1) plot the NACA 4414

NACA 4414

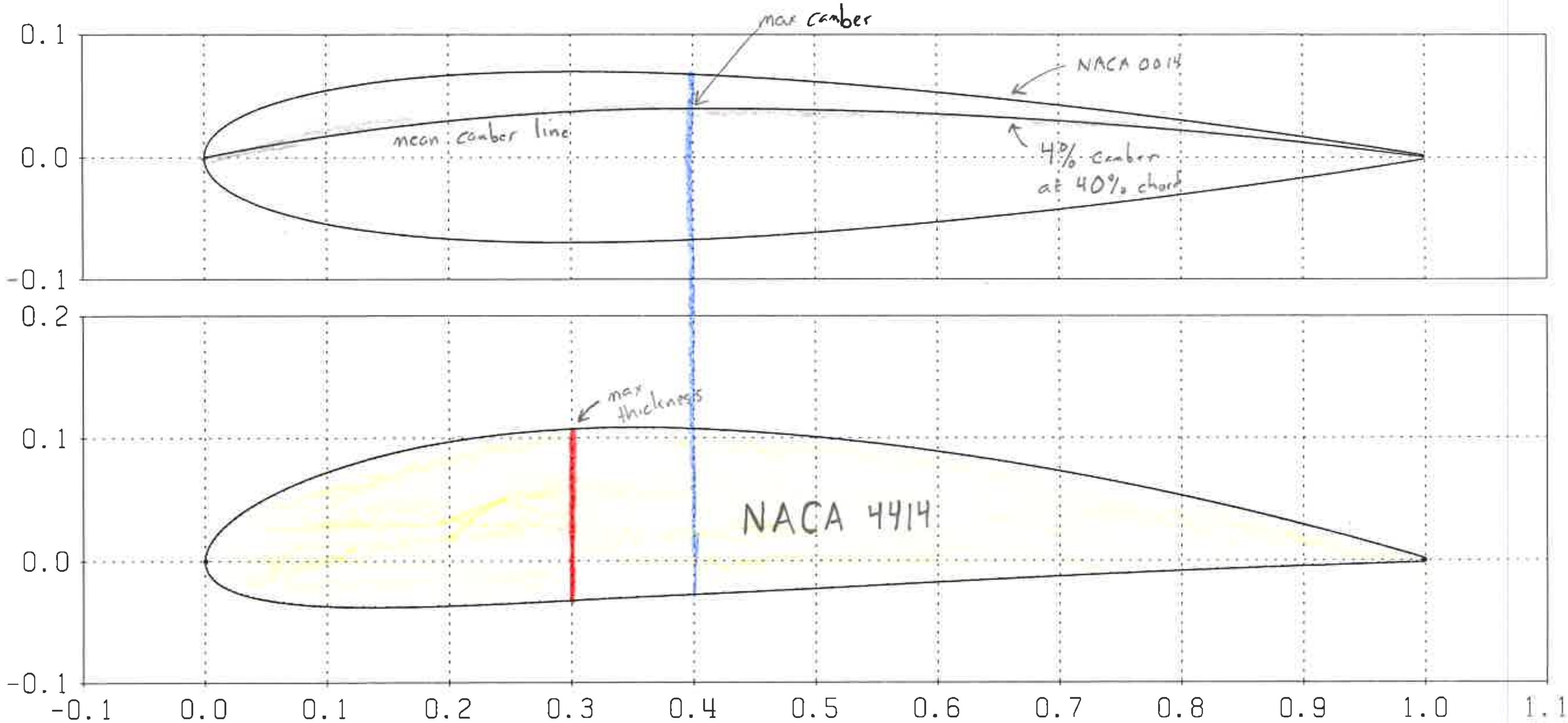
area = 0.09590

thick. = 0.14004

camber = 0.04000

r_{LE} = 0.02149

$\Delta\theta_{TE}$ = 18.28°



2) Compute the density of wet air at 80% rh and 80°F

From lesson 3 notes,

$$\rho = \frac{P M_{dry} + \phi P_{sat} \left(M_{vapor} - M_{dry}\right)}{\bar{R} T}$$

with
$M_{vapor} = 18$

$M_{dry} = 28.97$

$\phi = .80$

$\bar{R} = 1545.34 \ \frac{ft\ lbf}{R\ lbmol}$

$P_{SSL} = 14.7 \ psi$

$T = 80^F + 459.67 = 539.67^R$

From Arden-Buck,

$$P_{sat}^{[psi]} = 0.08865 \exp\left(\frac{-0.002369(539.67 - 8375.65)(539.67 - 491.67)}{539.67 - 28.818} \right)$$

