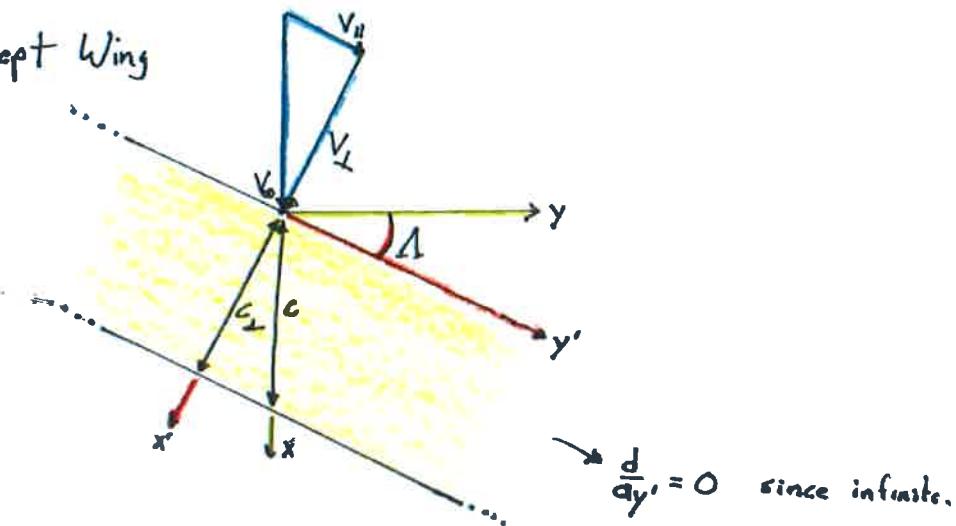


Lesson 27

Swept Wings

Infinite Swept Wing



Following a particle in the Lagrangian Frame of (x', y', z')

$$\rho \frac{DV'}{Dt} = -\nabla' p \Rightarrow \rho \frac{Du'}{Dt} = -\frac{dp}{dx'}$$

$$\rho \frac{Dv'}{Dt} = -\frac{dp}{dy'} = 0 \Rightarrow \frac{Dv'}{Dt} = 0$$

$$\rho \frac{Dw'}{Dt} = -\frac{dp}{dz'}$$

v' is constant in $x'y'z'$

What is v' ?

Far upstream or anywhere else, $v' = \text{constant}$

$$v' = V_{||} = V_\infty \sin \alpha$$

The spanwise velocity is constant
since the wing is infinite

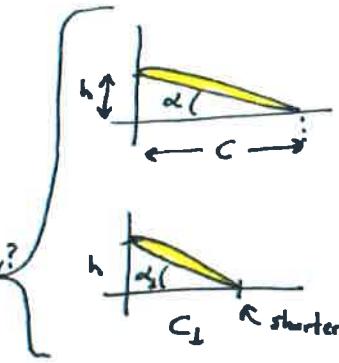
Geometry Terms

$$C_L = c \cos \Lambda$$

$$V_L = V_\infty \cos \Delta$$

$$\alpha_L = \alpha / \cos \Lambda$$

why?

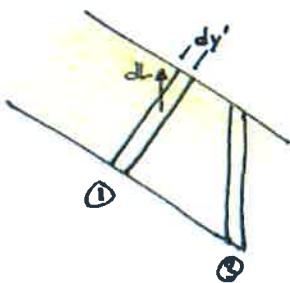


$$\alpha = \tan\left(\frac{h}{c}\right) \approx \frac{h}{c}$$

$$\alpha_L = \tan\left(\frac{h}{C_L}\right) \approx \frac{h}{C_L}$$

$$= \frac{h}{c \cos \Lambda} = \frac{h}{c} \frac{1}{\cos \Lambda} = \alpha \frac{1}{\cos \Lambda}$$

Lift



$$dL = \underbrace{\frac{1}{2} \rho V_L^2}_{g} \underbrace{C_{L\alpha}}_{C_L} \alpha_L dS$$

Integrate to get

$$L = \frac{1}{2} \rho V_L^2 C_{L\alpha} \alpha_L S$$

$$= \frac{1}{2} \rho V_\infty^2 \cos^2 \Lambda C_{L\alpha} \alpha \frac{1}{\cos \Lambda} S$$

Lift Coefficient

$$C_{L\alpha} = \frac{L/\alpha}{g S} = \frac{\frac{1}{2} \rho V_\infty^2 C_{L\alpha} \cos \Lambda S}{\frac{1}{2} \rho V_\infty^2 \cdot S} = C_{L\alpha} \cos \Lambda$$

Sweep reduces lift by the cosine of sweep.

