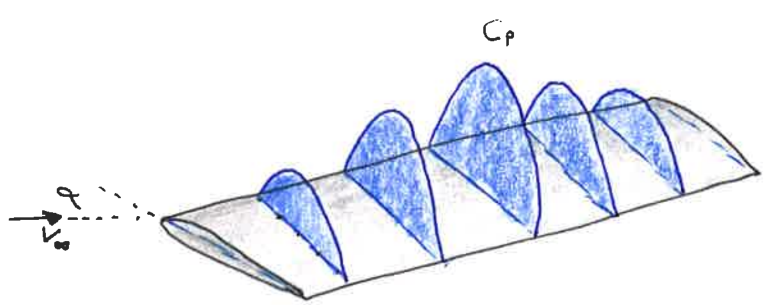
Lesson 17

Applications of Prandtl Lifting Line



Ex: Estimate C_{D_i} for a Cessna 172 wing cruising at 150 ft/s.

$W = 2450 \text{ lb}$ $\quad \lambda \approx 0.7$

$S = 174 \text{ ft}^2$

$b = 36 \text{ ft}$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{2450 \cancel{\text{lbf}}}{\frac{1}{2}} \left| \frac{\cancel{\text{ft}^3}}{0.00237 \cancel{\text{slugs}}} \right| \frac{\cancel{\text{s}^2}}{150^2 \cancel{\text{ft}^2}} \left| \frac{}{174 \cancel{\text{ft}^2}} \right| \frac{\cancel{\text{slugs ft}}}{\cancel{\text{lbf s}^2}} = 0.528$$

The 172 wing is not elliptical. Look up e from notes.

$$e \approx 1.78\left(1 - 0.045 A^{0.68}\right) - 0.64$$

$$\uparrow \frac{b^2}{S} = \frac{36^2}{174} = 7.45$$

$$\approx 0.826$$

$$C_{D_i} = \frac{C_L^2}{\pi AR\, e} = \frac{0.528^2}{\pi} \left| \frac{}{7.45} \right| \frac{}{0.826} = 0.01442 = 144 \text{ counts}$$

$$\boxed{C_{D_i} = 144 \text{ counts}}$$

How much power is necessary?

$$P = D \cdot V = \frac{1}{2}\rho V^2 S C_D V = \frac{1}{2} \left| \frac{0.00237 \cancel{\text{slugs}}}{\cancel{\text{ft}^3}} \right| \frac{150^2 \cancel{\text{ft}^2}}{\cancel{\text{s}^2}} \left| 174 \cancel{\text{ft}^2}\text{ft} \right| 0.01442 \left| \frac{150 \cancel{\text{ft}}}{\text{s}} \right| \frac{\text{lbf} \cancel{\text{s}^2}}{\cancel{\text{slugs ft}}}$$

$$= 10000 \frac{\text{ft-lbf}}{\text{s}} \left| \frac{\text{s Hp}}{550 \text{ ft lbf}} \right. = \boxed{18 \text{ Hp} = P}$$

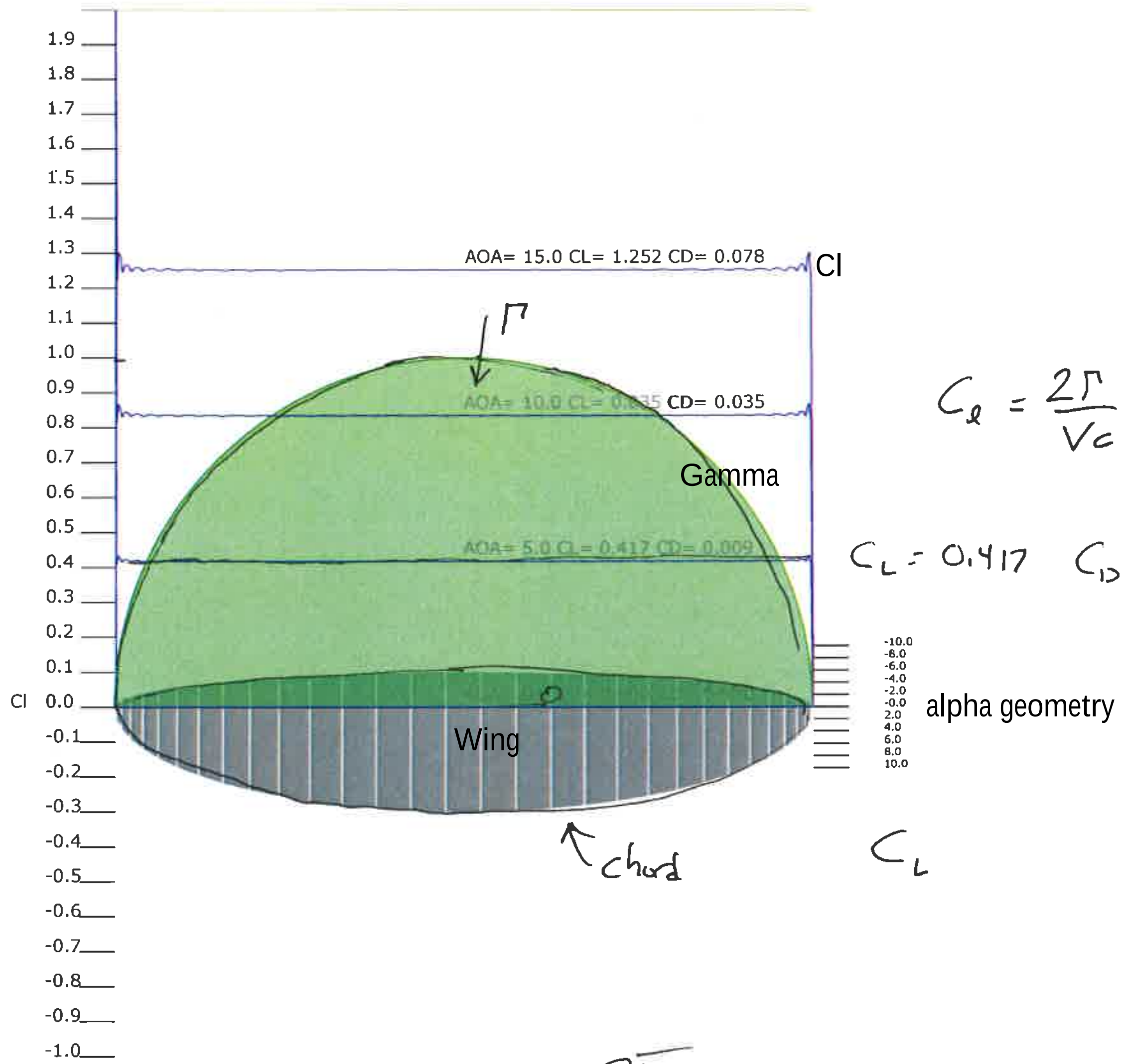
If you replaced the tapered 172 wing with an elliptical wing (same b, AR, S), what is the result for C_{D_i} and P?

$$C_{D_i} = \frac{C_L^2}{\pi AR\, e} = \frac{0.528^2}{\pi} \left| \frac{}{7.45} \right| \frac{}{1} = 0.0119 \Rightarrow C_{D_i} = 119 \text{ counts}$$

$$= 144 \cdot e = 144 \cdot 0.826$$

$$P_{ellip} = P \cdot e = 14.8 \text{ Hp}$$

$\approx 10\%$ of 140 Hp engine



$C_\ell = \frac{2\Gamma}{Vc}$

$C_L = 0.417 \quad C_D = 0.008$

C_L

$$\frac{dC_L}{d\alpha} = C_{L_\alpha} = \frac{2\pi}{1+\frac{2\pi}{\pi AR}}$$

$AR = 6.4$

$$= \frac{2\pi}{1+\frac{2}{AR}} = 4.78$$

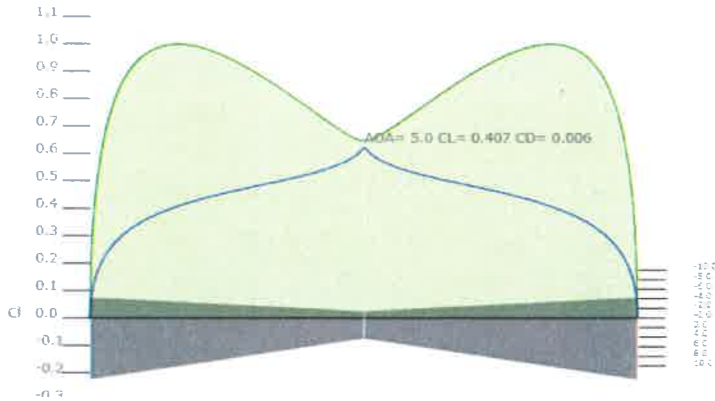
$C_{L_\alpha} = 0.083$

$$C_{D_i} = \frac{C_L^2}{\pi AR} = 0.0086$$

How does taper affect lift dist'?

