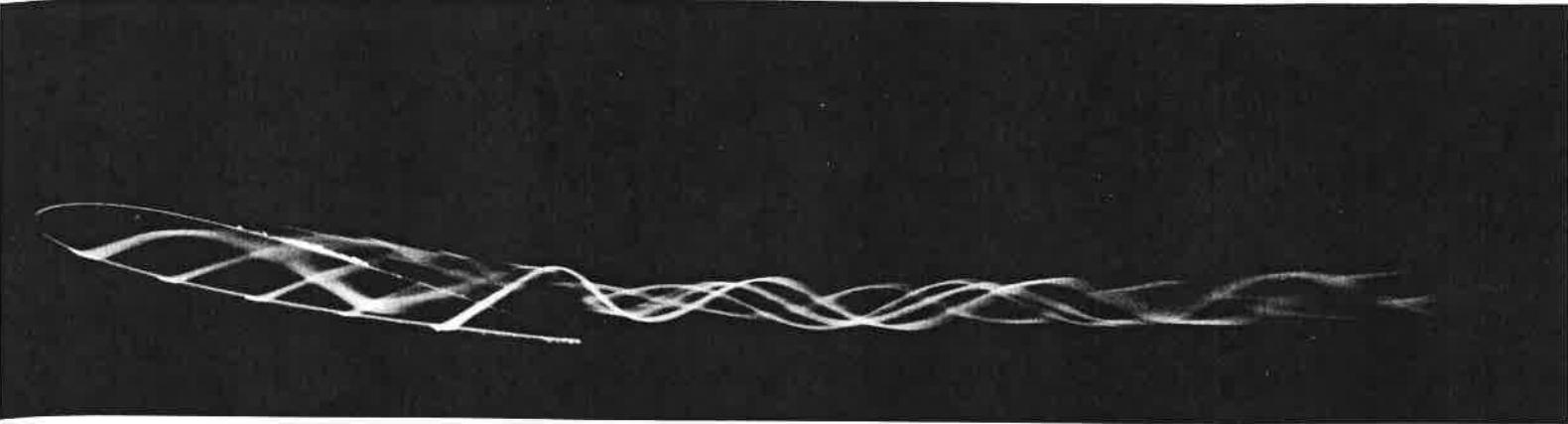


Lesson 15
Wing Aerodynamics
(Chapter 5 in textbook)

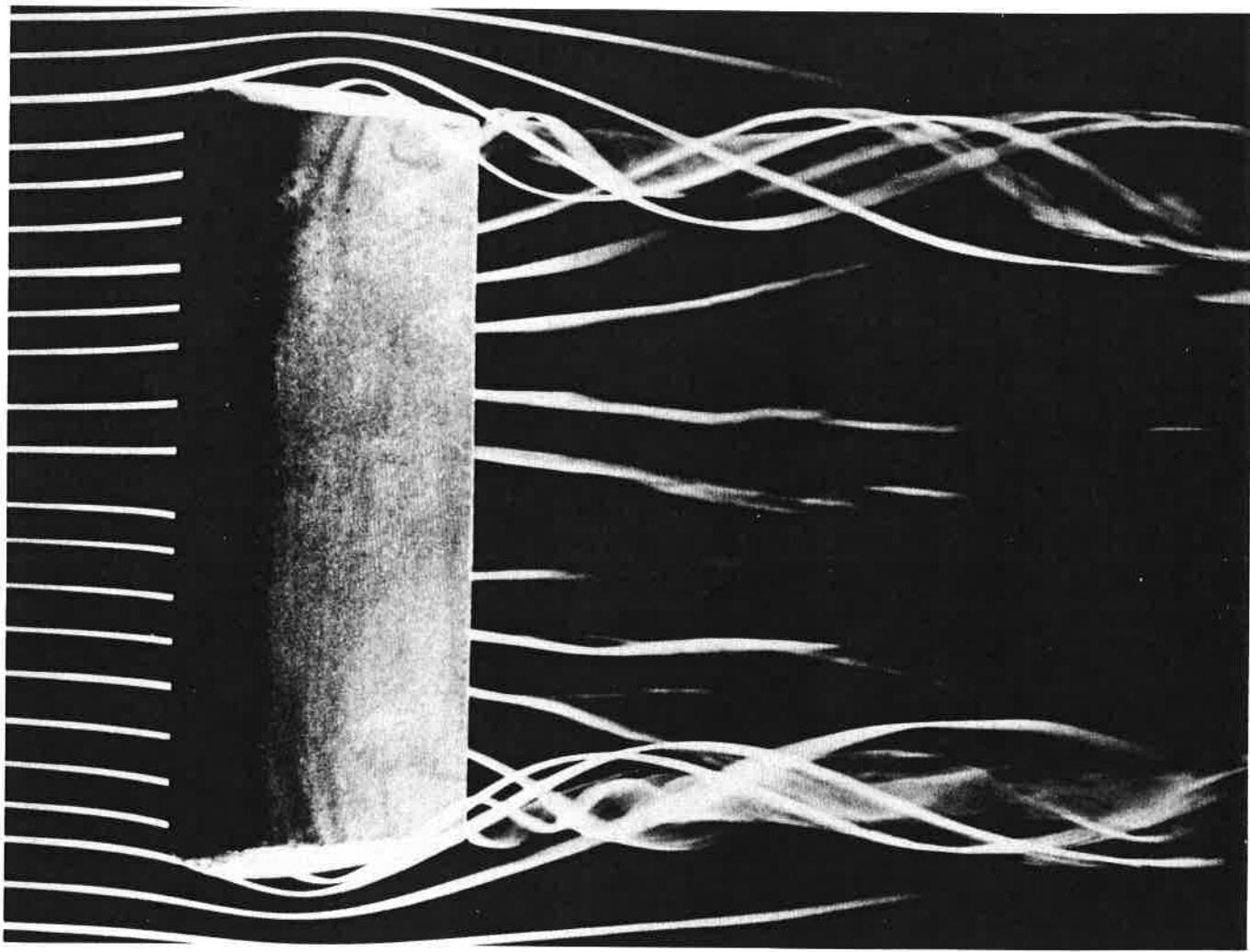
[tiny.cc/AEM Condit](http://tiny.cc/AEM_Condit)
[tiny.cc/AEM Wing Vortex](http://tiny.cc/AEM_Wing_Vortex)

