

University of Alabama Academic Honor Pledge:

I promise or affirm that I will not at any time be involved with cheating, plagiarism, fabrication, or misrepresentation while enrolled as a student at The University of Alabama. I have read the Academic Honor Code, which explains disciplinary procedures that will result from the aforementioned. I understand that violation of this code will result in penalties as severe as indefinite suspension from the University.

Signature: _____

Date: _____

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Multiple Choice Problems: Circle **EVERY** correct answer [5 pts each]

1. A linearly tapered wing has a span of 40 feet, an aspect ratio of 8, and a root chord of 10 feet. What is the taper ratio? $S = b^2/AR = 40^2/8 = 200$ $\bar{c} = \frac{200}{40} = 5$ $\bar{c} = \frac{c_r + c_t}{2} = \frac{c_t + 10}{2} \Rightarrow c_t = 0$

- | | | | | |
|--------|--------|--------|-------------|----------------------|
| A. 7/9 | B. 1.0 | C. 1.3 | D. 0 | E. None of the above |
|--------|--------|--------|-------------|----------------------|

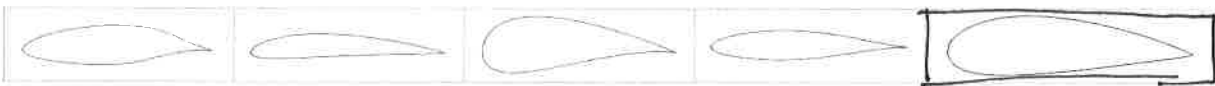
$\lambda = \frac{0}{10} = 0$

2. Estimate the SSL stall speed of a nearly empty Piper Tripacer (S=147 ft², W=1160 lbf) given the maximum lift coefficient is 2.2. $W = L = \frac{1}{2} \rho V^2 S C_{L_{max}} \Rightarrow V^2 = \frac{2W}{\rho S C_{L_{max}}} \Rightarrow V = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$

- | | | | | |
|------------|-----------|-------------------|-----------|----------------------|
| A. 64 ft/s | B. 41 mph | C. 55 ft/s | D. 33 mph | E. None of the above |
|------------|-----------|-------------------|-----------|----------------------|

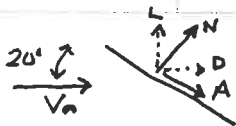
$V^2 = \frac{2(1160 \text{ lbf})}{0.00237 \text{ slug/ft}^3 \cdot 147 \text{ ft}^2 \cdot 2.2} \Rightarrow V = 55 \frac{ft}{s} = 55 \cdot \frac{60}{58} = 37 \text{ mph}$

3. Which airfoil is an NACA 4424? Hint: camber and thickness



4. In a wind tunnel, a wing is mounted at 20 degrees AOA. The normal force is 60 lbs. The axial force is 20 lbs. What is the lift to drag ratio?

- | | | | | |
|--------|--------|----------------|---------|----------------------|
| A. 3.0 | B. 1.6 | C. 1.25 | D. 0.79 | E. None of the above |
|--------|--------|----------------|---------|----------------------|



$L = N \cos(20^\circ) - A \sin(20^\circ) = 49.5 \text{ lbf}$
 $D = N \sin(20^\circ) + A \cos(20^\circ) = 39.3 \text{ lbf}$

$\frac{L}{D} = 1.26$
 within 1% of 1.25 ✓

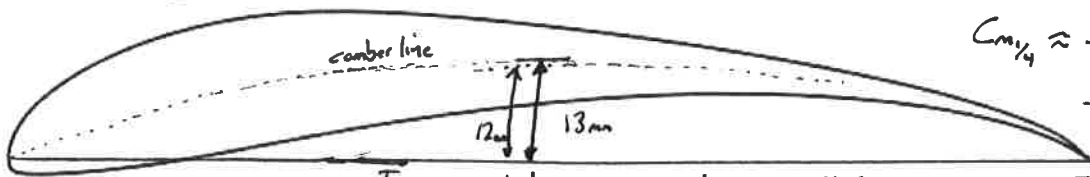
5. What is the air density in slug/ft³ at standard sea level and 59F given 90% humidity?