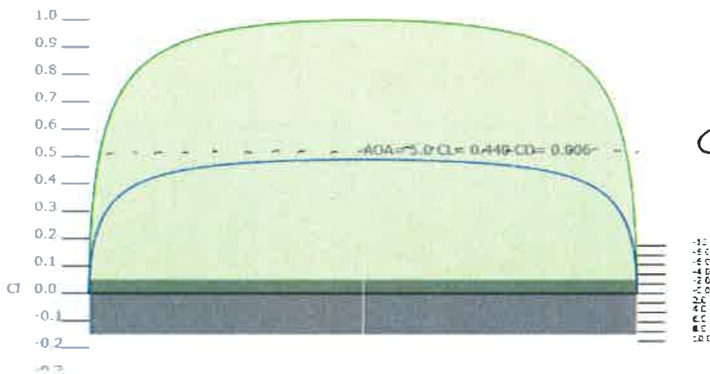
Geometry

AR = 10



$\lambda = 3$

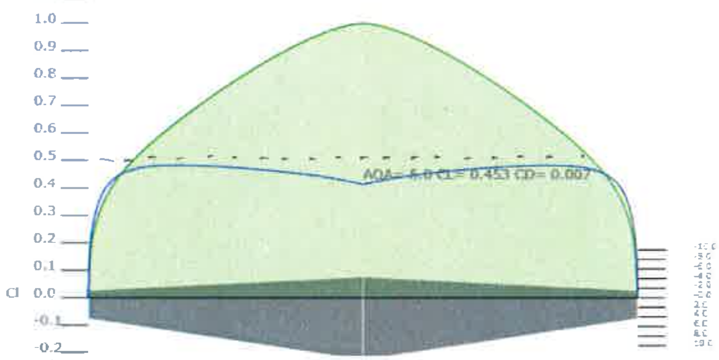
L/D=68



$C_{l_{max}}$

$\lambda = 1$

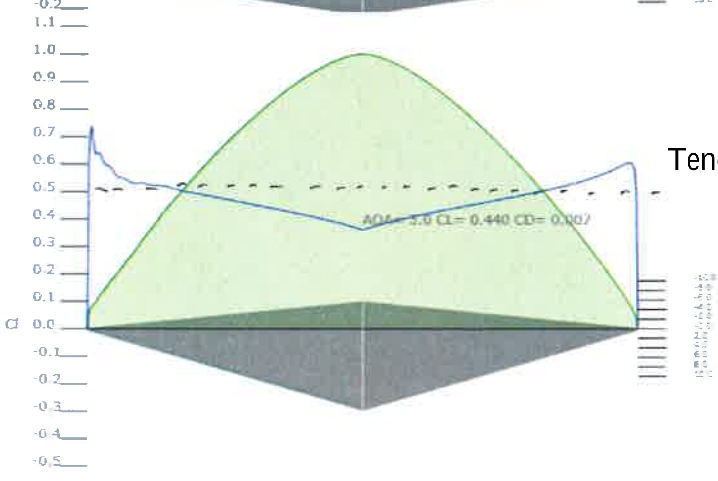
L/D=73



$\lambda = 1/3$

L/D=65

Tends to stall at wingtips



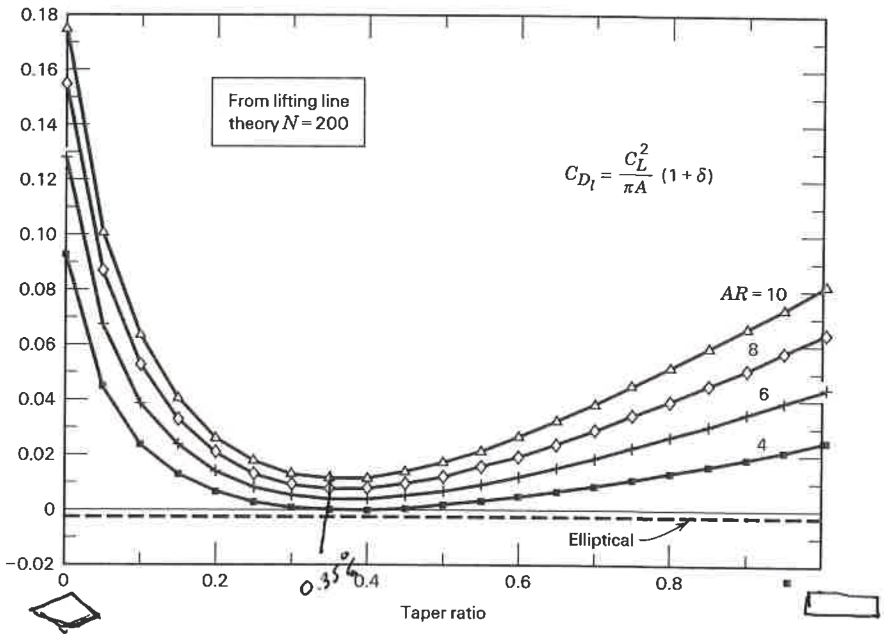
$\lambda = 0$

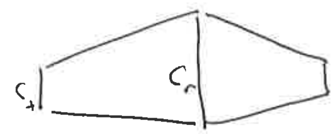
L/D=62

Wing Design

Is there an optimal taper ratio?