06:44 - 1:10
3:13 - 4:36
5:10 - 5:30



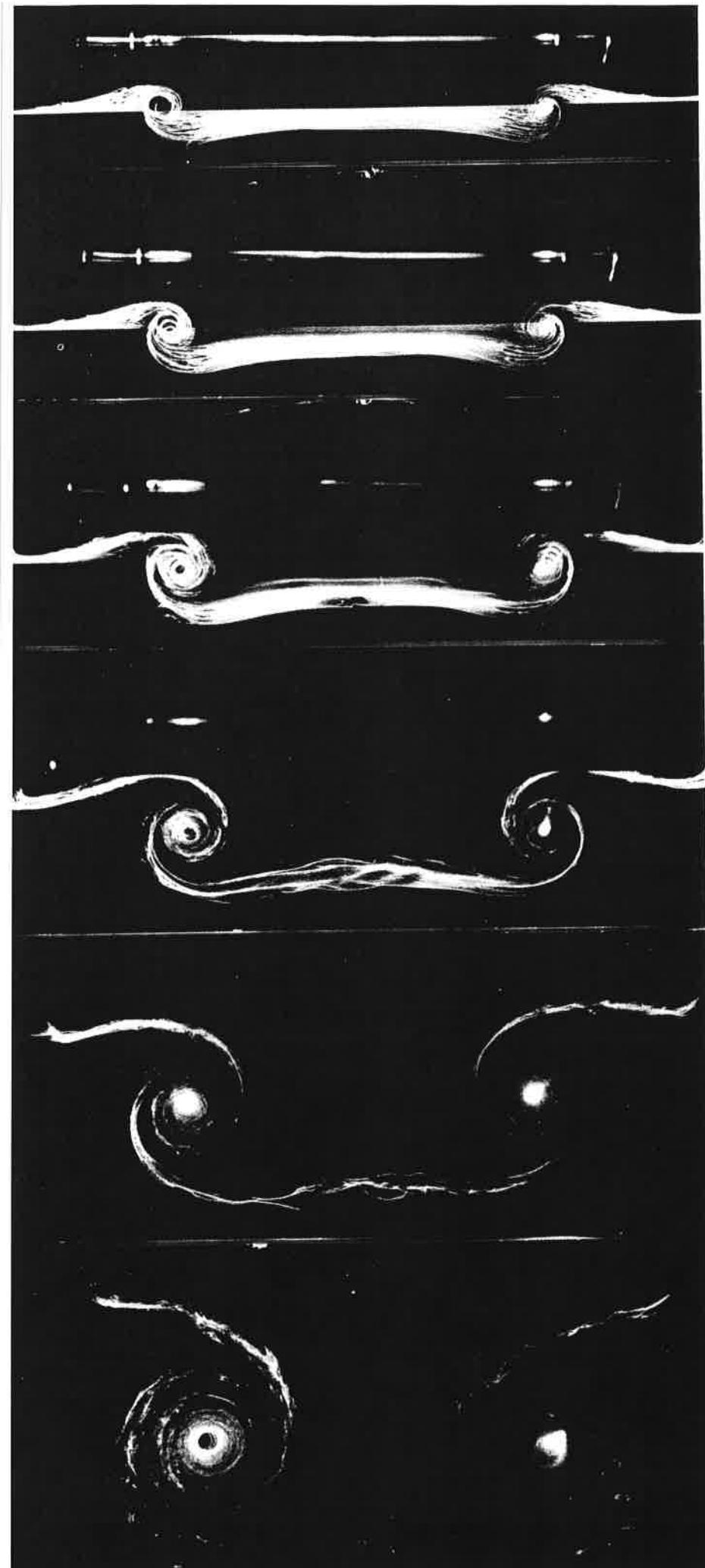
85. Trailing vortex from the tip of a rectangular wing. At 12.5° angle of attack the vortex is seen to separate well ahead of the trailing edge. The wing has an NACA 0012 profile and aspect ratio 4. At this Reynolds number of

10,000 the wake is laminar, in contrast to the opposite page. Visualization is by colored fluid in water. ONERA photograph, Werlé 1974

