

Lesson 23

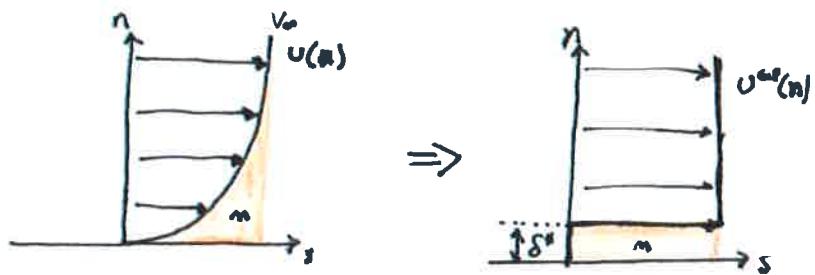
Turbulent Boundary Layers (Chap 19)

[tiny.cc/AEM313Turbulence](http://tiny.cc/AEM313Turbulence)

0800 - 0800  
0900 - 1400

Whole Aircraft Drag Prediction / Estimation

## Mass displacement thickness



$$\delta^* = \int_0^{n_e} \left(1 - \frac{\rho u}{\rho_e u_e}\right) dn \quad \text{and mass defect } m = \rho_e u_e \delta^*$$

$$= \int_0^{n_e} (1 - u) dn \quad (\text{incompressible}) \quad \Rightarrow \quad u = \frac{u}{u_e}$$

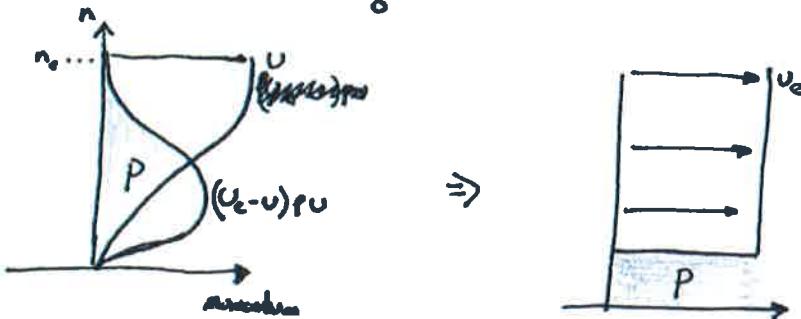
## Momentum Thickness

$$\Theta = \int_0^{n_e} \left(1 - \frac{u}{u_e}\right) \frac{\rho u}{\rho_e u_e} dn = \underbrace{\int_0^{n_e} (u - u^2) dn}_{\text{incomp' only}}$$

and the momentum defect

$$P = \int_0^{n_e} (u_e - u) \rho u dn = \rho_e u_e^2 \Theta$$

! Where have we seen  $\bar{P}$  before?  
Drag' =  $P$



## Energy Thickness

$$\Theta^* = \int_0^{n_e} \left(1 - \frac{u^2}{u_e^2}\right) \frac{\rho u}{\rho_e u_e} dn$$

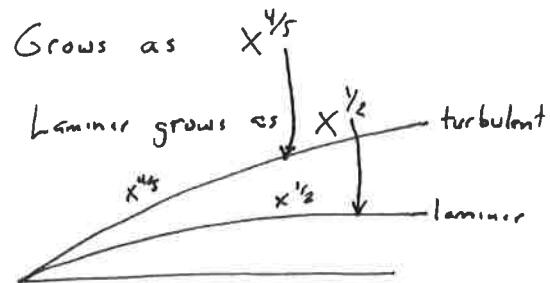
and the energy defect

$$K = \int_0^{n_e} \frac{1}{2} (u_e^2 - u^2) \rho u dn = \frac{1}{2} \rho_e u_e^3 \Theta^*$$