$\frac{C_{D_i}}{C_{D_{i,ell}}}$

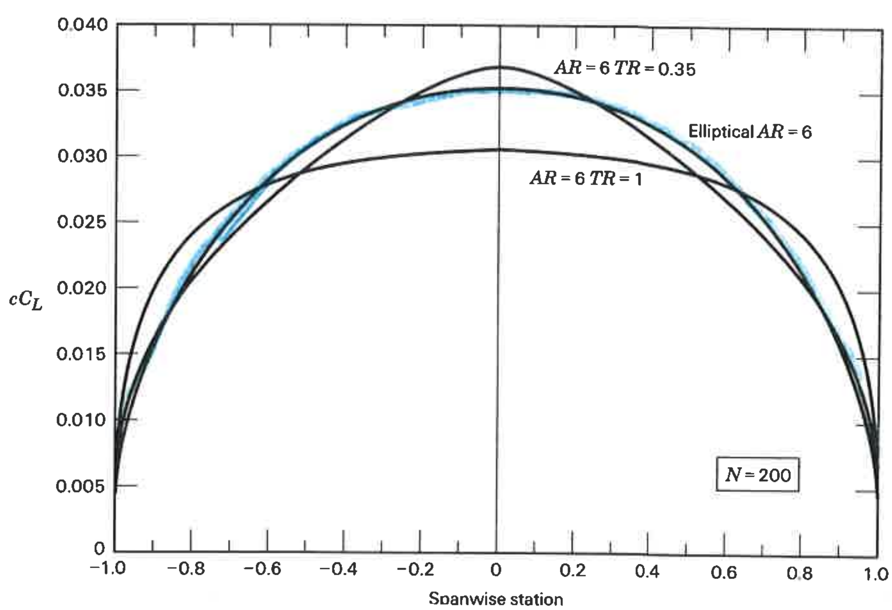




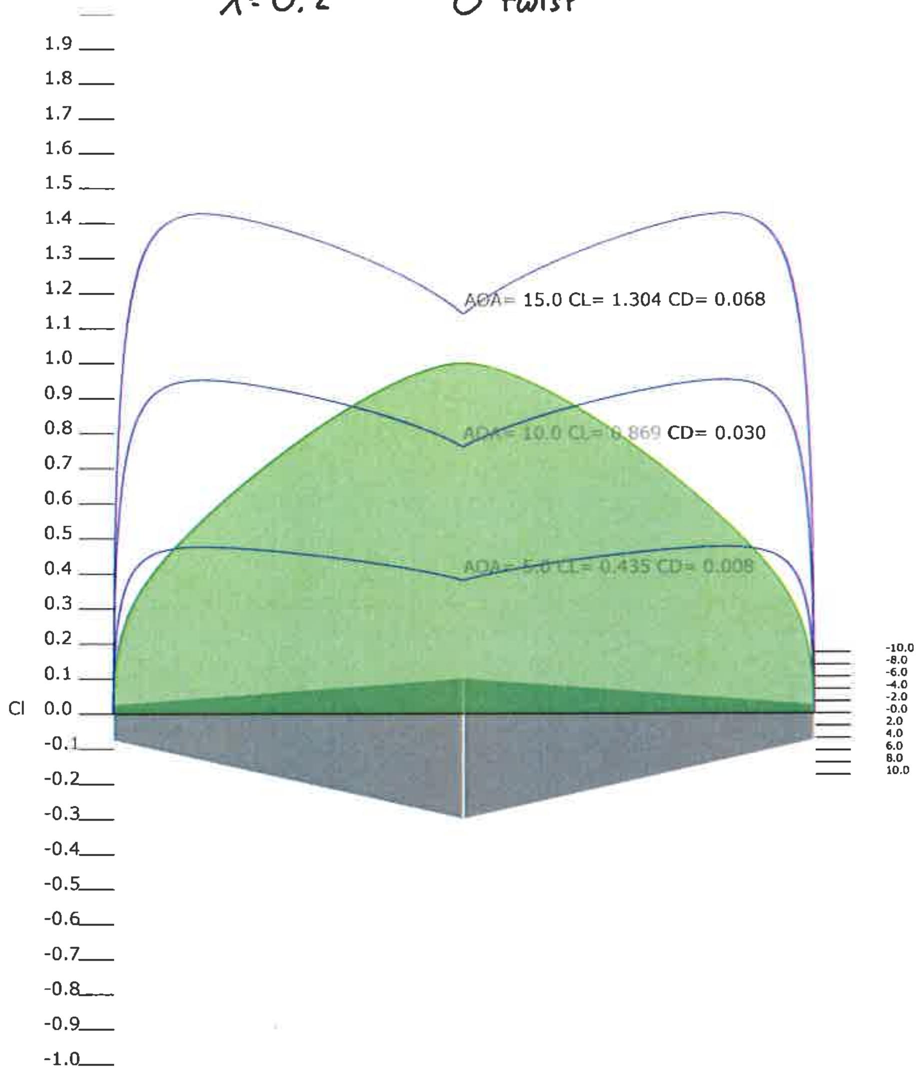
$\frac{C_t}{C_r} \approx 35\%$

Can a linearly tapered wing approach elliptical efficiency?