- | | | | | |
|------------|-------------------|------------|------------|----------------------|
| A. 0.00219 | B. 0.00236 | C. 0.00237 | D. 0.00238 | E. None of the above |
|------------|-------------------|------------|------------|----------------------|

Increasing humidity decreases density... only A, B, E

6. Given the following Selig 1223 airfoil, estimate $C_{m_{1/4}}$ at the quarter chord. After the exam, compare this estimate with the actual value to see amazing precision given the simple Joukowski estimate.

- | | | | | |
|----------|------------------|-----------|-----------|----------------------|
| A. 0.086 | B. -0.272 | C. -0.149 | D. -0.226 | E. None of the above |
|----------|------------------|-----------|-----------|----------------------|



$C_{m_{1/4}} \approx -\pi \frac{f}{c}$
 $-\pi \frac{12}{142} \approx -0.265$ (lower bound)
 $-\pi \frac{13}{142} \approx -0.287$ (upper bound)

I averaged two measurements of max f/c . $C = 142 \text{ mm}$

7. In a 2D flow, you notice a particle rotating at 1 rad/s. What is the vorticity?

- | | | | | |
|------|--------|------|-------------|----------------------|
| A. 0 | B. 1/2 | C. 1 | D. 2 | E. None of the above |
|------|--------|------|-------------|----------------------|

Twice angular rate

8. For an NACA 64₃-418 airfoil at Re=6 million, what is the drag coefficient at -6 degrees AOA? The experimental data is plotted below (source: Theory of Wing Sections)

- A. 60 counts B. 140 counts **C. 80 counts** D. 95 counts E. None of the above

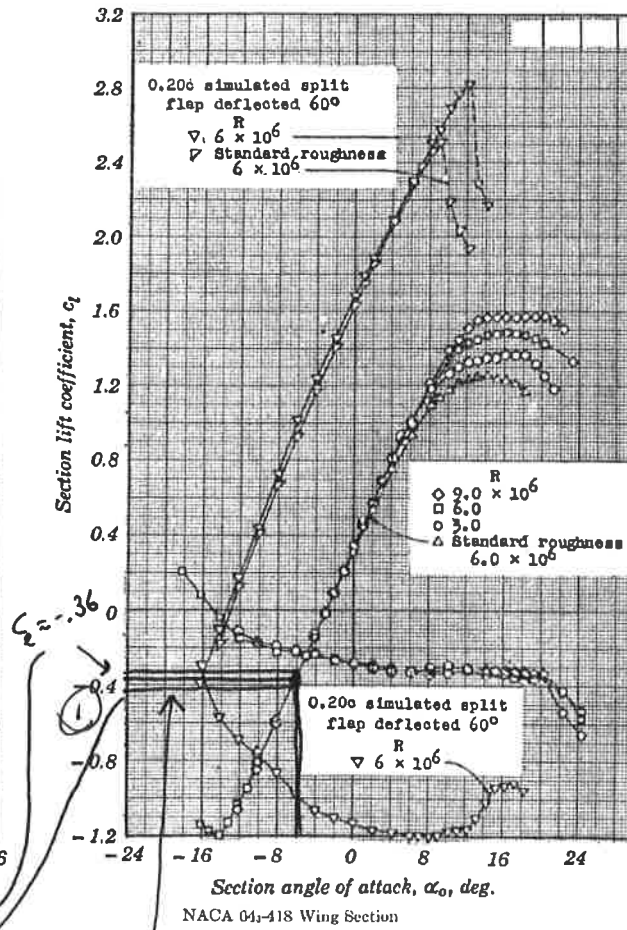
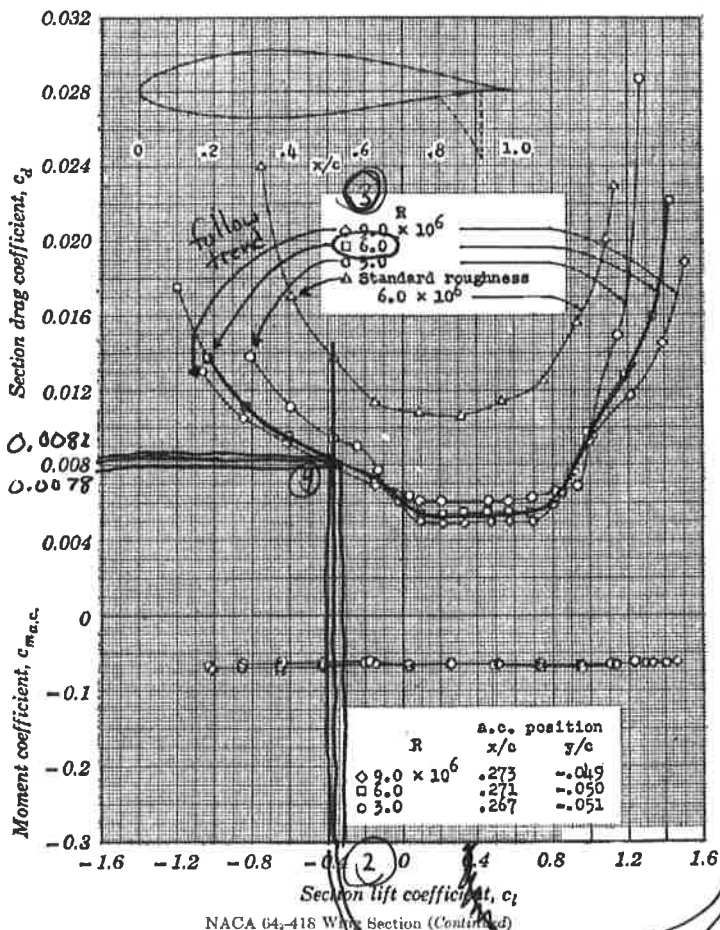
9. Given a **steady** flow, which of the following visualizes a trace of all fluid elements that flowed through a fixed location? Steady flow indicates $d/dt=0$.