# Turbulent plate

BL Thickness

$$\delta = \frac{0.37x}{Re_x^{1/5}}$$

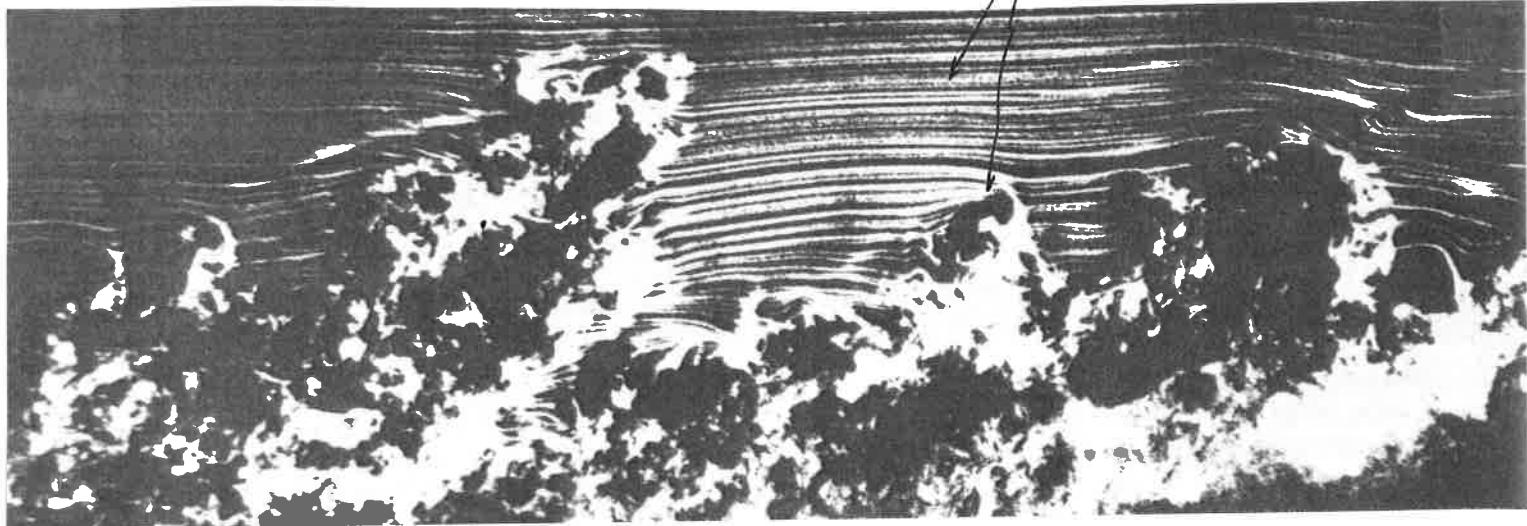


Fraction Coeff

$$C_f = \frac{0.074}{Re_c^{1/5}}$$

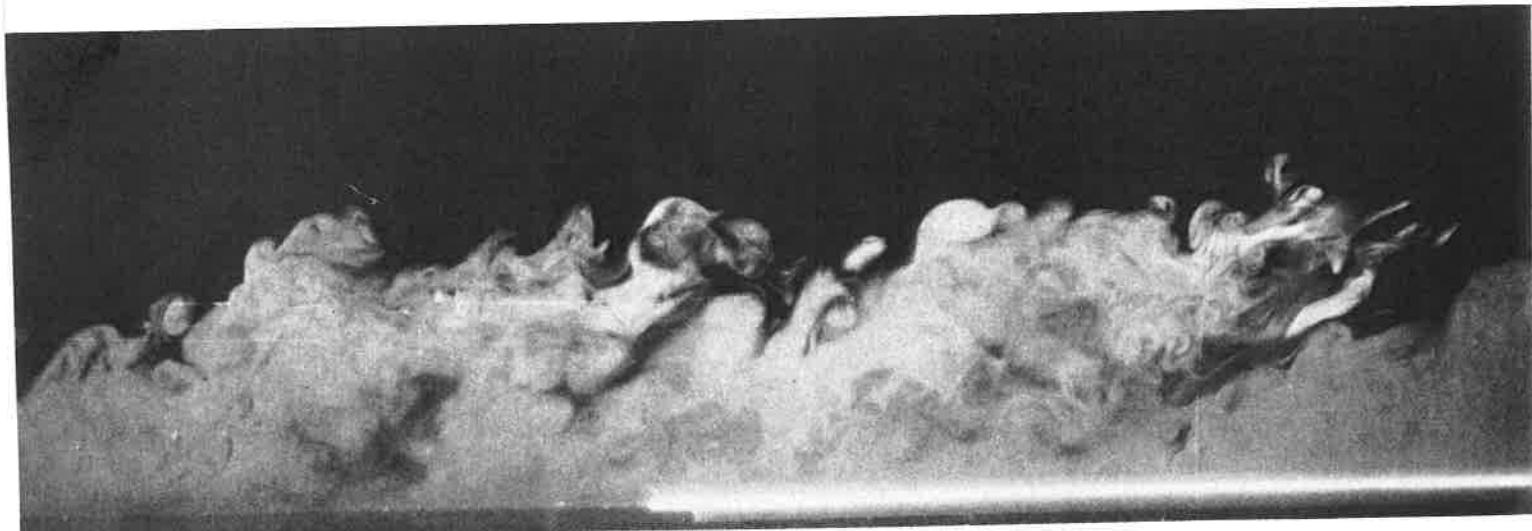
Governed by convection.  
No general analytical solution

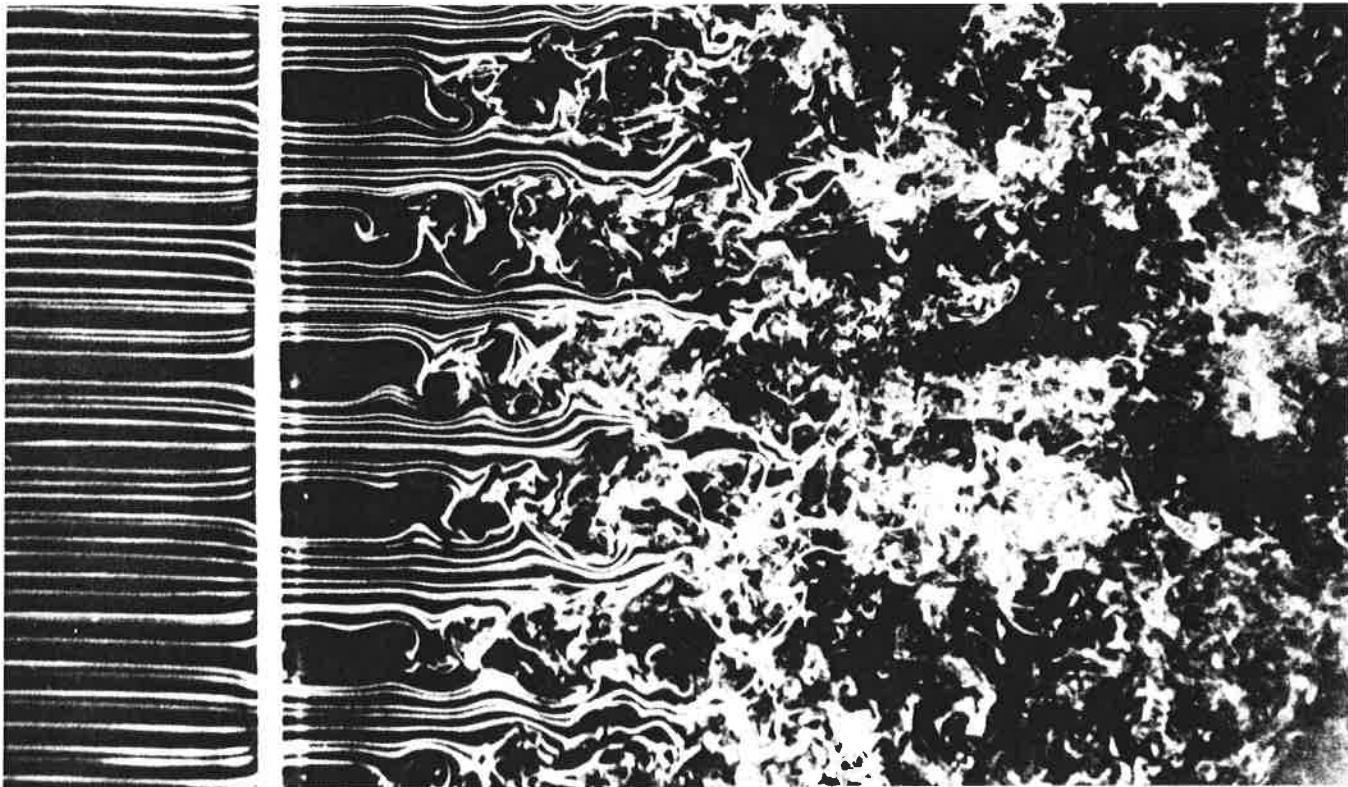
Intermittency



157. Side view of a turbulent boundary layer. Here a turbulent boundary layer develops naturally on a flat plate 3.3 m long suspended in a wind tunnel. Streaklines from a smoke wire near the sharp leading edge are illuminated by

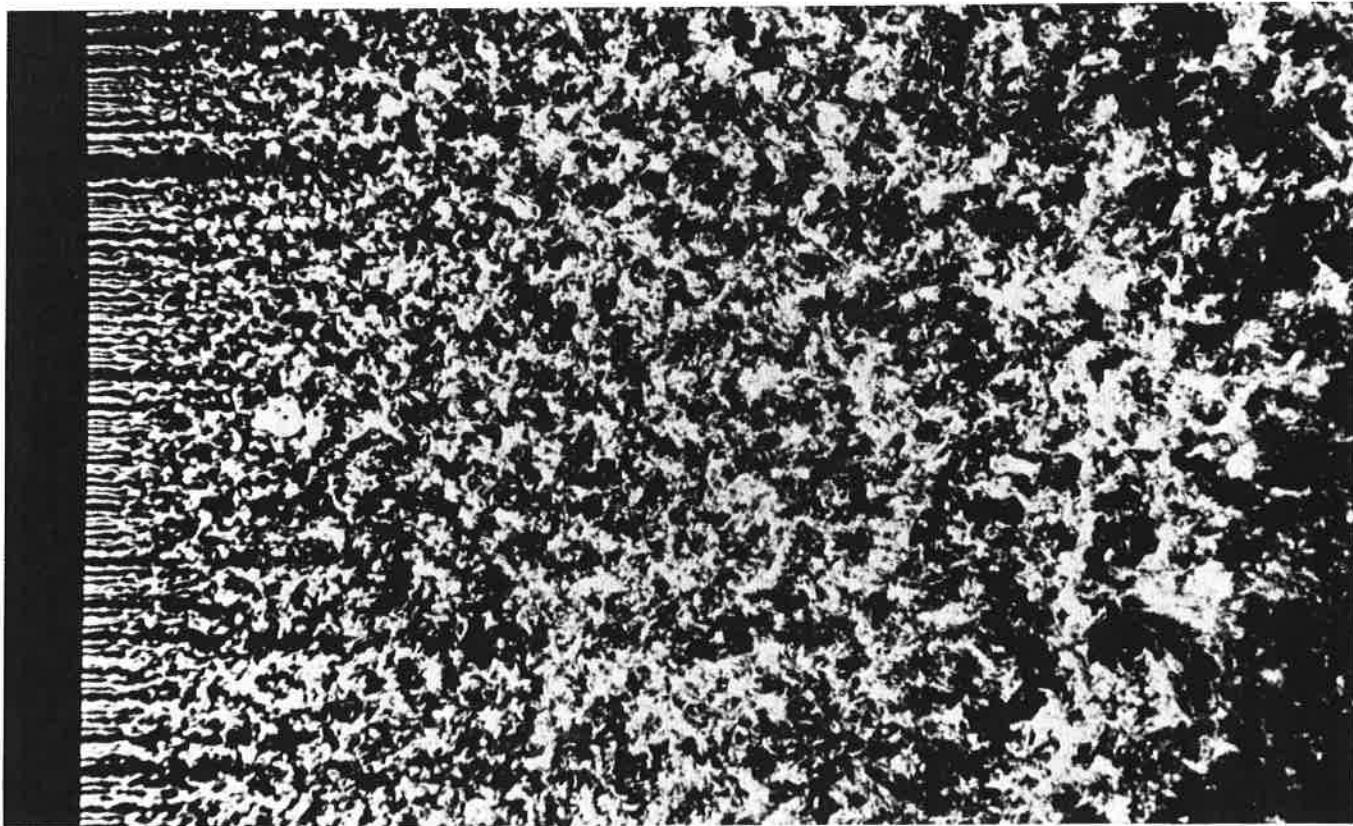
a vertical slice of light. The Reynolds number is 3500 based on the momentum thickness. The intermittent nature of the outer part of the layer is evident. Photograph by Thomas Corke, Y. Guezenec, and Hassan Nagib.