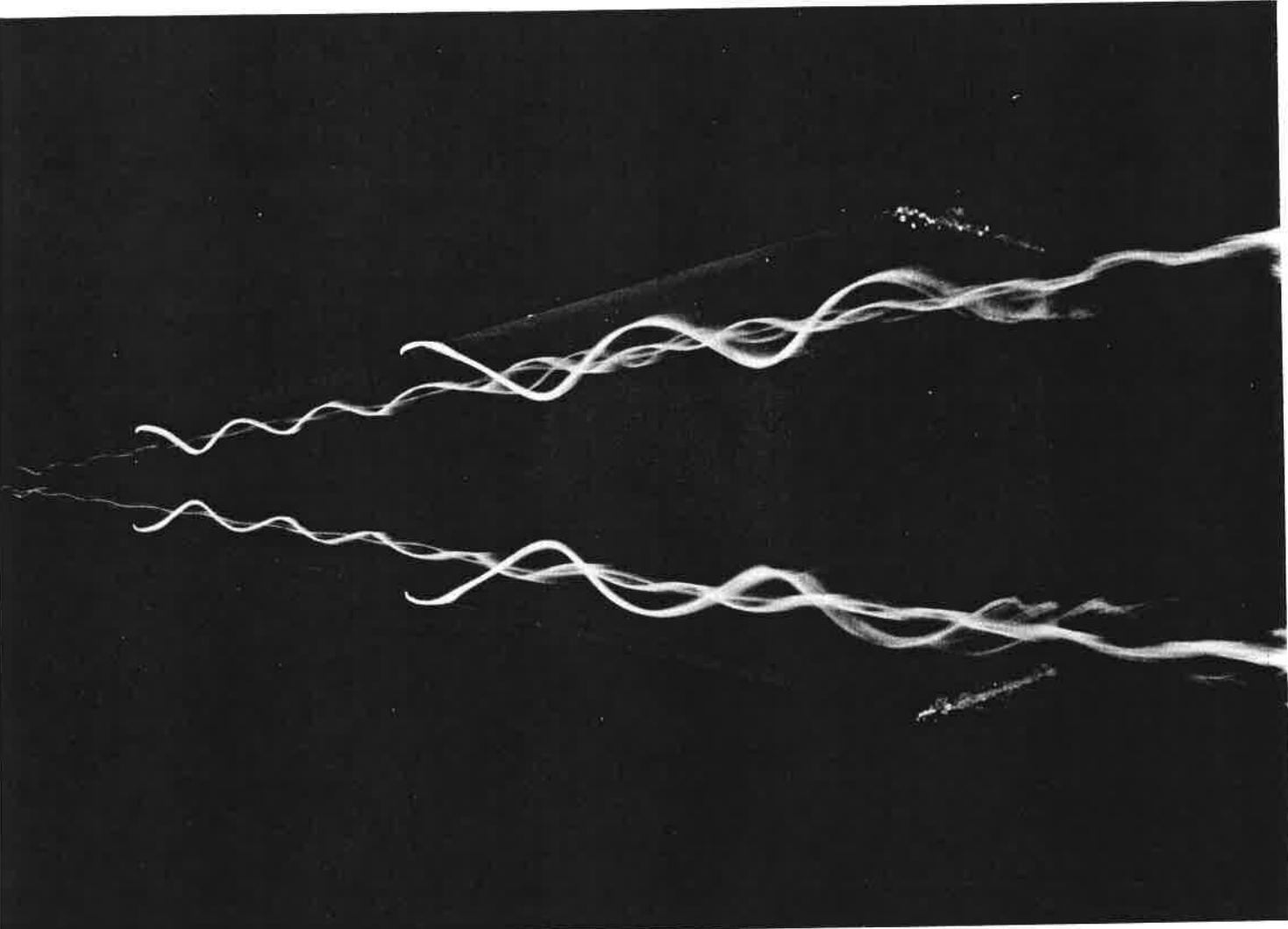


86. Trailing vortices from a rectangular wing. Suction is applied so that at 24° angle of attack the flow remains attached over the entire wing surface, in contrast to the preceding photograph. The centers of the vortex cores there-

fore spring from the trailing edge at the tips. The model is made of perforated metal covered with blotting paper, and tested in a smoke tunnel at Reynolds number 100,000. Head 1982