Ex: The Facebook Aquila program has developed a high AR swept wing (flying wing).

The aircraft has a sweep angle of about 20° . Estimate $C_{L\alpha}$ if AR is large (i.e. 2D flow)



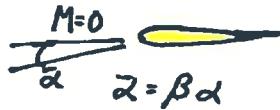
$$C_{L\alpha} \approx C_{L\alpha} \cos \Lambda = 2\pi \cos(20^\circ) = 94\% \cdot 2\pi$$

$C_{L\alpha} < 5.9 \text{ is}$

Compressible Swept Wing

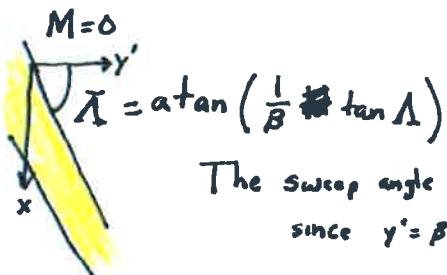
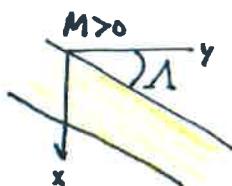
Given a non-zero Mach number, P-G creates an affine transform of y and z by β

AOA:



$$\beta = \sqrt{1 - M_\infty^2}$$

Sweep:



The sweep angle increased.
since $y' = \beta y$

Some math to get $\cos \bar{\lambda}$ from $\tan \bar{\lambda}$

~~note down~~

$$c^2 + s^2 = 1 \quad \text{and} \quad \frac{s^2}{c^2} = t^2 \Rightarrow \bar{t}^2 + 1 = \frac{1}{\cos^2 \bar{\lambda}} \Rightarrow \bar{t}^2 = \frac{1}{\bar{c}^2} - 1$$

$$\therefore \sqrt{\bar{t}^2} = \bar{t} = \sqrt{\frac{1}{\bar{c}^2} - 1} = \underbrace{\frac{1}{\beta} \tan}_{\text{from above}}$$

Solve for \bar{c}

$$\frac{1}{\bar{c}^2} - 1 = \frac{1}{\beta^2} \bar{t}^2 \Rightarrow \frac{1}{\bar{c}^2} = \sqrt{\frac{1}{\beta^2} \bar{t}^2 + 1}$$

$$\bar{c} = \frac{1}{\sqrt{\frac{1}{\beta^2} \bar{t}^2 + 1}} \cdot \frac{\beta c}{\beta c} = \frac{\beta c}{\sqrt{\bar{t}^2 c^2 + \beta^2 c^2}} = \frac{\beta c}{\sqrt{s^2 + \beta^2 c^2}}$$

$$\cos \bar{\lambda} = \frac{\beta \cos \lambda}{\sqrt{s^2 + \beta^2 c^2}}$$

From Incompressible case

$$\bar{C}_L = C_{L\alpha} \bar{\alpha} \cos \bar{\Lambda}$$

Apply Goethert's Rule $C_L = \frac{1}{\beta^2} \bar{C}_L$

$$C_L = \frac{1}{\beta^2} C_{L\alpha} \bar{\alpha} \cos \bar{\Lambda} = \frac{1}{\beta^2} C_{L\alpha} \beta \alpha \frac{\beta \cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}}$$

$$= \frac{C_{L\alpha} \cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}} \cdot \alpha$$

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}}$$

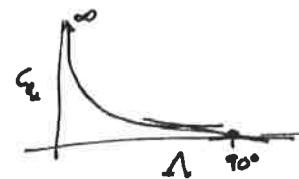
$$M=0: \beta = \sqrt{1-M^2}$$

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{\cos^2 \Lambda + \sin^2 \Lambda}} = C_{L\alpha} \cos \Lambda \quad \text{recovered previous page's result } \checkmark$$