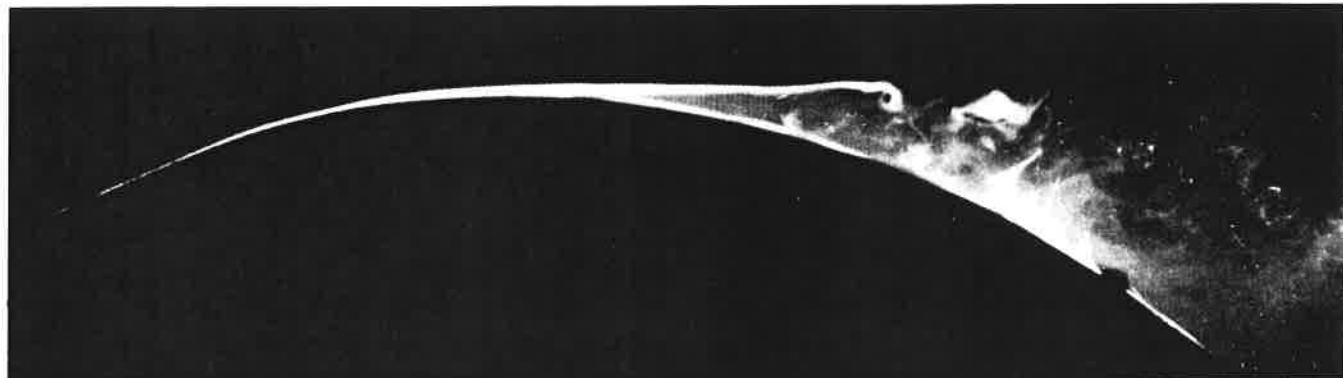
**152. Generation of turbulence by a grid.** Smoke wires show a uniform laminar stream passing through a  $\frac{1}{16}$ -inch plate with  $\frac{3}{4}$ -inch square perforations. The Reynolds num-

ber is 1500 based on the 1-inch mesh size. Instability of the shear layers leads to turbulent flow downstream. *Photograph by Thomas Corke and Hassan Nagib*



**153. Homogeneous turbulence behind a grid.** Behind a finer grid than above, the merging unstable wakes quickly form a homogeneous field. As it decays down-

stream, it provides a useful approximation to the idealization of isotropic turbulence. *Photograph by Thomas Corke and Hassan Nagib*



156. Comparison of laminar and turbulent boundary layers. The laminar boundary layer in the upper photograph separates from the crest of a convex surface (cf. figure 38), whereas the turbulent layer in the second

photograph remains attached; similar behavior is shown below for a sharp corner. (Cf. figures 55-58 for a sphere.) Titanium tetrachloride is painted on the forepart of the model in a wind tunnel. Head 1982

Turbulent BLs are more resistant to imperfections and shapes (usually)



# Transition from Laminar to Turbulent (and back?)

For details refer to Turbulent Fluids (AEM 622).

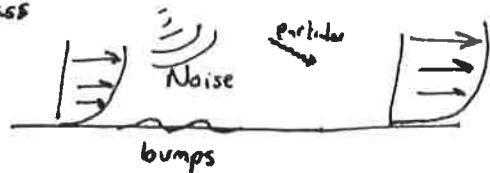
3 types of transition



Natural



Bypass



Tollmien-Schlichting Waves (TS)

$$U = \bar{U} + U' \Rightarrow \text{Flux} = \rho U^2 + p \quad (\text{nonlinear}) \\ = \rho (\bar{U}^2 + 2\bar{U}U' + U'^2) + p$$

- TS waves are small perturbations in the flow that slowly begin affecting the nonlinear terms of the N.S. equation (convection).

wave amplitude ratio  $\propto e^N$

$$N \approx 9 \quad \text{wind tunnel}$$
$$N \approx 4 \quad \text{noisy environment}$$

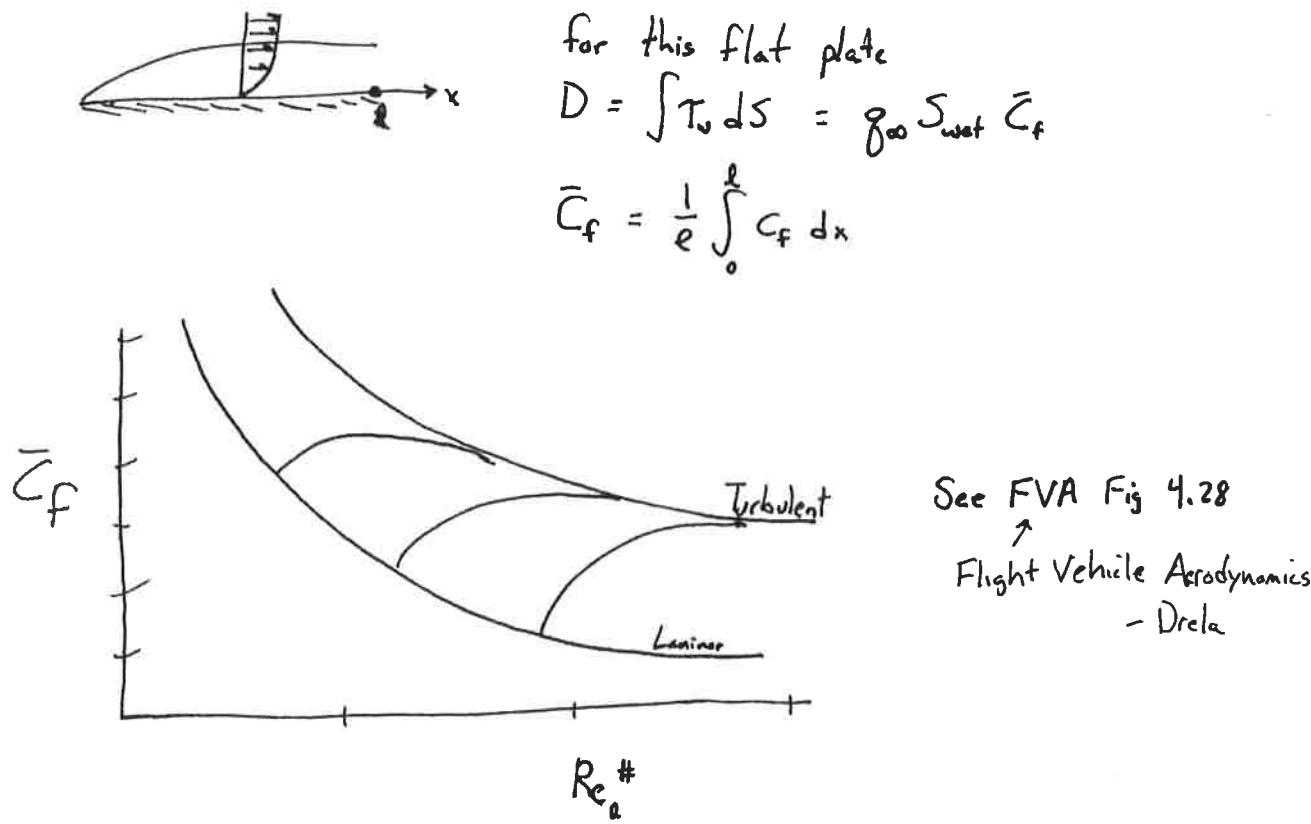
When  $N > N_{\text{crit}}$ , the flow is turbulent.