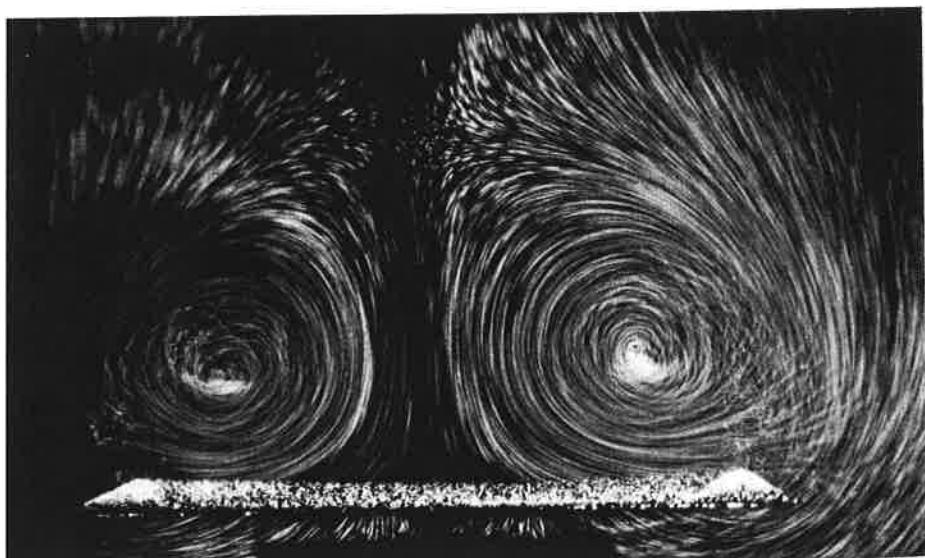


84. Cross-sections of the vortex sheet behind a rectangular wing. The wake rolling up behind a rectangular wing at 90° angle of attack is seen at distances behind the trailing edge of 1.0, 1.6, 2.9, 5.5, 11.2, and 21 chord lengths. The wing has a Clark Y section 10-percent thick, a chord of 0.125 m, and a span of 0.3 m, with tips cut off square. It is supported by fine wires and towed through water. The wake is visualized by hydrogen bubbles emitted from a $30 \mu\text{m}$ wire located just behind the trailing edge and illuminated by xenon lamps. The vortices separate from the wing surface just behind mid-chord. The Reynolds number is 100,000 based on chord. The vortex sheet is initially turbulent, but is relaminarized farther downstream. Photograph by H. Bippe



90. Vortices above an inclined triangular wing. Lines of colored fluid in water show the symmetrical pair of vortices behind a thin wing of 15° semi-vertex angle at 20° angle of attack. The Reynolds number is 20,000 based on

chord. Although the Mach number is very low, the flow field is practically conical over most of the wing, quantities being constant along rays from the apex. ONERA photograph, Werlé 1963

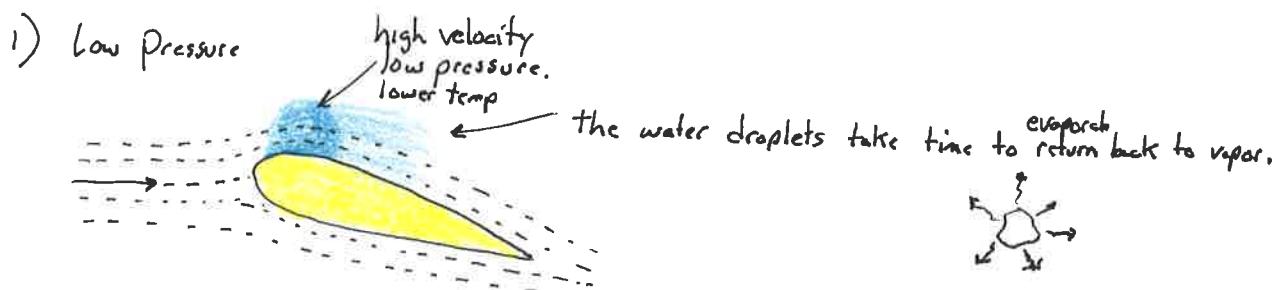


91. Cross section of vortices on a triangular wing. Tiny air bubbles in water show the vortex pair for the flow above in a section at the trailing edge of the wing. ONERA photograph, Werlé 1963

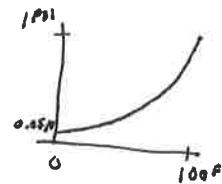
Why Contrails?

Air has water. As the local pressure drops, the humidity ratio increases until the water vapor condenses. We call this a cloud.

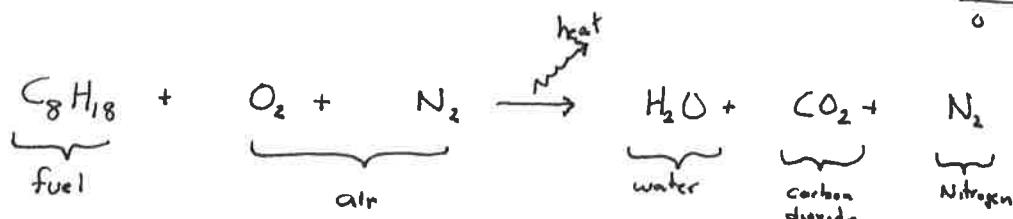
- ① An aircraft or rocket has areas of low pressure such that clouds form.
- ③ Water is added to the air such that $\phi \rightarrow 100\%$ and clouds form.



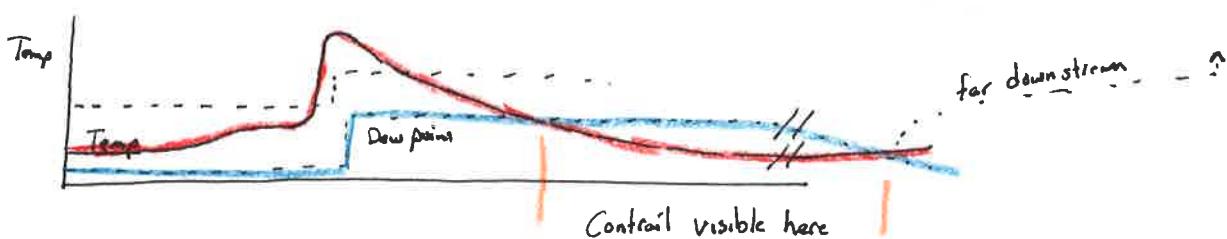
Remember the Arden-Buck eqn for $P_{sat}(T)$



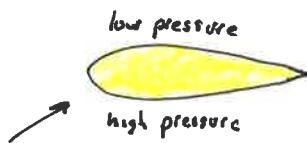
- 2) Combustion



to balance $12.5\text{ O}_2, 50\text{ N}_2, 9\text{ H}_2\text{O}, 8\text{ CO}_2$ and 50 N_2
every mole of fuel (octane) gives 9 moles of water vapor