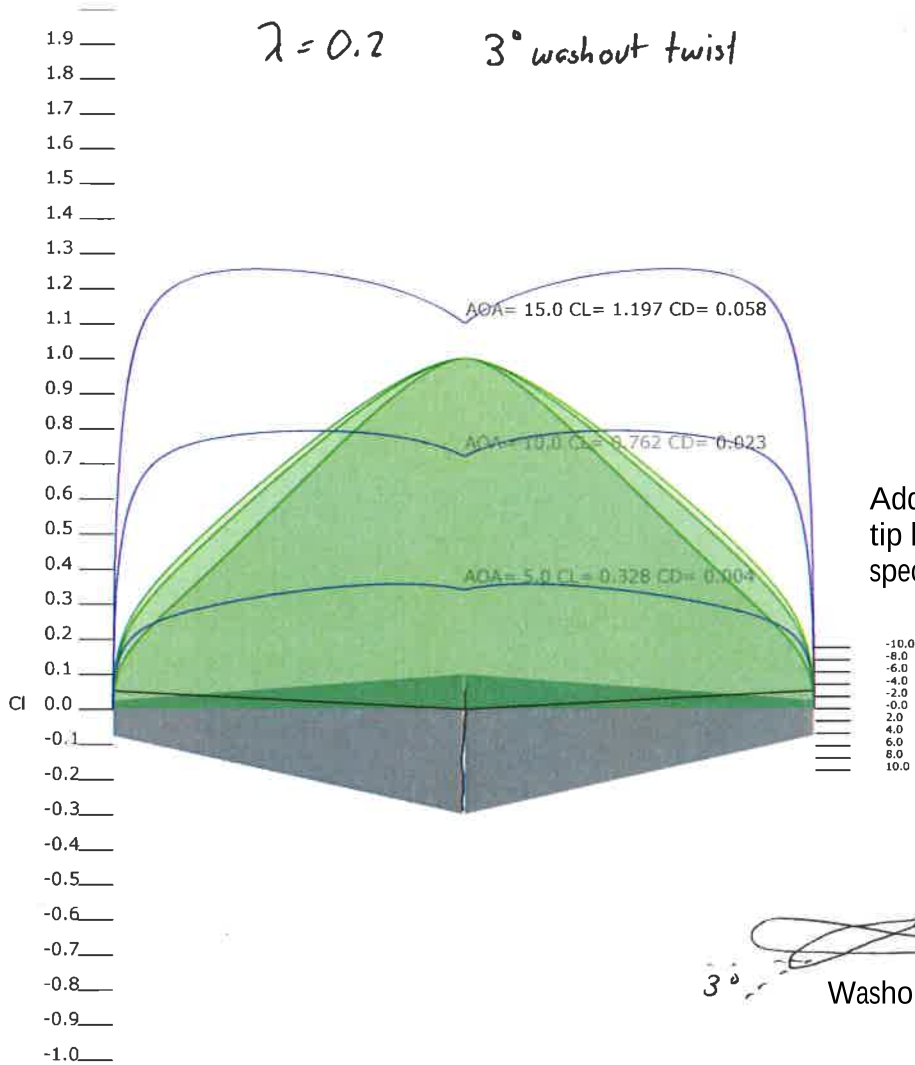
Yes



$\lambda = 0.2$ 0 twist

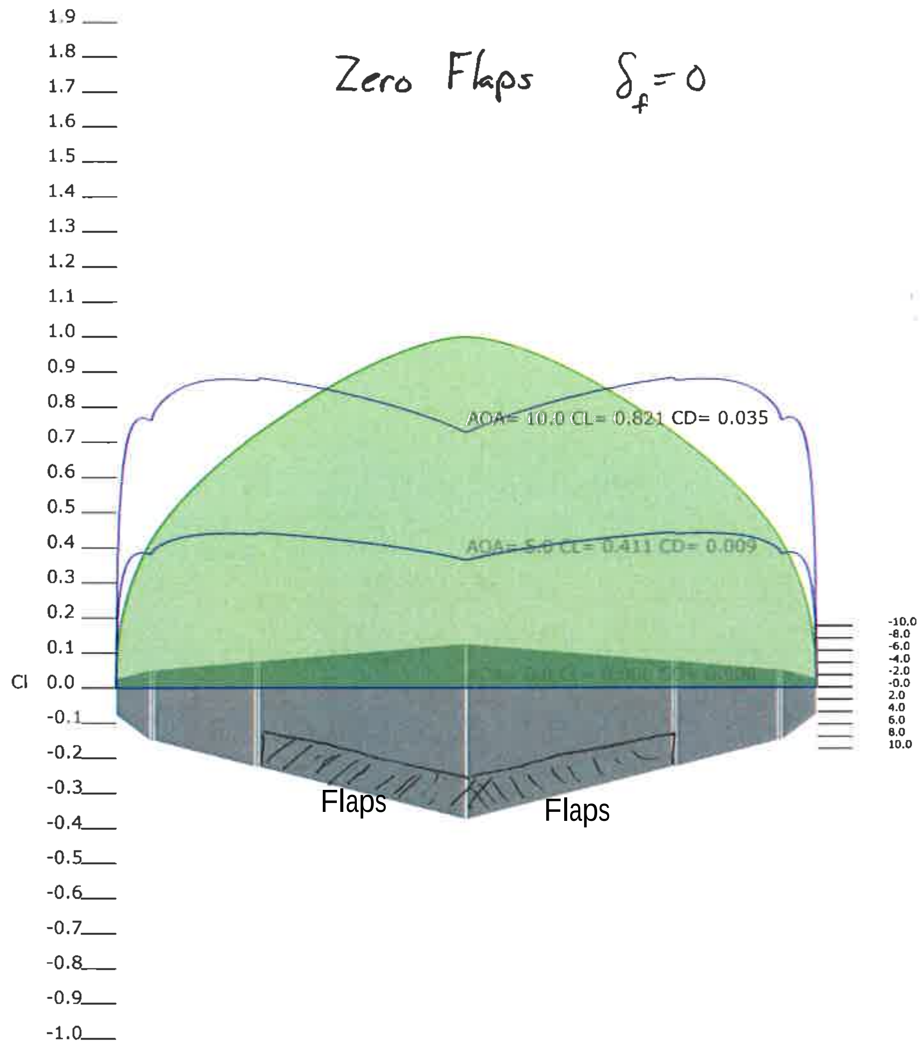


λ = 0.2 3° washout twist



Adding twist improves the tip loading, but only near a specified CL

Washout is common

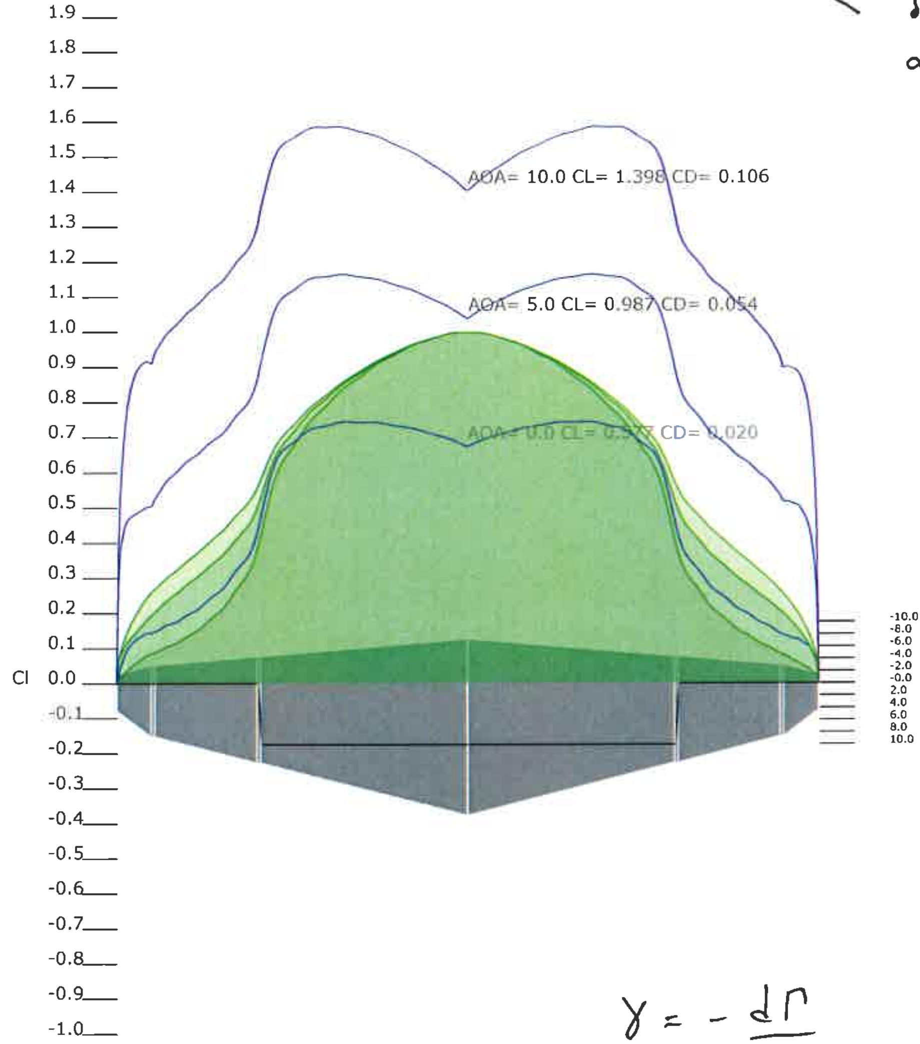

Zero Flaps $\delta_f = 0$
Cl
AOA= 10.0 CL= 0.821 CD= 0.035
AOA= 5.0 CL= 0.411 CD= 0.009
Flaps
Flaps

$\alpha_{L} = 10^\circ$ Flaps

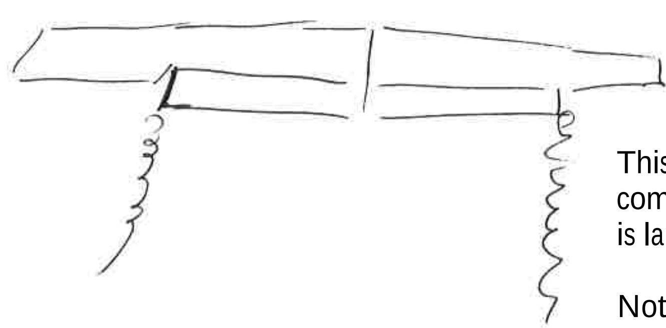
20%

$\delta = 20^\circ$

$\alpha_{zL} \approx -10^\circ$



$$\gamma = -\frac{d\Gamma}{dy}$$

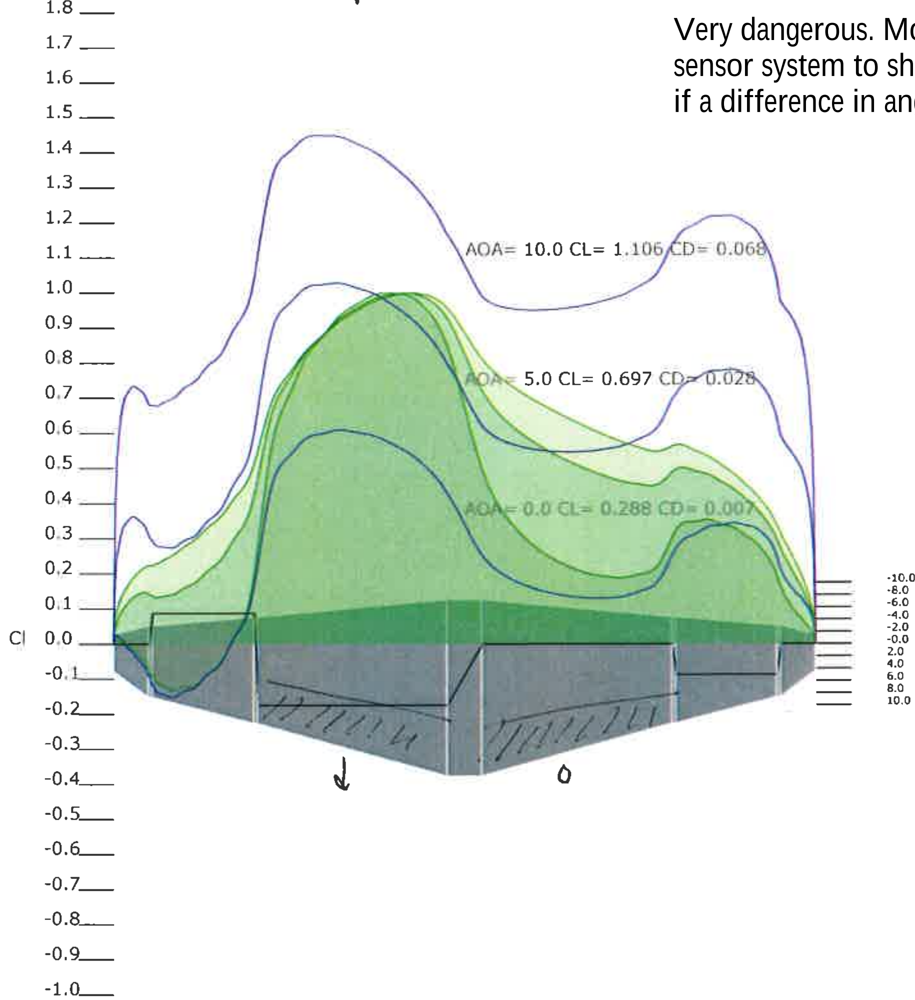


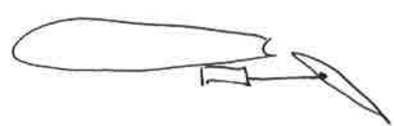
This is why we saw vortex contrails coming off the flap regions. dG/dy is larger here than at tip.

Note that it is INCORRECT to say that the vortices are ONLY at the wingtips. In fact, the entire wing sheds vorticity, but that vorticity only "rolls up" and coalesces downstream.