## Profile Drag

Earlier, we found that profile drag depends on the momentum defect in the wake.

$$D' = P_{\infty} = \underbrace{\int T_w ds}_{\text{shear stress at wall}} + \underbrace{\int -m \frac{du_e}{ds} ds}_{\text{"pressure drag"}}$$

Wetted area



For non-flat-plates or any particular body,

$$C_D = \frac{S_{\text{wet}}}{S_{\text{ref}}} \bar{C}_f K_f$$

dim      low      low  
 skin      form  
 coeff      factor  
 for flat      plate

Refer to Hoerner's Fluid-Dynamic Drag.

Q: What is the drag of a 4'x8' plate at 60 mph and SSL?

Compare w/ Blasius.

$$Re \approx 6350 \cdot \left(\frac{88}{60} \cdot 60\right)(4) = 2.24 \times 10^6$$

$$C_f = \frac{0.074}{Re_c^{1/5}} = \frac{0.074}{(2.24 \times 10^6)^{1/5}} = 0.00397$$

$$D = \frac{1}{2} \rho V^2 C_f A = \frac{1}{2} \left| \frac{0.00237 \text{ slugs}}{\text{ft}^3} \right| \frac{88^2 \text{ ft}^2}{\text{s}^2} \left| \frac{0.00397}{4 \text{ ft}} \right| \frac{8 \text{ ft}}{1}$$
$$= 1.165 \text{ lbf per side}$$

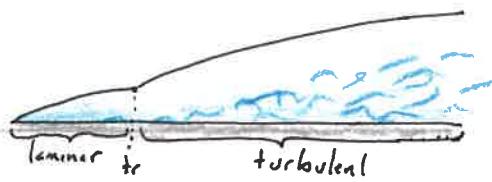
$\approx 4.5 \times$  higher than Blasius

Q: How thick is the BL?

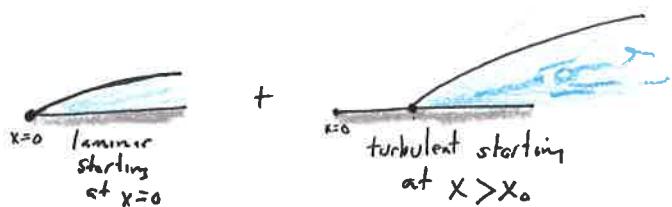
$$\delta = \frac{0.37 x}{Re_x^{1/5}} = \frac{0.37}{(2.24 \times 10^6)^{1/5}} \left| \frac{4 \text{ ft}}{\text{s}^2} \right| = 0.079 \text{ ft} \approx 0.95 \text{ in}$$

$\approx 4.5 \times$  higher than Blasius.

Q: What is a better approximation of the drag of a 4'x8' plate?



Approximate as 2 parts



- Transition (~~boundary layer~~)

$$Re_{tr} \approx 500,000 \quad \text{for a flat plate.}$$

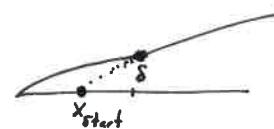
Find  $Re_x$  in laminar BL

$$500,000 = \frac{\rho V x}{\mu} \approx 6350 \cdot V \cdot x$$

$$x_{tr} \approx \frac{78.7}{V^{1/8}}$$

- Match BL properties at  $x_{tr}$

$$\approx \frac{78.7}{88} \approx 0.9 \text{ ft}$$

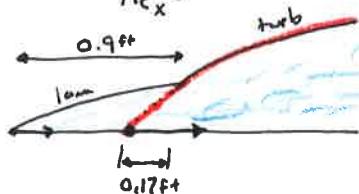


Now find the height at  $x_{tr}$ . (only an approximation,  $\delta$  is a fair way.  $\theta$  is better)

$$\delta = \frac{5.0}{\sqrt{Re_x}} x = \frac{5.0 / 0.9 \text{ ft}}{\sqrt{6350 \cdot 88 / 0.9 \text{ ft}}} = 0.006345 \text{ ft}$$

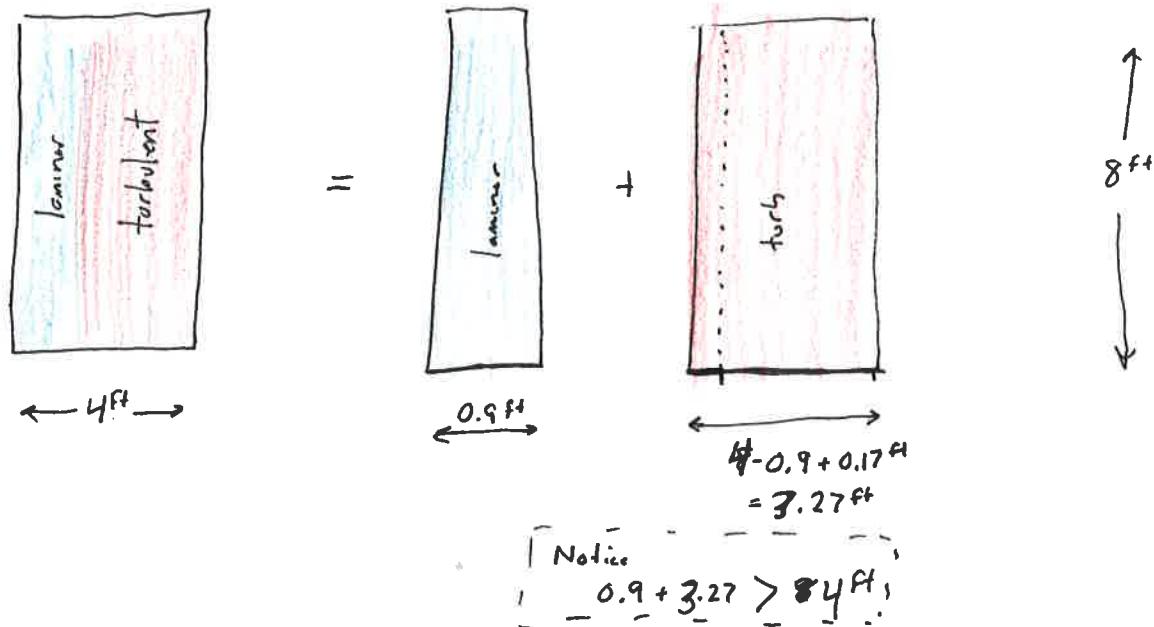
Now find virtual starting point of turbulent BL.

$$\delta = \frac{0.37 x}{Re_x^{1/5}} = 0.006345 \text{ ft} \quad \text{solve for } x.$$



$$x_{start} = 0.17 \text{ ft}$$

So a better approximation is



$$g(88 \frac{ft}{s}, SSL) = 9.17 \text{ psf}$$

• Drag

$$D = g C_{f_{lam}} A_{lam} + g C_{f_{turb}} A_{turb} - g C_{f_{turb}} A_{turb}$$

$\frac{8 \times 0.9}{\sqrt{R_{cuto}}}$        $\frac{8 \times 3.27}{(R_{turb})^{1/5}}$        $\frac{0.074}{(6350 \cdot 88 \cdot 0.9)^{1/5}} = 0.00537$   
 $\frac{1.328}{\sqrt{500000}} = 0.00188$        $\frac{0.074}{(6350 \cdot 88 \cdot 3.27)^{1/5}} = 0.00414$

$$D = 9.17 \text{ psf} \left( 0.00188 \cdot 8^{ft} \cdot 0.9^{ft} + 0.00414 \cdot 8^{ft} \cdot 3.27^{ft} \right) - 0.00537 \cdot 8^{ft} \cdot 0.17^{ft}$$

$$D = 1.04 \text{ lb f}$$

Compare to Blasius and Turb approx.