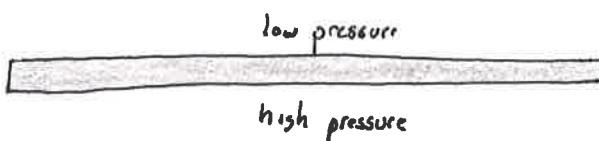
Initial Concept



side view



Iso-view

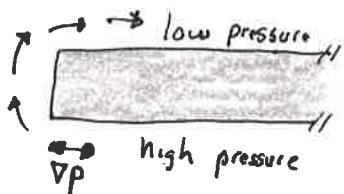


back view

$$\text{In Lagrangian frame, } \rho \frac{DV}{Dt} = -\nabla p \Rightarrow \frac{DV}{Dt} = -\frac{1}{\rho}(\nabla p)$$

$$\frac{DU}{Dt} = -\frac{1}{\rho} \frac{dp}{dx} \quad \frac{Dr}{Dt} = -\frac{1}{\rho} \frac{dp}{dy}$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{dp}{dz}$$



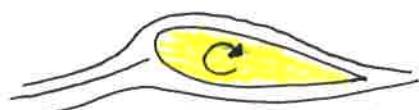
1) Fluid flow is driven from high to low pressure.

2) At the wingtip the pressure difference must be finite (zero!) so $\frac{Dw}{Dt}$ is finite.

$\frac{p^+}{p^-}$ gives $\nabla p = \infty$ and thus infinite acceleration.
(a hint of Kutta condition ?!)

3) A wing is not an airfoil. $C_L(\alpha) \neq C_e(\alpha)$

Recall that the 2D airfoil generates lift proportional to circulation



Model \Rightarrow

$\rightarrow + \Gamma$

$L' = \rho V_\infty \Gamma$

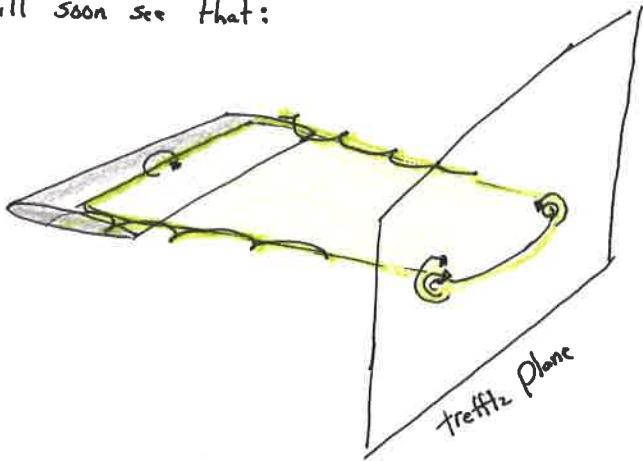
The diagram shows a 2D airfoil with a yellow shaded region representing circulation C . To its right is a model equation \Rightarrow followed by four horizontal arrows pointing right, plus a symbol representing circulation Γ , and finally the formula $L' = \rho V_\infty \Gamma$.

On the previous slide, we saw that a pressure differential at the wing tip tends to create flow going around the tip.



This is not a coincidence!

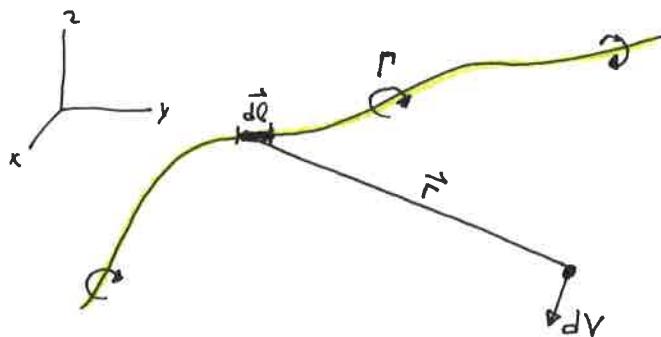
In fact, we will soon see that:



Vortex model of wing aerodynamics

↑ why model? A vortex is only a simplified model of an actual distributed vorticity field.
A vortex can't really exist since $V(r=0) = \infty$ (in other words, the vorticity was squashed into a single point)

Vortex Filament (3D space)



Strength Γ

$$\text{Biot Savart Law} \quad d\vec{V} = \frac{\Gamma}{4\pi} \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

A small segment dl has a consistent velocity of dV at vector distance \vec{r} with Γ .

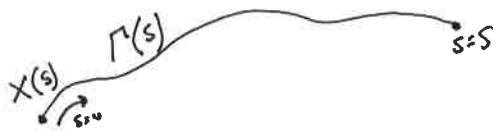
Aside: Most references will say " Γ imposes a velocity dV with segment dl at \vec{r} "
 { induces }

- We know this is not quite right, since the definition of a vortex's velocity is just a consistent velocity field (irrotational, etc)