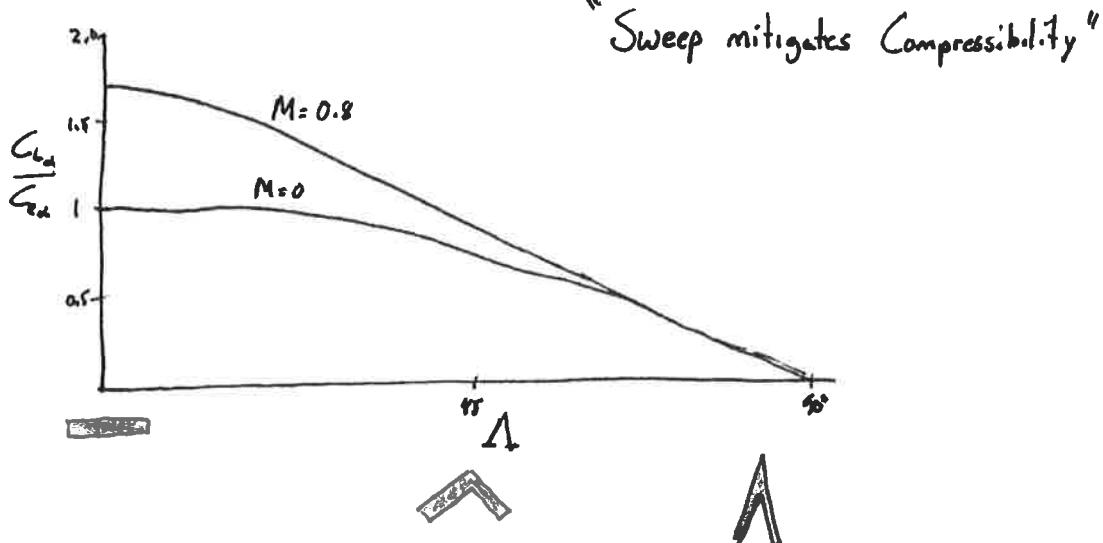
$M \approx 1$: PG is nonsense.

$$C_{L\alpha} = C_{L\alpha} \cdot \frac{\cos \Lambda}{\sqrt{\beta^2 \cos^2 \Lambda + \sin^2 \Lambda}} = C_{L\alpha} \frac{\cos \Lambda}{\sin \Lambda} = C_{L\alpha} \frac{1}{\tan \Lambda}$$

$$\Lambda = 0 \Rightarrow C_{L\alpha} \text{ is } \infty \quad X$$

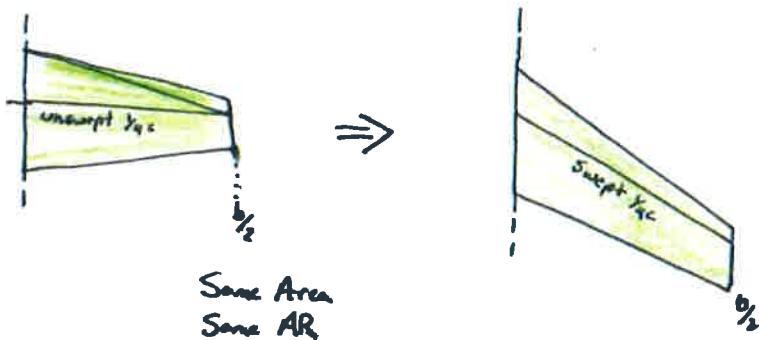


From NACA Fig. 8.18
FVA



Swept Wings Chapters 20 - 24 in ADTA

Given the infinite swept wing results (mitigation of compressibility), apply to wings.



Historical note:

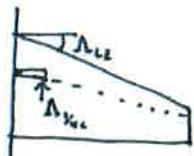
The 1st operational jet fighter, the Me-262, had 18° swept wings. This sweep corrected a CG issue (heavy engines) rather than an aerodynamic reason.