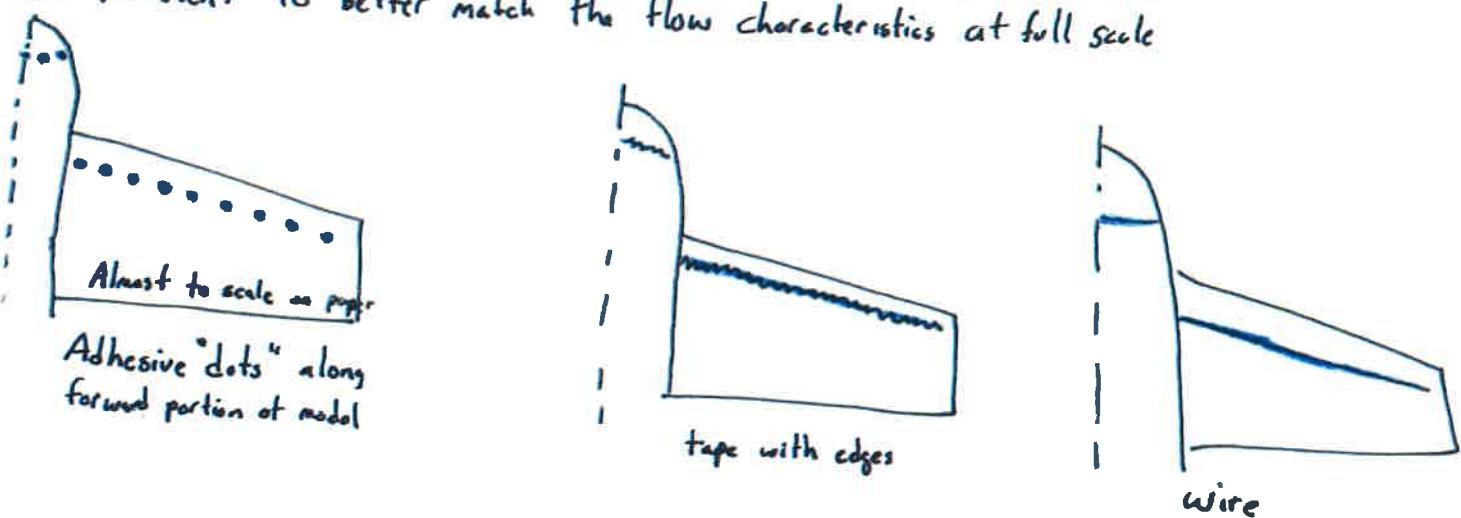
$$0.26 \text{ lb f} \quad 1.16 \text{ lb f}$$

For rapid analysis at higher  $Re$ , just assume a fully turbulent BL

## Transition strips/dots for wind tunnel testing

Often in a wind tunnel, the model is subscale (e.g. 1:10) and the resulting  $Re$  is low. Since the full scale aircraft operates at higher  $Re$ , the model results and flow behavior ~~may~~ will not exactly match the actual aircraft.

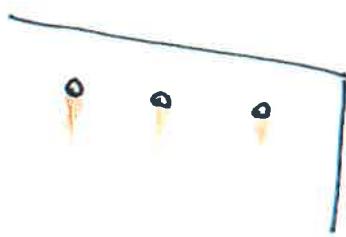
To prevent laminar separation from completely ruining any correlation, a boundary layer trip is applied to the model. This trips the flow to turbulent to better match the flow characteristics at full scale



Where should the trips be placed? Why?

- Far enough aft to trip flow
- Far enough forward to be ahead of bubbles/separation
- Large enough to trip reliably
- Small enough to not completely change drag

When seen via surface flow visualization, the dots/trips have a separation drag associated with their height/geometry.

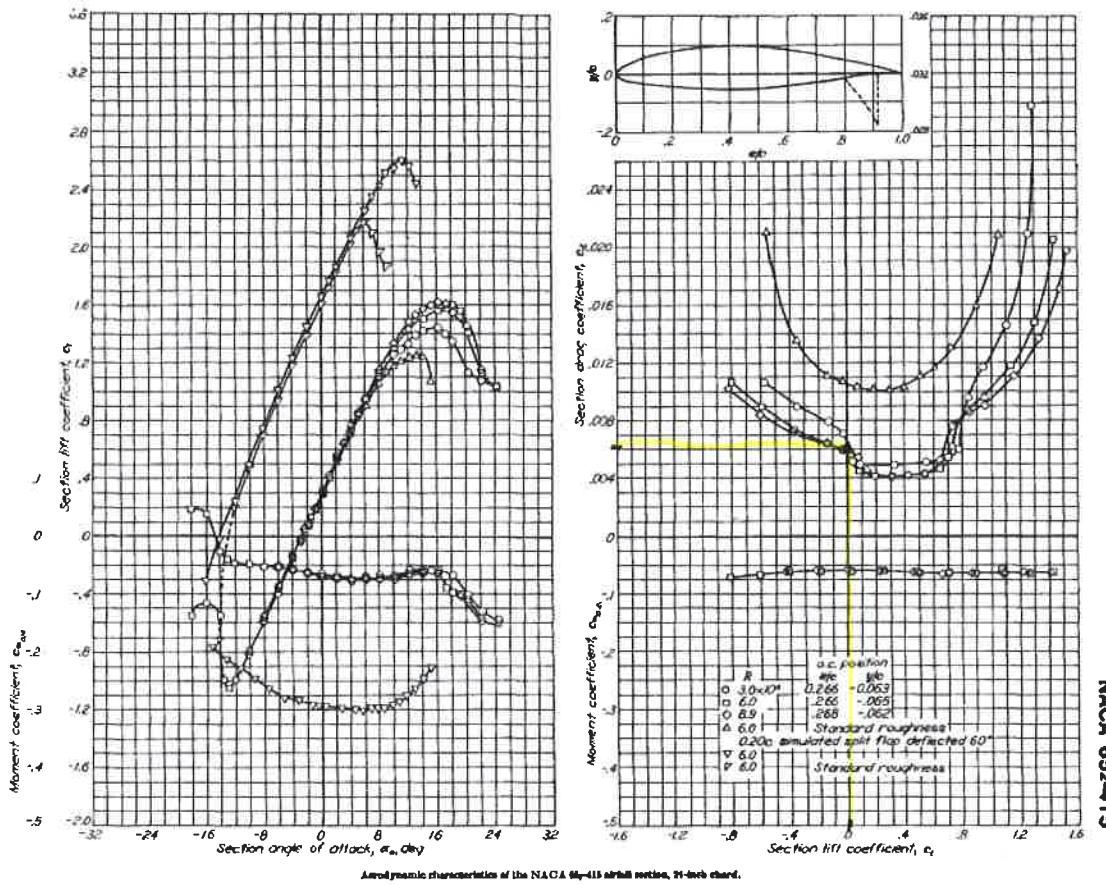
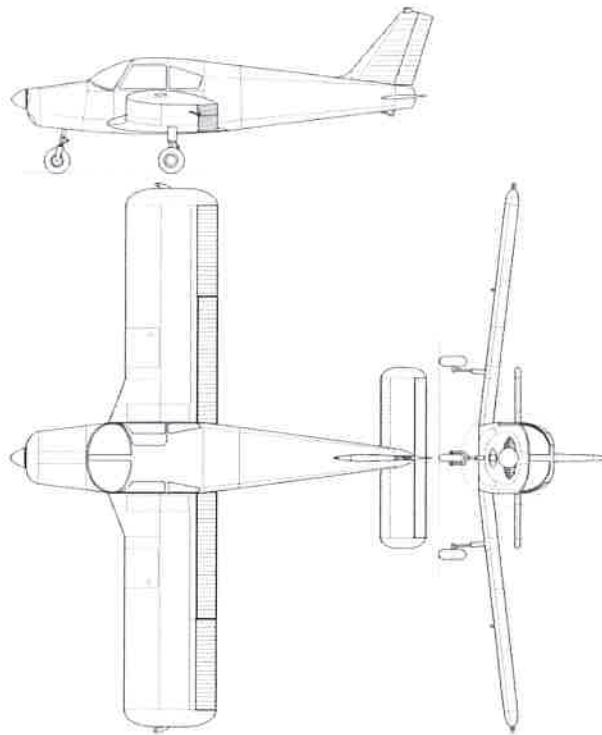