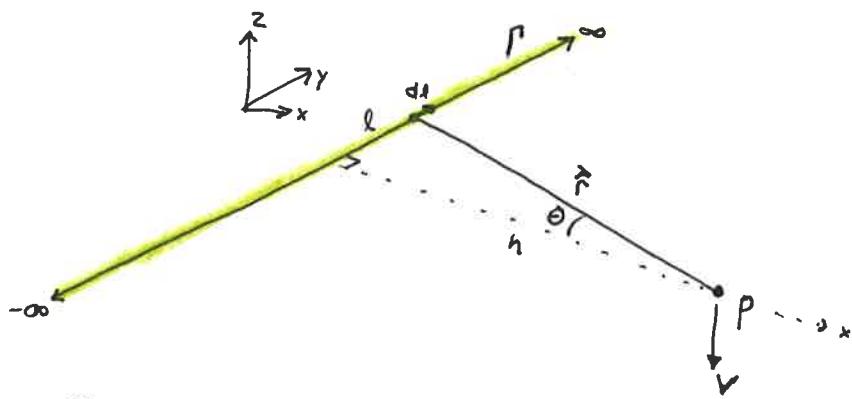
Ex: Given a vortex filament of strength Γ and a ~~written~~ path (x, y, z) , how can you find the velocity consistent with that vortex at point (x_p, y_p, z_p) ?

$$V = \int_0^V dV \quad \text{Integrate}$$

$$= \int_{\text{path}} \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3} \quad \text{where } r \text{ is the } \cancel{\text{distance}} \text{ vector from the } dl \text{ segment} \\ \bullet p \text{ to } (x_p, y_p, z_p)$$



Ex: Infinite vortex filament



Notice θ angle in this derivation.

$$V = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3}$$

From geometry/trig:

$$h = r \cos \theta \quad \text{and} \quad \tan \theta = \frac{l}{h}$$

Since the filament is flat and in the y direction, V is in the z direction ($dl \times r$)

$$\text{So, } dl \times r = dl \cdot h$$

Find differential dl .

$$l = h \tan \theta \Rightarrow dl = d(h \tan \theta) = h d(\tan \theta) = h \frac{1}{\cos^2 \theta} d\theta$$

$$V = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3} = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{dl \cdot h}{r^3} = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{dl \cdot r \cos \theta}{r^3} = \underbrace{\int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \left(\frac{h}{\cos^2 \theta} \right) \left(\frac{r}{r^3} \right) \cos \theta}_{\text{Notice that } l=\infty \Rightarrow \theta=\pi/2, l=-\infty \Rightarrow \theta=-\pi/2 \text{ and } r = \frac{h}{\cos \theta}}$$

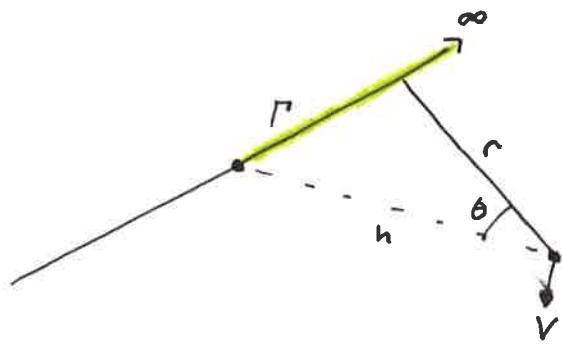
$$V = \int_{-\pi/2}^{\pi/2} \frac{\Gamma}{4\pi} \frac{h d\theta}{\cos^2 \theta} \frac{\cos \theta}{h^2} \cos \theta = \int_{-\pi/2}^{\pi/2} \frac{\Gamma}{4\pi} \frac{\cos \theta d\theta}{h} = \frac{\Gamma}{4\pi h} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

Symmetric

$$= \frac{2\Gamma}{4\pi h} \int_0^{\pi/2} \cos \theta d\theta = \frac{\Gamma}{2\pi h}$$

$$V_{\text{infinite filament}} = \frac{\Gamma}{2\pi h}$$

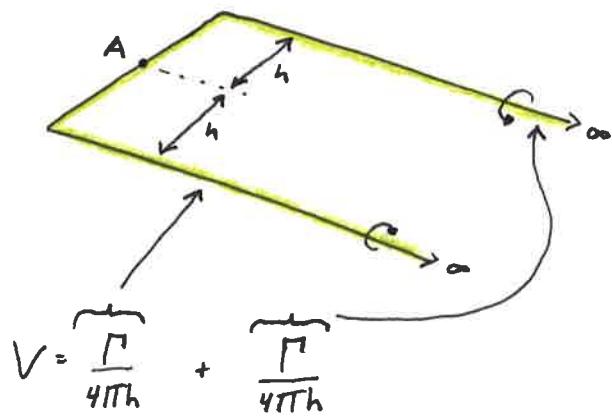
Ex: Sem: Infinite Filament



By inspection, half the velocity of the infinite filament

$$V = \frac{\Gamma}{4\pi h}$$

Ex: Horseshoe Vortex: What is the velocity at point A?



Q: What about the connecting left-hand portion? Doesn't it contribute?!?

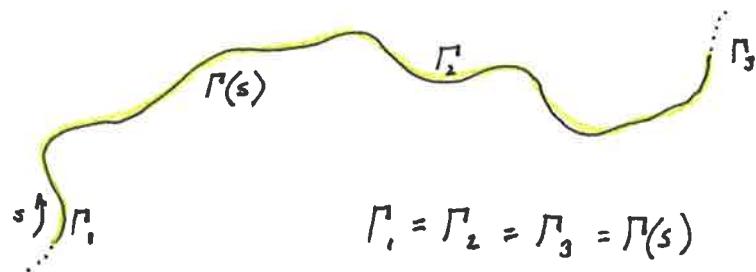
No. The bound vortex "induces" zero velocity on itself.

Remember, this is only a model

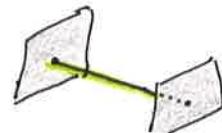
Helmholtz Vortex Theorems.