$$\cos(18^\circ) \approx 95\%$$

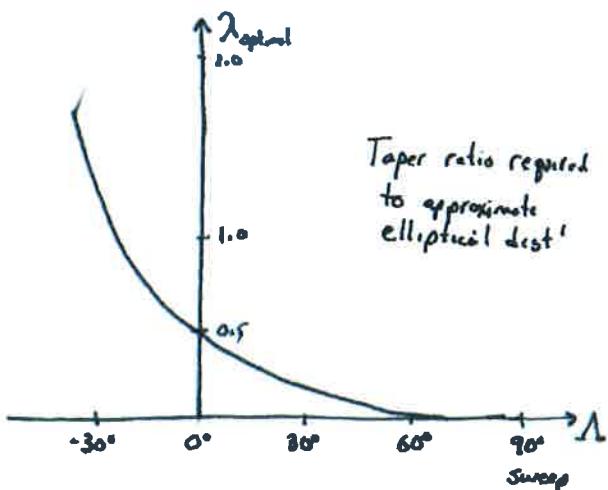
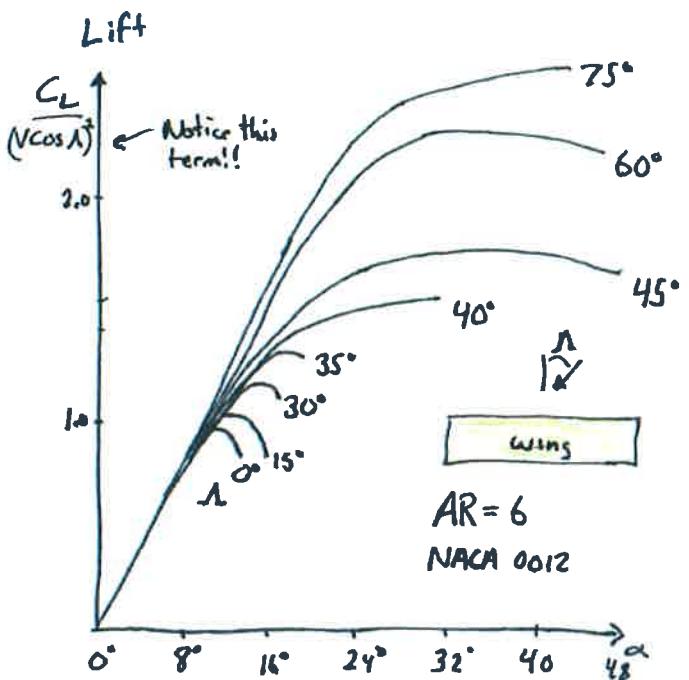
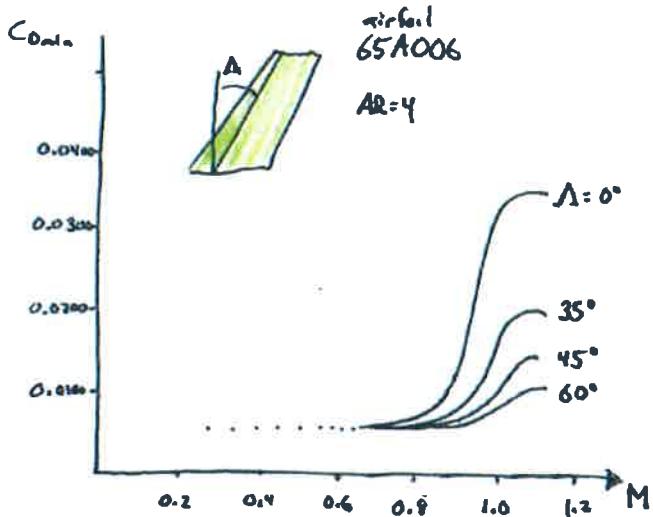
A 35° sweep concept was proposed but rejected. On par with F-86?
No. Dream on, unless you are an excellent pilot!!

Geometry note

$$\Lambda_{LE} \neq \Lambda_{yc} \text{ when } \lambda \neq 1$$

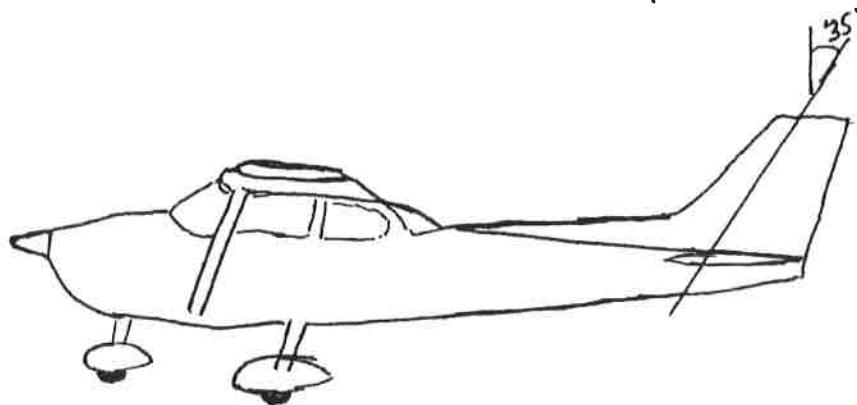


Swept Wings Drag



Hoerner Lift , Drag

Q: Why is the C-172 vertical swept?



A: Advantage in high speed flight (High Mach)? No.

A: Slightly more robust stalling characteristics. Less control authority? Area further aft, but less effective.

$\frac{C_{L_{max}}}{C_{L_{stall}}}$ ≈ 1.3 vs 1.0

$\alpha_{stall} \approx 16^\circ$ vs 10°

87% of straight tail (approx)
60% increase in effective angle

A: Marketing.

"Swept = fast"