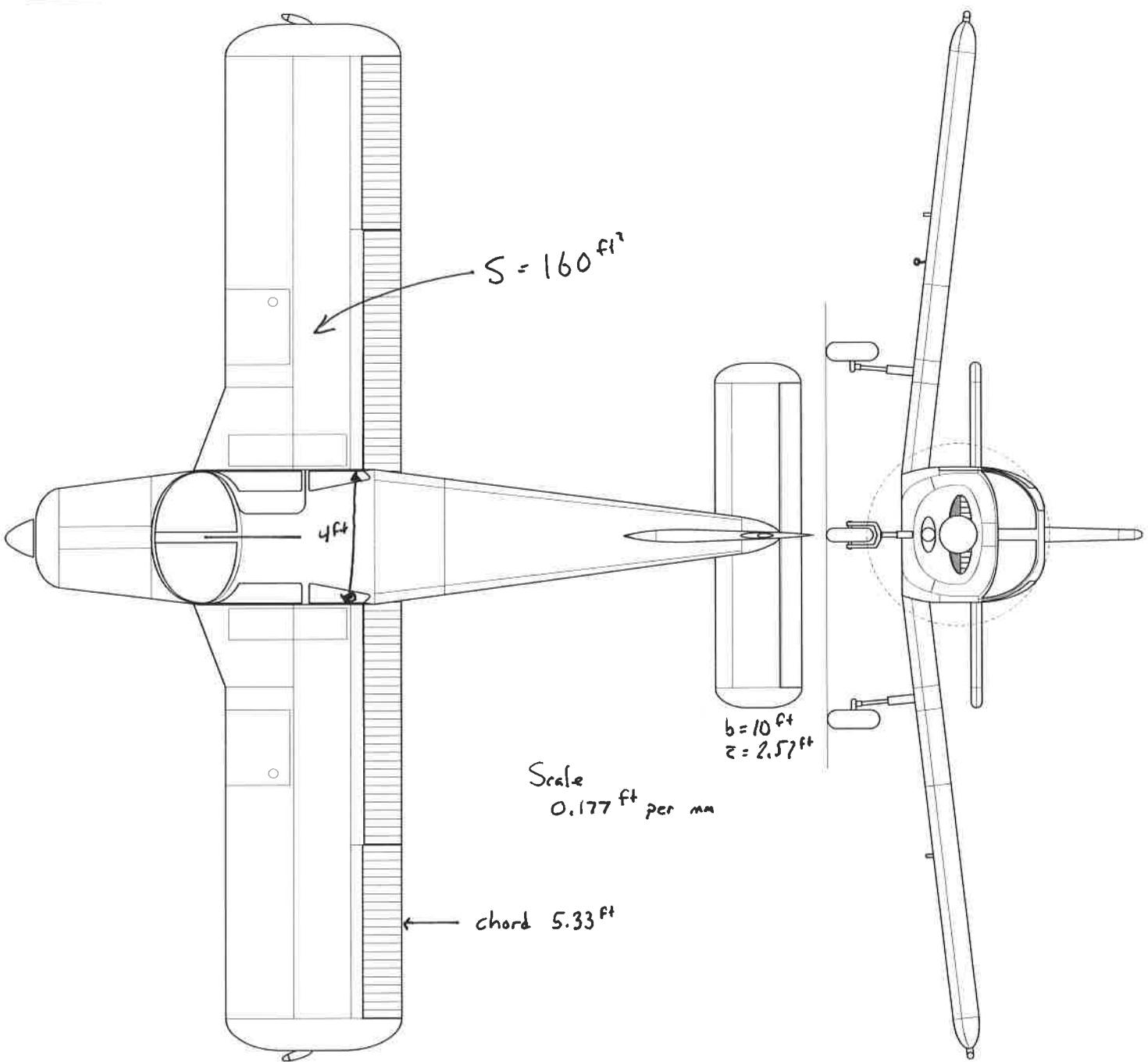
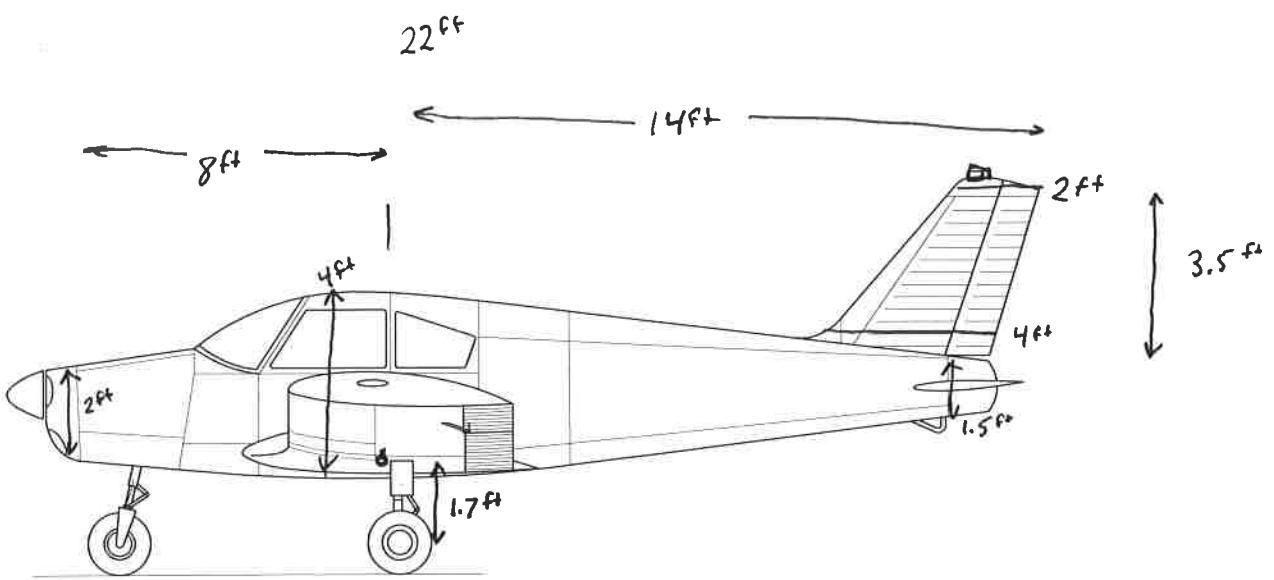
- Dots can be around 0.1 in in diameter and a few thousandths thick ( $\approx 10$  thousandths).
- Spacing  $\approx 0.25$  in.
- Dots come as tape to be applied. *L.L. Bean*

## AEM 313 Problem Set #7

Due: 15<sup>th</sup> November 2016 by 5:00pm

1. Estimate the total wetted area of the Piper Cherokee ( $b=30$  ft,  $S=160$  sq-ft) below.
2. Estimate the drag of the landing gear (reference area  $S=160$  sq-ft).
3. Estimate  $C_{D0}$  given the NACA 65-215 airfoil and a flat plate approximation elsewhere.