- F. Pathline** **G. Streakline** H. Timeline **I. Streamline** J. None of the above

10. Given a 15% thick symmetrical Joukowski airfoil at 10 degrees AOA, estimate $C_{l\alpha}$ [1/rad]?

- K. 6.9** L. 20π M. 0.12 N. 2π O. None of the above

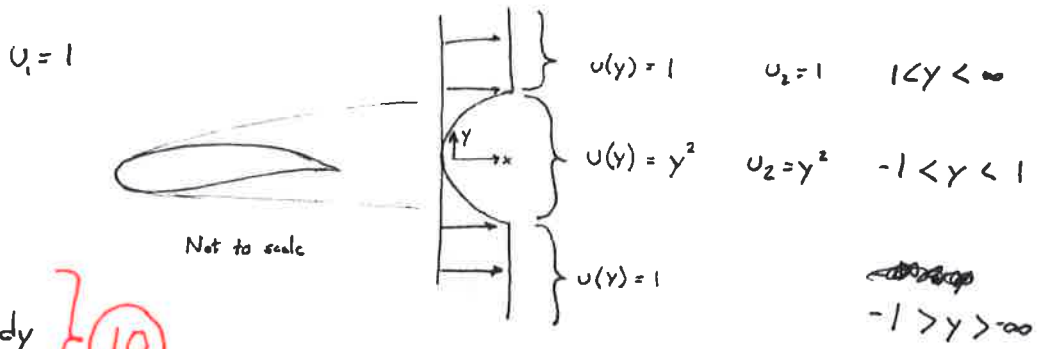
$$C_{e_d} \approx 2\pi \frac{1+\epsilon}{1+\epsilon^2} \text{ with } \epsilon \approx 0.77 \cdot 15\% \Rightarrow C_{e_d} \approx 6.9$$



uncertainty band to ensure that I bound the correct value.

When you are working with experimental hard-to-read data, improve your estimate with multiple samples.

11. [25 pts] An airfoil with a 100 inch chord creates the following velocity profile measured in inches. The upstream velocity is $U_1=1$. Determine the sectional drag coefficient C_d .



$$D = \int_{-\infty}^{\infty} \rho_2 u_2 (U_1 - u_2) dy \quad \text{⑩}$$

Notice that the profile is symmetric about $y=0$

$$D = \int_{-\infty}^{\infty} (\dots) = 2 \int_{-\infty}^{\infty} (\dots)$$

Integral ⑩

Notice that the u_2 function switches from y^2 to 1 at $y^2 = 1 \Rightarrow y = \pm 1$. New integration limits/functions are:
split into two portions ≈ 3pts

$$\begin{aligned}
 D_{1/2} &= \int_0^1 \rho_2 y^2 (1 - y^2) dy + \int_1^{\infty} \rho_2 \cdot 1 \cdot (1 - 1) dy = \int_0^1 \rho_2 y^2 (1 - y^2) dy \\
 &= \rho_2 \int_0^1 (y^2 - y^4) dy = \rho_2 \left(\frac{y^3}{3} - \frac{y^5}{5} \right) \Big|_0^1 = \rho_2 \left(\frac{1}{3} - \frac{1}{5} \right) = \rho_2 \frac{2}{15} \quad \text{Value 1pt} \\
 &\quad \text{apply velocities and defail 3pts} \quad \text{Integral 3pts}
 \end{aligned}$$