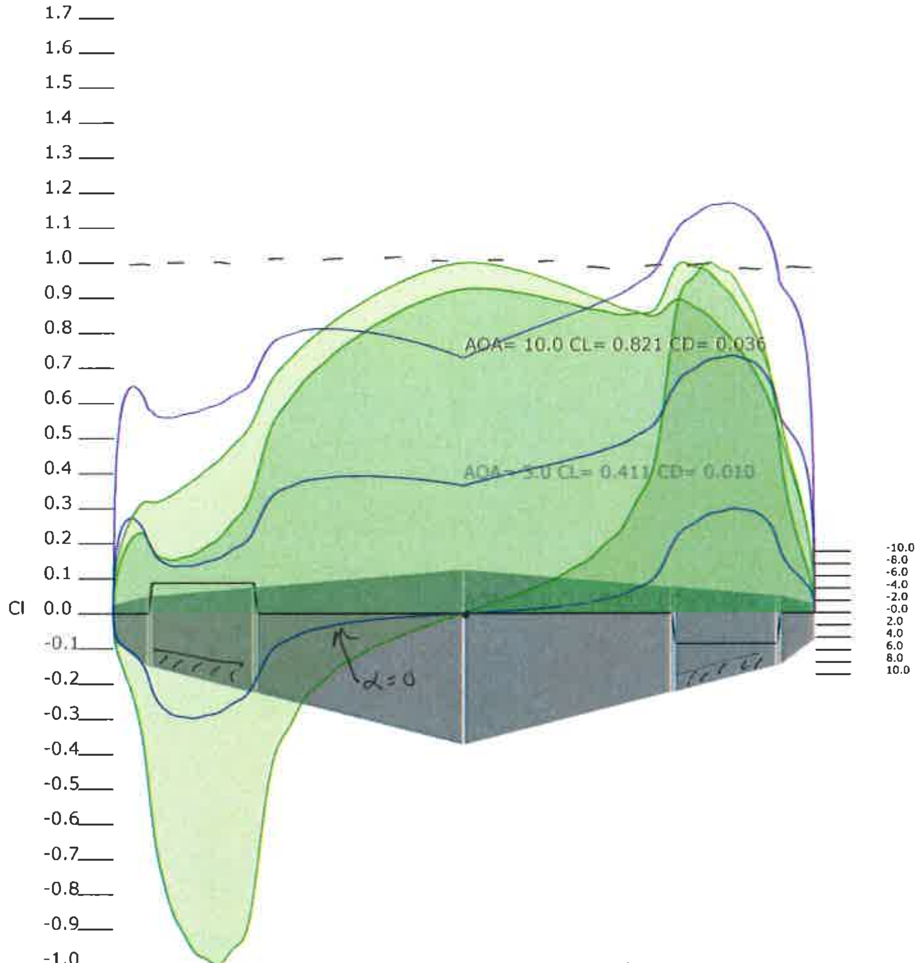
Split Flap Condition

Very dangerous. Most aircraft have a sensor system to shut down the flaps if a difference in angle is detected.





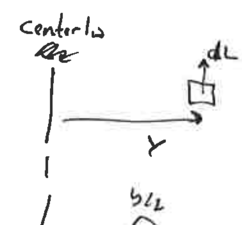
Aileron Deflection ≈ ±10% for 20% c



C_L = Lift Coeff

$\vec{C}_L$ = Roll Moment Coeff

=



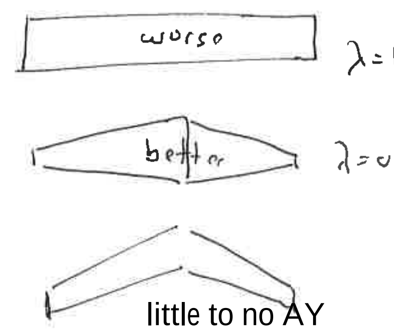
$$\int_{-b/2}^{b/2} y\,dL$$

Adverse Yaw

Asymmetrical lift distributions often have asymmetrical drag distributions.

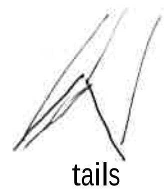
This often shows up as a yaw moment when ailerons are deflected.

$\left\{\begin{array}{l}\text{Higher } \lambda \\ \text{Tip loaded}\end{array}\right\}$ have stronger adverse yaw.



Certain distributions can have proverse yaw.

Ex: Birds.

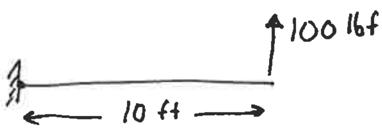


Bending Moment

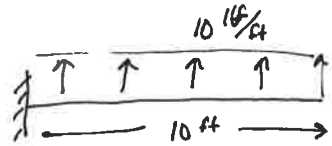
The structural design is critical to aircraft weight.

Q: which is easier to create a structure for.

1)



2)



Root bending Moment

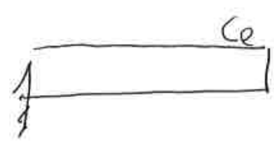
$M = F \cdot d = 1000 \text{ ft-lbf}$

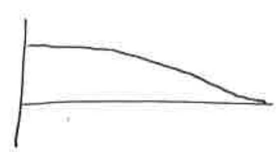
$$M = \int dM = \int_0^{10} \left(10 \tfrac{\text{lbf}}{\text{ft}}\right) y \, dy$$

$$= 500 \text{ ft-lbf}$$

Q: Is an elliptical wing <u>always</u> <u>best</u>?

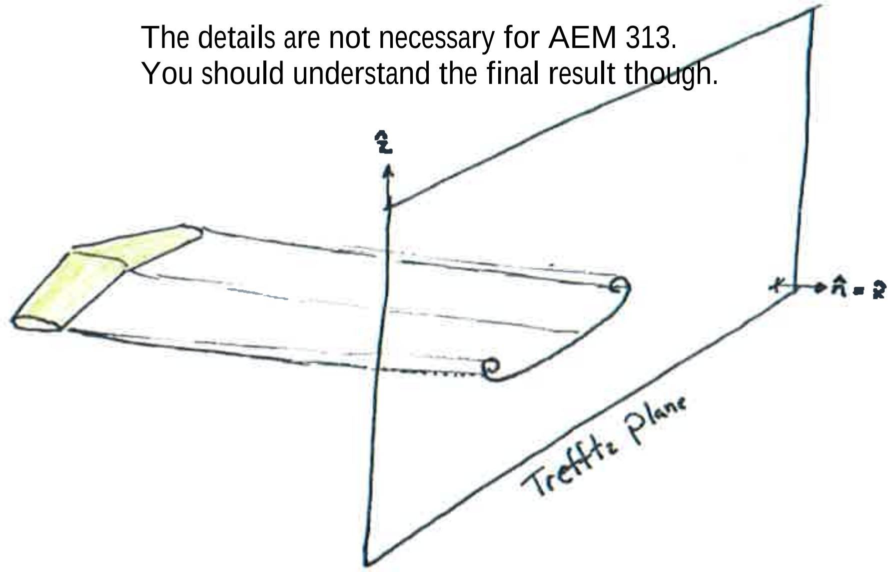
No.





Trefftz Plane

The details are not necessary for AEM 313.
You should understand the final result though.



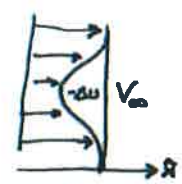
Drag computed on Trefftz plane

$$D = F \cdot \hat{x} = \iint_{\text{outer}} (P_\infty - p)\,\hat{n}\cdot\hat{x} - \rho\,(V\cdot\hat{n})(u - V_\infty)\, dS$$

$$= \iint_{TP} P_\infty - p - \rho u (u - V_\infty)\, dy\, dz$$

Decompose velocity at TP into viscous + shock velocity defect and streamwise vorticity.

$$V = (\underbrace{V_\infty}_{\text{freestream}} + \underbrace{\Delta U}_{\text{defect}})\,\hat{x} + \underbrace{\nabla \phi}_{\text{potential gradient}}$$



$\underbrace{\text{side view}}_{\text{defect}}$

$\underbrace{\text{front view}}_{\text{vorticity in streamwise dir.}}$

Low perturbation velocities (Bernoulli) in y-z plane

$$P_0 = P_\infty + \tfrac{1}{2}\rho_\infty V_\infty^2 = p + \tfrac{1}{2}\rho_\infty V^2$$

rearrange to

$$p = P_\infty + \tfrac{1}{2}\rho_\infty V_\infty^2 - \tfrac{1}{2}\rho_\infty V^2$$

$$P = P_\infty + \tfrac{1}{2}\rho_\infty V_\infty^2 - \tfrac{1}{2}\rho_\infty (V_\infty \hat{x} + \nabla\phi)^2$$

Notice that only the potential portion of the TP velocity impacts pressure. The defect portion does <u>not</u>!

