frac{\rho V_\infty (\Gamma_1 + \Gamma_2)}{\frac{1}{2} \rho V^2 c}$$

The aerodynamic center is the location where changing α does not change the moment. Given the moment and lift at a particular location, we can find the a.c..



$$M_{\text{New}} = M_{\text{Old}} + \frac{\partial M}{\partial \alpha} L \cdot x$$

We know that the A.C. is where $\frac{dM_{\text{New}}}{d\alpha} = 0$

So just apply to $M_{\text{New}} = M_{\text{Old}} + L \cdot x$

$$\frac{d(M_{\text{Old}} + L \cdot x)}{d\alpha} = 0 = \frac{dM_{\text{Old}}}{d\alpha} + \frac{dL}{d\alpha} x + L \frac{dx}{d\alpha} \quad x \neq f(\alpha)$$

Solve for x

$$\frac{dM_{\text{Old}}}{d\alpha} = - \frac{dL}{d\alpha} x \Rightarrow x = - \frac{\frac{dM_{\text{Old}}}{d\alpha}}{\frac{dL}{d\alpha}}$$

The offset is

$$x = - \frac{dM}{dL}$$

So if the moment is measured at $\frac{c}{4}$, then the aerodynamic center is at

$$\begin{aligned} X_{\text{ac}} &= \frac{c}{4} - \frac{dM_{\text{Old}}}{dL} = \frac{c}{4} - \frac{d(\frac{1}{2}\rho v^2 c^2 C_m)}{d(\frac{1}{2}\rho v^2 c C_e)} \\ &= \frac{c}{4} - \frac{dC_m}{dC_e} \cdot c \end{aligned}$$

$$\boxed{\frac{X_{\text{ac}}}{c} = \frac{1}{4} - \frac{dC_m}{dC_e}}$$

More about this in AEM 368.

For a multi surface aircraft, the A.C. is called the "Neutral Point"