Assuming an incompressible and inviscid flow (and implies vorticity lumping into vortex)

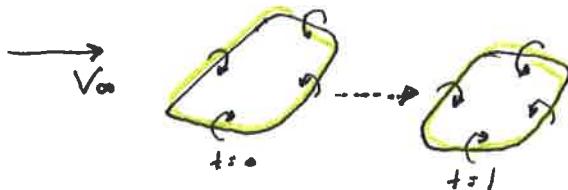
- 1) The strength of a vortex is constant along its length



- 2) A vortex filament can not start or end in a fluid. A filament must either be
 - closed
 - Start/stop on a solid surface
 - Extend to infinity



- 3) A vortex element convects downstream while remaining a vortex element.

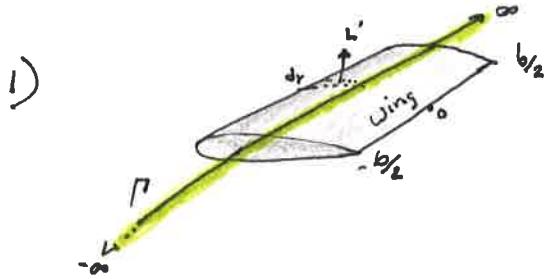


Wing Aerodynamics (with a vortex filament model)

We already know that a 2D airfoil generates lift with a circulation (vortex)



We will follow the historical development by considering possibilities.



Circulation from a wing extends to $\pm \infty$ out the tips.

$$L' = \rho V R \quad \text{and} \quad \int_{-\infty}^{\infty} L' dy = \infty \quad \text{Infinite lift!}$$

$$C_L = \int C_\theta dy \quad \underline{\text{No}}$$

2) Constant circulation on wing surface

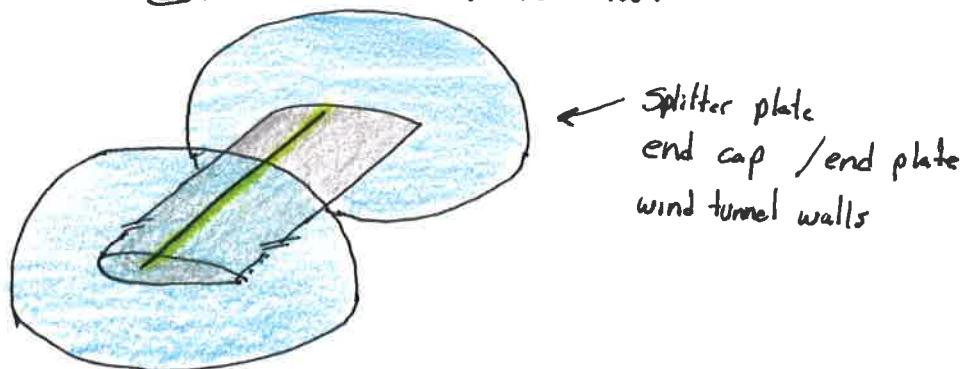


$$C_L = \int C_\theta dy = \int_{-b/2}^{b/2} \frac{2R}{z} \quad \underline{\text{Finite!}} \quad \underline{\text{Wrong!}}$$

Fundamental physics error... Helmholtz theorem.

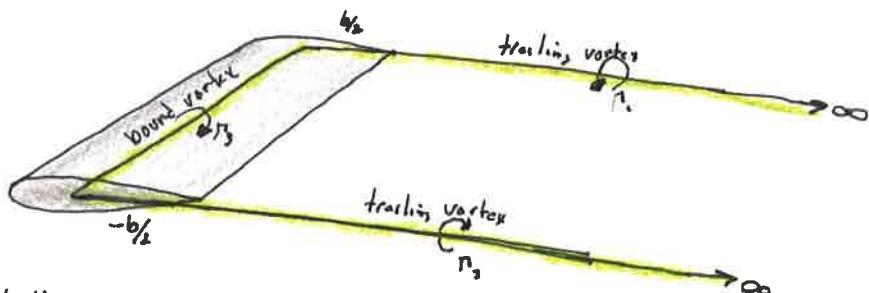
The vortex can not end in a fluid (i.e. at wingtip)

- For this to be possible, we would need to add a solid surface at the tips.
This is exactly how 2D airfoils are tested.

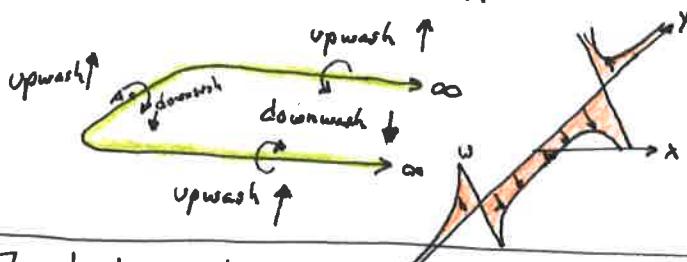


Clearly, we don't have airplanes flying around with those plates!

3) Vortex at wingtip is convected downstream (Helmholtz)



- Satisfies Helmholtz.
- Trailing vortices create a downwash within the wake region behind the wing and upwash outside of the wake.