Look at the term $\left(V_\infty \hat{x} + \nabla \phi\right)^2$

$$V_\infty \hat{x} + \nabla\phi = \begin{pmatrix} V_\infty \hat{x} \\ 0\,\hat{y} \\ 0\,\hat{z} \end{pmatrix} + \begin{pmatrix} \phi_x\,\hat{x} \\ \phi_y\,\hat{y} \\ \phi_z\,\hat{z} \end{pmatrix} = \begin{pmatrix} (V_\infty + \phi_x)\,\hat{x} \\ \phi_y\quad \hat{y} \\ \phi_z\quad \hat{z} \end{pmatrix}$$

Square this (Keep vector terms seperate!!)

$$\left(V_\infty \hat{x} + \nabla\phi\right)^2 = \begin{pmatrix} (V_\infty + \phi_x)^2 \\ \phi_y^2 \\ \phi_z^2 \end{pmatrix} = V_\infty^2 + 2V_\infty\phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2$$

Substitute into p

$$p = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 - \frac{1}{2}\rho_\infty\left(V_\infty^2 + 2V_\infty\phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2\right) \xrightarrow{} 0$$

$$= p_\infty - \rho_\infty V_\infty \phi_x - \frac{1}{2}\rho_\infty\left(\phi_x^2 + \phi_y^2 + \phi_z^2\right)$$

Substitute into TP Drag equ

$$D = \iint_{TP} p_\infty - p - \rho u\,(u - V_\infty)\,dy\,dz \qquad u = V\cdot\hat{x} = V_\infty + \phi_x + \Delta u$$

$$= \iint_{TP} \underbrace{p_\infty - p_\infty}_{=0} + \rho_\infty V_\infty \phi_x + \frac{1}{2}\rho_\infty\left(\phi_x^2 + \phi_y^2 + \phi_z^2\right) - \rho\left(V_\infty + \Delta u + \phi_x\right)\underbrace{\left(V_\infty + \Delta u + \phi_x - V_\infty\right)}_{\Delta u + \phi_x}\,dy\,dz$$

$$= \iint_{TP} \rho_\infty V_\infty \phi_x + \frac{1}{2}\rho_\infty\left(\phi_x^2 + \phi_y^2 + \phi_z^2\right) - \rho V_\infty \Delta u - \rho V_\infty \phi_x - \rho\Delta u^2 \underbrace{- \rho\Delta u\phi_x - \rho\phi_x\Delta u}_{-2\rho\Delta u\phi_x} - \rho\phi_x^2\;dy\,dz$$

$$= \iint_{TP} \underbrace{\frac{1}{2}\rho_\infty\left(\phi_y^2 + \phi_z^2 - \phi_x^2\right)}_{\substack{\text{potential only term} \\ \text{"vorticity"}}} + \underbrace{\rho(-\Delta u)\left(V_\infty + \overbrace{2\phi_x}^{\text{vorticity term mult' by viscous!}} + \Delta u\right)}_{\substack{\text{Velocity defect term} \\ \text{"viscous"}}}\,dy\,dz$$

Separate into induced drag and profile drag components

$$\text{Drag} \equiv D_i + D_p$$

$$D_i = \iint_{TP} \tfrac{1}{2}\rho_\infty \left(\phi_y^2 + \phi_z^2 - \phi_x^2\right) dTP$$

ϕ

$\phi_x = \frac{d\phi}{dx} =$

$$D_p = \iint_{TP} \rho\left(V_\infty + 2\phi_x + \Delta u\right)(-\Delta u)\, dTP$$

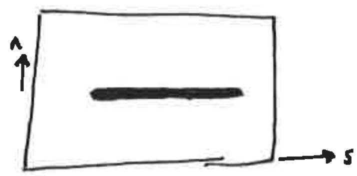
when $\phi_x \ll V_\infty + \Delta u$

$$D_p \approx \underbrace{\iint \rho u (V_\infty - u)\, dS}_{\text{seen this before?}}$$

Momentum defect term P.

$$= \int P\, ds$$

$$= \int\int \rho u (V_\infty - u)\, dn\, ds$$



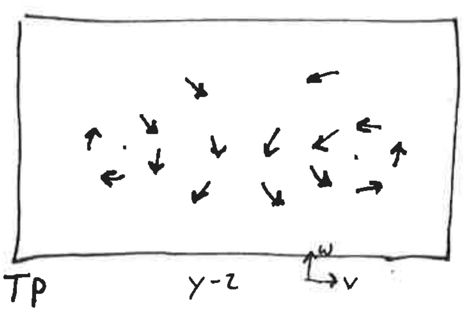
$$\boxed{D_p = \rho_\infty V_\infty \theta_\infty}$$

Induced drag components

What is ϕ_x ? $\phi_y = ?$ $\phi_z = ?$

perturbation velocities. $\phi_x = u$ $\phi_y = v$ $\phi_z = w$

$$D_i = \iint_{TP} \tfrac{1}{2}\rho_\infty \left(v^2 + w^2 - u^2\right) dTP \approx \iint_{TP} \underbrace{\tfrac{1}{2}\rho_\infty \left(v^2 + w^2\right) dTP}_{\text{Kinetic energy of non streamwise flow}}$$

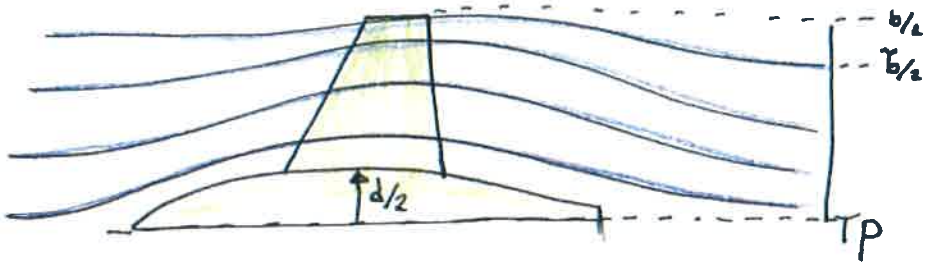


Induced drag is what we call the kinetic energy contained in ~~stream~~ non-streamwise flow.

"Rotational Flow Drag"

Fuselage Wake Contraction

Fact: Wings are usually mounted on a fuselage of a certain width.



The aft fuselage contraction <u>reduces</u> the effective span at the TP.

Geometric span b $\Rightarrow$ Trefftz span $\tilde{b}$

From mass conservation at the wing and at the TP: $\dot{m}_{wing} = \dot{m}_{TP}$

$$y = \sqrt{\tilde{y}^2 + (d/2)^2}$$

for a ratio

$$\left(\frac{\tilde{b}}{b}\right)^2 = 1 - \left(\frac{d}{b}\right)^2 \quad \Rightarrow \quad AR = \frac{b^2}{S} \Rightarrow \tilde{AR} = \frac{\left(\frac{\tilde{b}}{b}\right)^2 b^2}{S} = AR\left(1 - \left(\frac{d}{b}\right)^2\right)$$

Replace all occurences of span b^2 in previous theory by $\tilde{b}^2$

For an elliptical wing

$$C_{L_\alpha} \approx \frac{2\pi}{1 + \frac{2}{AR}} \quad \Rightarrow \quad C_{L_{\alpha_{fuselage}}} \approx \frac{2\pi}{1 + \frac{2}{AR}\left(1 - \left(\frac{d}{b}\right)^2\right)^{-1}}$$

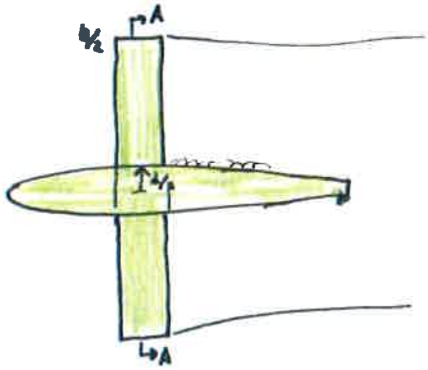
$$C_{D_i} = \frac{C_L^2}{\pi AR e} \quad \Rightarrow \quad \frac{C_L^2}{\pi AR\left(1 - \left(\frac{d}{b}\right)^2\right) e}$$

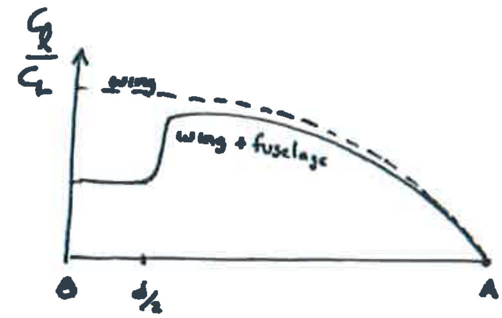
A fuselage slightly decreases aero performance of a raw wing.