Non-dim

$$C_d = \frac{2D_{1/2}}{\frac{1}{2} \rho U_1^2 c} = \frac{2 \rho_2 \frac{2}{15}}{\frac{1}{2} \rho U_1^2 c} = \frac{2 \rho_2 \frac{2}{15}}{\frac{1}{2} \rho U_1^2 c} = \frac{1}{375} = \frac{8}{15} \frac{1}{100}$$

$0.2667 \rho_2$

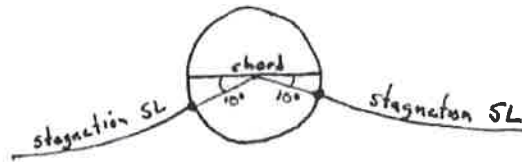
⑤

$C_d = \frac{2}{375} \approx 26 \text{ counts}$

53 counts

Challenge problem.

[25 pts] Given a cylinder of radius 10 inches in a freestream flow of 100 ft/s at SSL, you measure stagnation points at -10 degrees below the chordline. What is the pressure coefficient at the leading edge of the cylinder where $r=10$ and $\theta = \pi$?



Streamfunction: $\Psi = V_{\infty} r \sin \theta \left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi} \ln\left(\frac{r}{R}\right)$ (5)

Velocity at stagnation pt is zero. On surface, $V_r = 0$ already. We want $V_{\theta} = 0$ too.

(5)
$$V_{\theta} = -\frac{d\Psi}{dr} = -V_{\infty} \sin \theta \left(1 - \frac{R^2}{r^2}\right) + V_{\infty} r \sin \theta \left(+\frac{2R^2}{r^3}\right) + \frac{-\Gamma}{2\pi} \frac{1}{r}$$

$$= -V_{\infty} \sin \theta \left(1 - \frac{R^2}{r^2} + \frac{2R^2}{r^2}\right) + \frac{-\Gamma}{2\pi} \frac{1}{r}$$

$$= -V_{\infty} \sin \theta \left(1 + \frac{R^2}{r^2}\right) + \frac{-\Gamma}{2\pi r}$$

(5)
$$V_{\theta}(\theta = -10^\circ) = 0 = -V_{\infty} \sin(-10^\circ) \left(1 + 1\right) - \frac{\Gamma}{2\pi \cdot 10}$$

$$\Gamma = -V_{\infty} \sin(-10^\circ) (2) (2\pi) (10 \dots)$$

$$= -100 \frac{\text{ft}}{\text{s}} \cdot -0.1736 \cdot 4\pi \cdot \frac{10}{12} \text{ft} = 181.8 \frac{\text{ft}^2}{\text{s}}$$

Pressure Coefficient

(3)
$$C_p = 1 - \frac{V^2}{V_{\infty}^2}$$

(5)
$$V_{\theta} = -V_{\infty} \sin\left(\frac{\pi}{2}\right) \left(1 + \frac{R^2}{R^2}\right) - \frac{\Gamma}{2\pi R} = -\frac{\Gamma}{2\pi R} = \frac{-181.8 \text{ft}^2/\text{s}}{2\pi \cdot 10 \text{in}} \cdot \frac{12 \text{in}}{\text{ft}}$$

$$= 34.7 \frac{\text{ft}}{\text{s}}$$

$$C_p = 1 - \frac{34.7^2}{100^2} = 0.88$$

(82)
$$C_p = 0.88$$

Alternative work is fine

Shortcut:

$$C_p = 1 - 4 \sin^2 \theta - \frac{2\Gamma}{\pi R V_{\infty}} \sin \theta - \frac{\Gamma^2}{4\pi^2 R^2 V_{\infty}^2}$$

$$= 1 - 0.12$$

$$= \underline{\underline{0.88}}$$