Drag Breakdown: Reference area is wing area =  $S_{ref} = 160 \text{ ft}^2$

Wing:  $S = 160 \text{ ft}^2$ ,  $c = 5.33 \text{ ft}$ ,  $b = 30 \text{ ft}$ , NACA 65<sub>2</sub>415

From airfoil polar,  $C_D(C_L=0) = 0.006$

$$C_D = \frac{C_D S}{S_{ref}} = C_D \underbrace{\frac{NACA}{65_2415}}_{\text{f}} = 0.006 \Rightarrow \underbrace{S_{ref} C_D}_{\text{f}} = 0.006 \cdot 160 \text{ ft}^2 = 0.96 \text{ ft}^2$$

Equivalent flat plate area

Wing (alternative "flat plate")

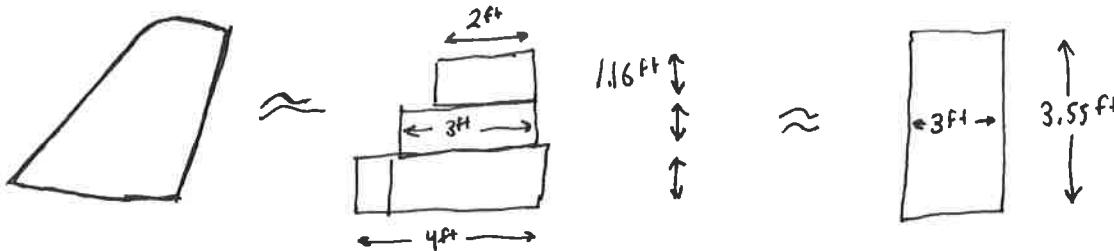
$$C_D = C_{f,turb} \cdot 2 \quad \text{top and bottom!} \\ C_D = C_{f,turb} \cdot 2 = \frac{0.074}{(6350 \cdot 176 \text{ ft} \cdot 5.33 \text{ ft})^{1/5}} = 0.0065 \quad \underline{\text{not bad!}}$$

Horizontal (flat plate)

$$C_D = C_{f,turb} \cdot 2 \cdot \frac{S_h}{S_{ref}} = \frac{0.074}{(6350 \cdot 176 \text{ ft} \cdot 2.57 \text{ ft})^{1/5}} \cdot 2 \cdot \frac{25.7 \text{ ft}^2}{160 \text{ ft}^2}$$

$$C_D = 0.0012 \quad \text{or} \quad f = 0.192 \text{ ft}^2$$

Vertical (flat plates)



$$C_D = C_{f,turb} \cdot 2 \cdot \frac{10.65 \text{ ft}^2}{160 \text{ ft}^2} = \frac{0.074}{(6350 \cdot 176 \cdot 3)^{1/5}} \cdot 2 \cdot \frac{10.65 \text{ ft}^2}{160 \text{ ft}^2} = 0.0005$$

*I don't believe this #!*

$$f = 0.078 \text{ ft}^2$$

## Fuselage

Area Sides  $\approx$



Top Area



Total area  $\approx 266 \text{ ft}^2$

Frontal Area  $\approx 4' \times 4' = 16 \text{ ft}^2$



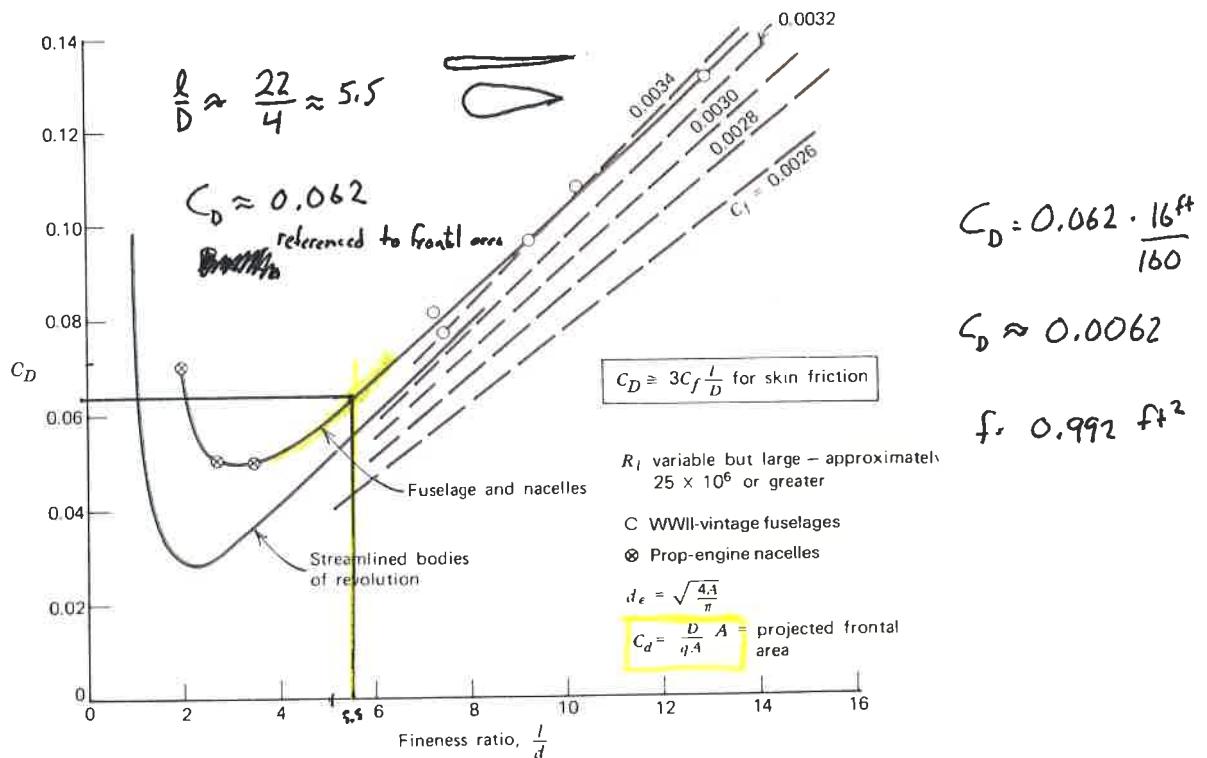
- Flat plate approx.

$$Re \approx 6350 \cdot 176 \cdot 22^4 = 24 \times 10^6$$

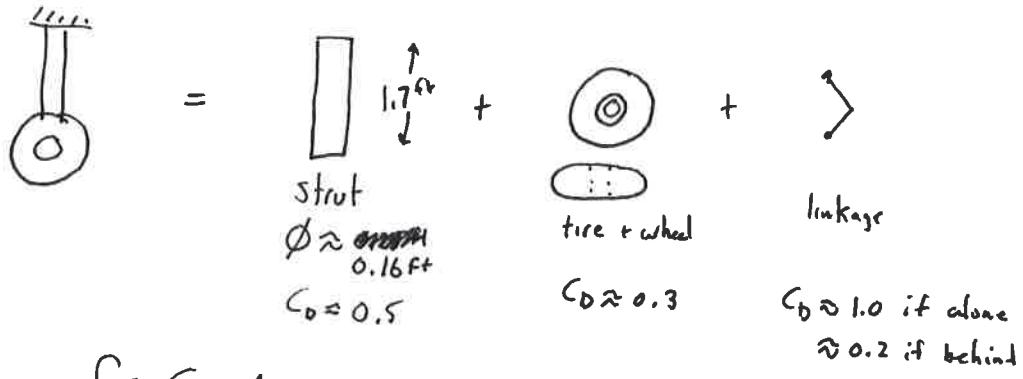
$$C_D = C_{f,turb} \cdot \frac{A}{S_{ref}} = \frac{0.074}{(24 \times 10^6)^{1/5}} \cdot \frac{266 \text{ ft}^2}{160 \text{ ft}^2} \approx 0.0040$$

$$f \approx 0.65 \text{ ft}^2$$

- Empirical Data



## Landing Gear



$$= 0.5 \cdot 0.16 \cdot 1.7 + 0.3 \cdot 1.3^2 \cdot 0.5^2 + 0.2 \cdot 1^2 \cdot 0.1^2$$

$$\approx 0.35 \text{ ft}^2$$

Hoerner!