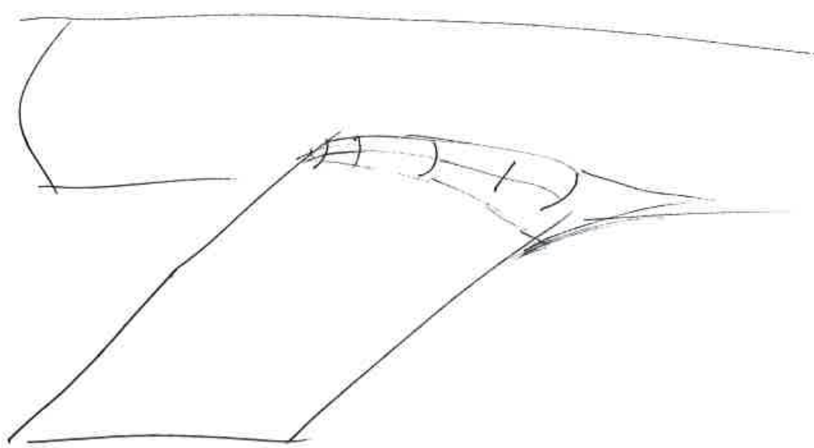
$$\left(\frac{\tilde{b}}{b}\right)^2 = 1 - \left(\frac{d}{b}\right)^2 \quad \text{for } \frac{d}{b} \approx 10\% \Rightarrow \left(\frac{\tilde{b}}{b}\right)^2 = 99\%$$

Worse, the fuselage is less efficient at lift production.

The spanwise loading ~~may~~ will decrease efficiency.



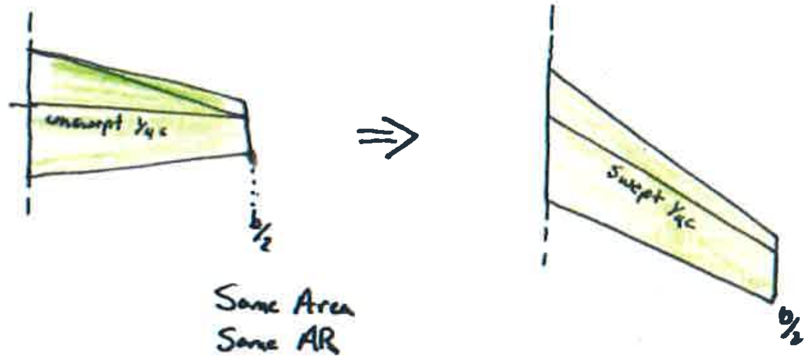




Swept Wings Chapters 20 - 24 in ADTA

Aerodynamic Design of Transport Aircraft
Obert

Given the infinite swept wing results (mitigation of compressibility), apply to wings.



Same Area
Same AR

Historical note:

The 1st operational jet fighter, the Me-262, had 18° swept wings. This sweep corrected a CG issue (heavy engines) rather than an aerodynamic reason.

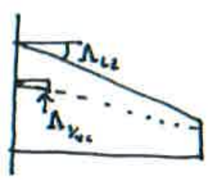
$$\cos(18°) \approx 95\%$$

A 35° sweep concept was proposed but rejected. On par with F-86?

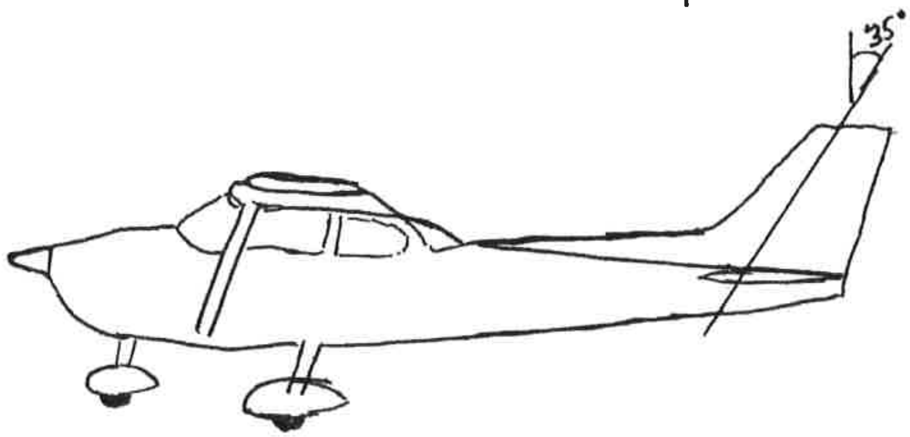
No. Dream on, unless you are an excellent pilot!

Geometry note

$\Lambda_{LE} \neq \Lambda_{1/4c}$ when $\lambda \neq 1$



Q: Why is the C-172 Vertical swept?



A: Advantage in high speed flight (High Mach)? Ha. No!

A: Slightly more robust stalling charactertistics. Less control authority? Area further aft but less effective.

$$\frac{C_{L_{max}}}{\cos\Lambda^2} \approx 1.3/\cos 35° \text{ vs } 1.0/1$$

$$\alpha_{C_{L_{max}}} \approx 16° \text{ vs } 10°$$

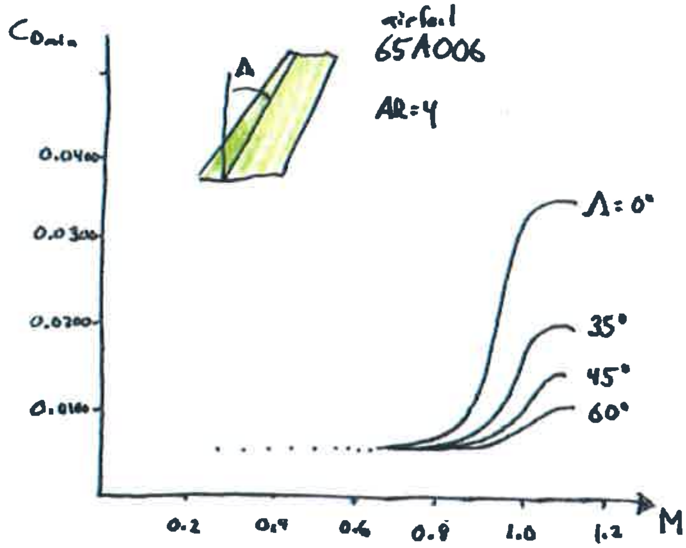
87% of straight tail (approx)