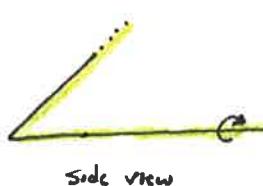
The trailing vortices change the velocity components at the bound vortex

How much? We already calculated this for the center point "A".

$$V_{\text{bound}} = V_\infty + \frac{\Gamma}{2\pi b}$$

Still a problem....

$$V_p = \frac{\Gamma}{4\pi b}$$



The velocity at the wing tip is infinite!

$$V = \frac{\Gamma}{4\pi b} \rightarrow \pm \infty$$

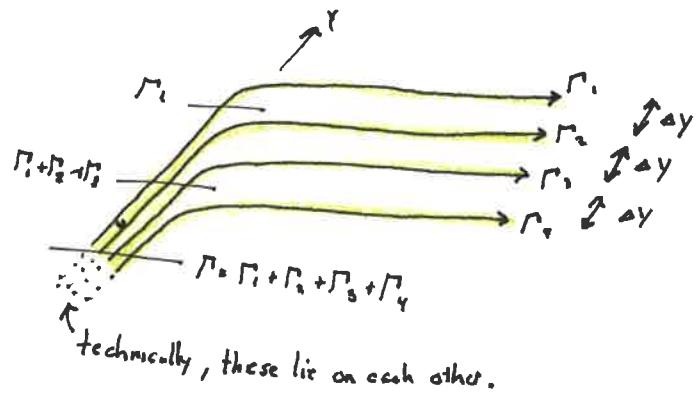
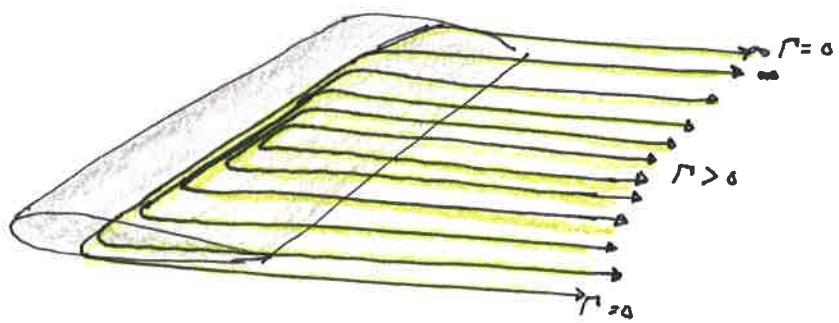
Recall the Kutta condition for airfoils

$$\gamma_{\text{TE}} = 0$$



The vortex strength at the wingtips must also be zero.

4) Stack discrete horseshoes along wingspan.



In the limit as the number of horseshoes $\rightarrow \infty$ and $\Delta y \rightarrow 0$



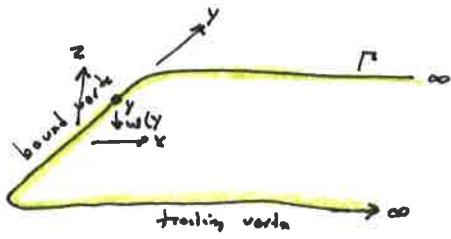
$$C_L = \int_{-b/2}^{b/2} C_e(y) dy \quad \text{also} \quad L = \int_{-b/2}^{b/2} L' dy = \int_{-b/2}^{b/2} \rho V_\infty \Gamma(y) dy$$

Correct Physics

Prondtl Lifting Line Theory

Germany 1910s

Prandtl Lifting Line Theory



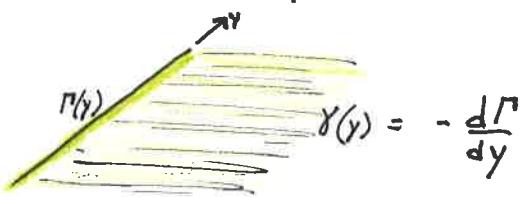
Given a vortex filament of strength Γ , the velocity along the bound vortex is

$$w(y) = -\frac{\Gamma}{4\pi(b/2+y)} - \frac{\Gamma}{4\pi(b/2-y)}$$

$$= -\frac{\Gamma}{4\pi} \frac{b}{(b/2)^2 - y^2}$$

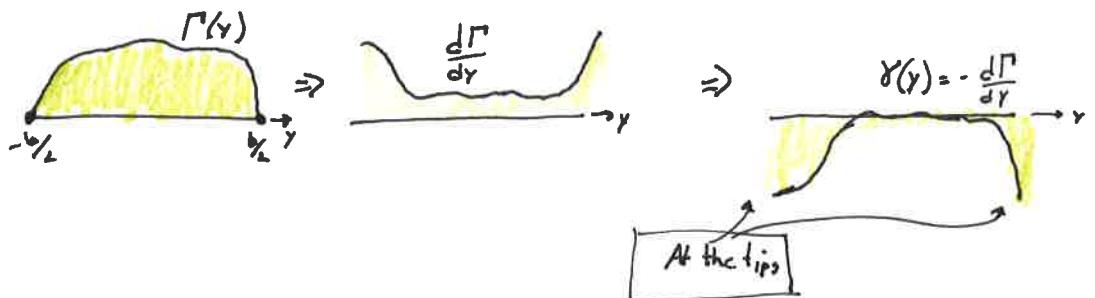


In the limit as the horseshoe $\Delta y \rightarrow 0$



The strength of the wake vortex depends on the rate of change of the bound vortex strength

Ex: Given that the bound vortex $\Gamma(y)$ strength changes rapidly at the wing tip for a particular wing, where is $\gamma(y)$ in the wake strongest?



Ex: Given that we see vapor clouds corresponding to γ magnitudes, can you identify the strongest gradients in the lift distribution?