What else?

- Cooling drag  $\approx 0.5 \text{ ft}^2 = f$
- Gaps
- Rivets
- Antennas

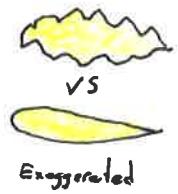
Total:

Wing + Horz + Vert + Fus + LG

$$f = 0.96 + 0.192 + 0.078 + 0.992 + 0.35$$

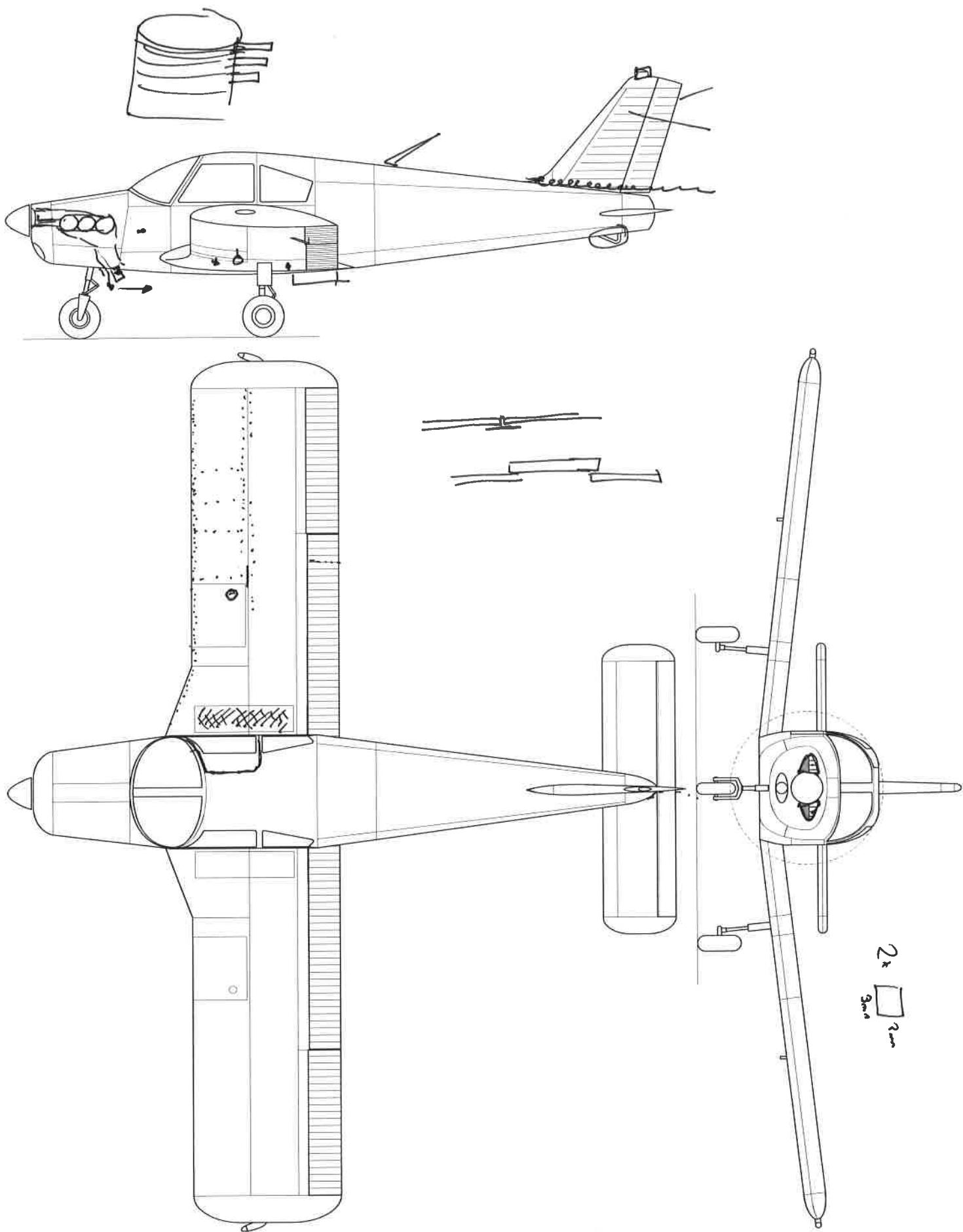
$$= 2.57 \text{ ft}^2 \approx 160 \text{ counts}$$

- Insects
- Dirty airplane / Paint
- Wavy Wing Panels
- Trim Drag  
(wait for 360°)



Way too low

Why?



Q: A flight test of a PA-28 gives  $\frac{L}{D} = 10$  at  $75 \text{ kts}$  at  $2040 \text{ lbf}$ .  
 What is  $C_D$ ?

$$C_L = \frac{W}{\frac{1}{2} \rho V^2 S}$$

$= 9$  at  $60 \text{ kts}$

$= 8$  at  $90 \text{ kts}$

$= 9.5$  at  $65 \text{ kts}$

$= 9.5$  at  $80 \text{ kts}$

$$V = 60 \text{ kts} = 101 \text{ ft/s}$$

$$V = 75 \text{ kts} = 126.6 \text{ ft/s}$$

$$V = 90 \text{ kts} = 152 \text{ ft/s}$$

$$V = 65 \text{ kts} = 109.7 \text{ ft/s}$$

$$V = 80 \text{ kts} = 135 \text{ ft/s}$$

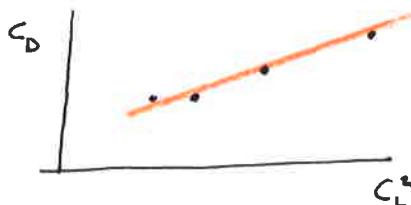
$$\Rightarrow C_L = \begin{cases} 1.055 \\ 0.671 \\ 0.467 \end{cases} \Rightarrow C_D = \begin{cases} 0.1172 \\ 0.0671 \\ 0.0584 \end{cases}$$

$$0.894 \\ 0.590$$

Given  $C_D = C_{D_0} + KC_L^2$ , can we find  $C_{D_0}$  and  $K$ ?

$$\begin{pmatrix} 0.1172 \\ 0.0671 \\ 0.0584 \end{pmatrix} = \begin{pmatrix} C_{D_0} \end{pmatrix} + K \begin{pmatrix} 1.055^2 \\ 0.671^2 \\ 0.467^2 \\ 0.894^2 \\ 0.590^2 \end{pmatrix} = \begin{pmatrix} 0.1172 \\ 0.0671 \\ 0.0584 \\ 0.0941 \\ 0.0621 \end{pmatrix}$$

plotting  $C_D$  vs  $C_L^2$



Best fit equation is.

$$C_D = \underbrace{0.0685}_{K} C_L^2 + \underbrace{0.0396}_{C_{D_0}}$$

Compare with theory and predictions

$$K \approx \frac{0.1}{\pi AR_e} = \frac{1}{\pi \cdot 5.625} \cdot (1+8)^{-1} \approx 1.035$$

$$= 0.0586$$

Why is there a difference?

- Fuselage negatively affects  $K$ . Trim drag affects  $K$ .
- Glide test of  $C_D$  included propeller (windmilling)