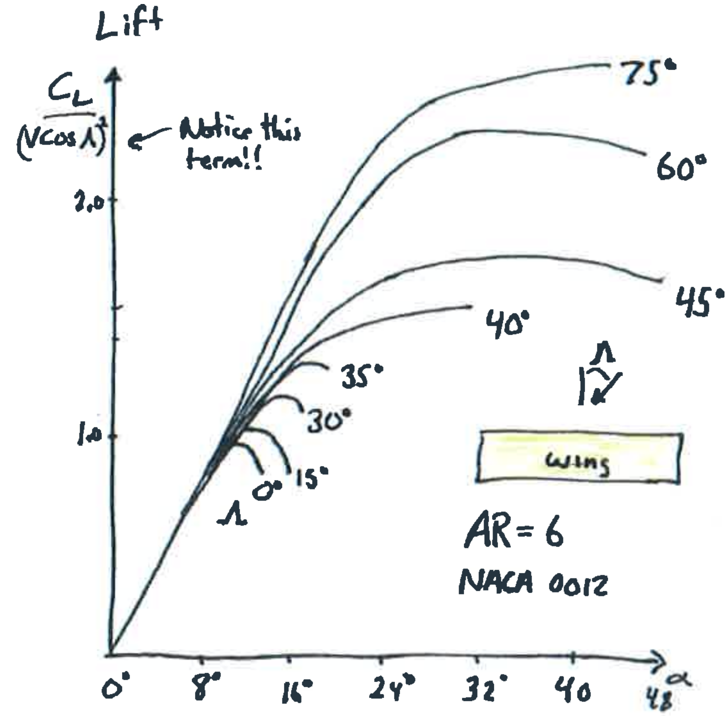
60% increase in effective angle

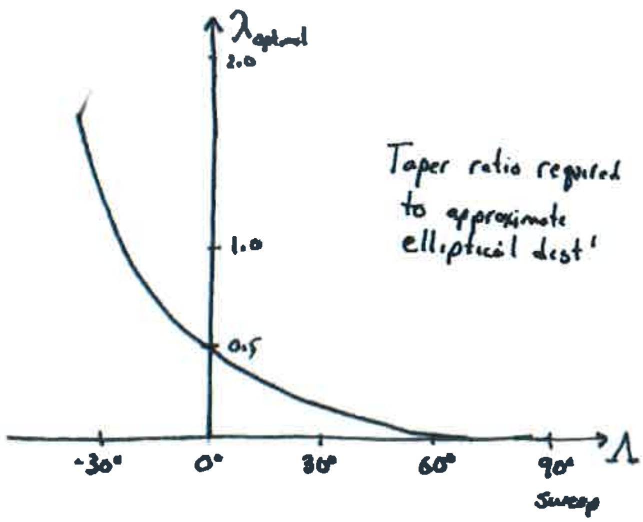
A: Marketing.

"Swept = fast"

Swept Wing Drag

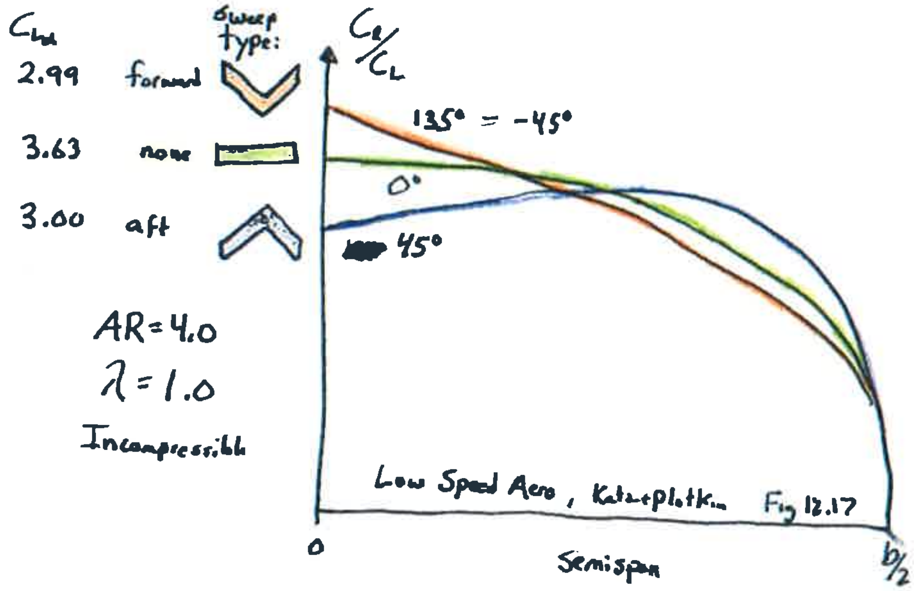






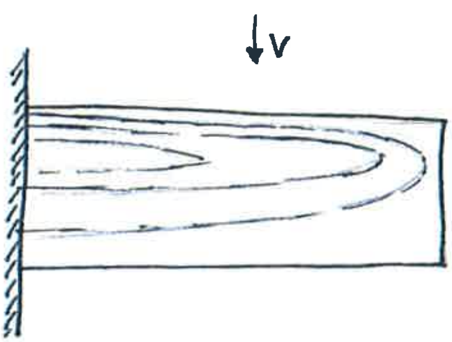
Hoerner Lift, Drag

Lift Distribution of Swept Wings

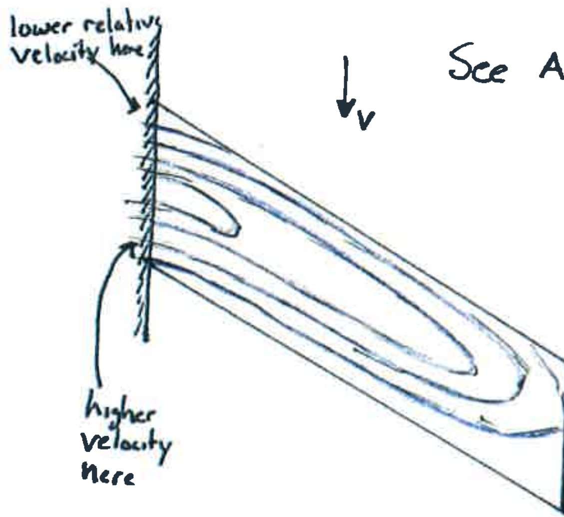


Aft sweep loads the tips more than zero sweep.

Comparison of Unswept and Swept Pressure Contours



Symmetry requires pressure contours are normal to symmetry plane at y=0.



See ADTA Fig 22.1

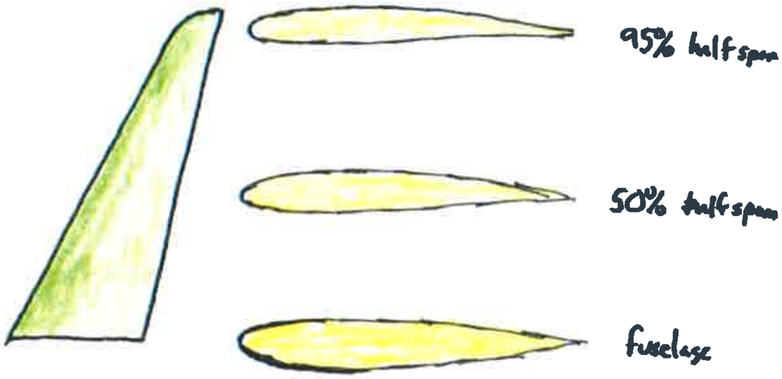
Decrease in C_l near root.

General guidelines:

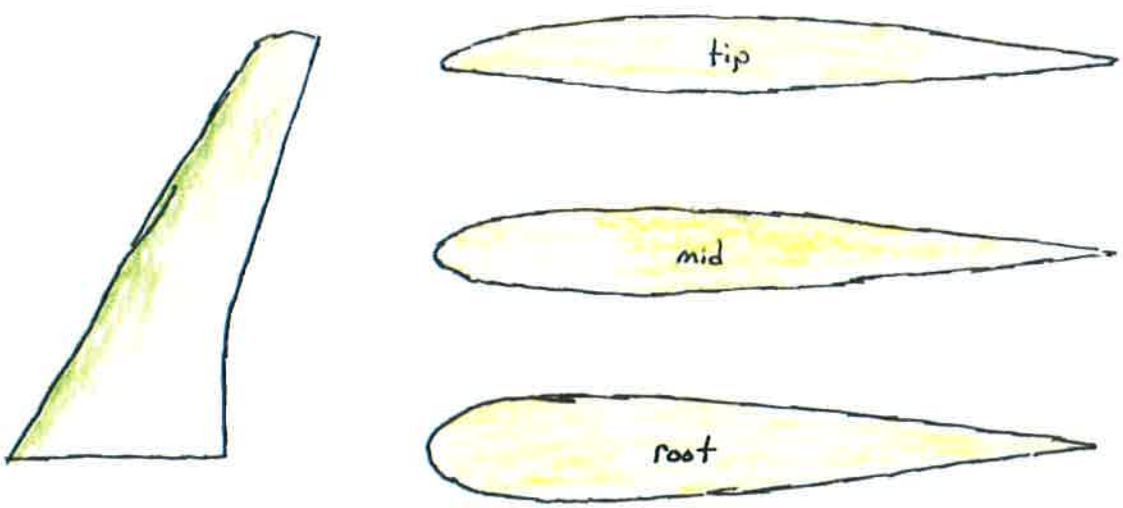
- Tip C_l is relatively high. Requires camber to achieve.  exaggerated.
- Root C_l needs to be increased for spanwise C_l efficiency.
  To "fill" the LE C_p, requires rapid increase in velocity (large round nose)
  Reduce C_p magnitude on aft portion with less curvature in airfoil. 

- Washout twist at tip to create a more elliptical lift distribution

Comparison to actual wings:



DC9 1966 ADTA Fig 24.15



Boeing 737

Also see multiple examples in ADTA Chapter 24