$$= 0.507 \ psi$$

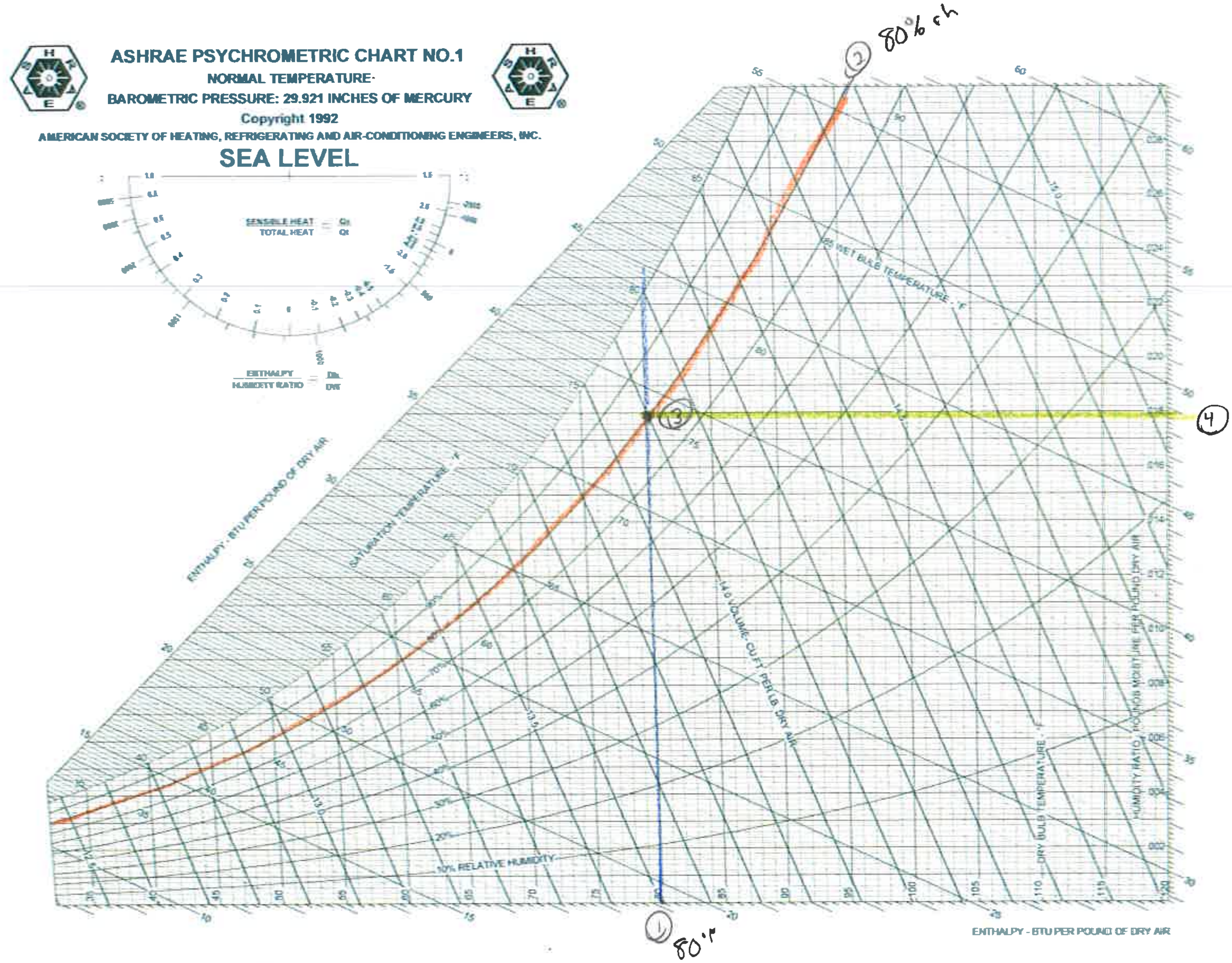
Compares well with thermo book $P_{sat} \approx 0.5073$ psi

Compute density

$$\rho = \frac{14.7 \frac{\cancel{lbf}}{\cancel{in^2}} \Big| \frac{28.97\ \cancel{lbm}}{\cancel{lbmol}} \Big| \frac{slug}{\cancel{lbm}\ 32.174} \Big| \frac{144\ \cancel{in^2}}{ft^2} + 0.80 \Big| \frac{slug}{\cancel{lbm}\ 32.174} \Big| \frac{0.5073\ \cancel{lbf}}{\cancel{in^2}} \Big| \frac{18\ lbm - 28.97\ \cancel{lbm}}{\cancel{lbmol}} \Big| \frac{144\ \cancel{in^2}}{ft^2}}{\frac{1545.34\ ft\ \cancel{lbf}}{\cancel{R}\ \cancel{lbmol}} \Big| 539.67\ \cancel{R}}$$

$$\boxed{\rho = 0.002262 \ \frac{slug}{ft^3}}$$

- Be very careful of the units.
- Track units or you will make a mistake.



1) 80° F
2) 80% rh
3) intersection at $14 \frac{ft^3}{lb}$ of dry air $= v$
4) Humidity ratio 0.0178

Calculate

$$\rho = \frac{1}{v} \cdot (1 + hr) = \frac{lb}{14\ ft^3} \cdot \frac{slug}{32.174\ lb} \cdot (1 + 0.0178)$$

$$\boxed{\rho = 0.00226\ \frac{slug}{ft^3}}$$

3) Compute Re and q for $V_\infty = 100\ ft/s$, $L = 4\ ft$ at SSL.

$$Re \equiv \frac{\rho V L}{\mu}$$

$\rho_{SSL} = 0.0023769\ \frac{slug}{ft^3}$

$\mu = 3.7383 \times 10^{-7}\ \frac{slug}{ft \cdot s}$

$$= \frac{0.0023769\ \cancel{slug}}{\cancel{ft^3}} \Big| \frac{100\ \cancel{ft}}{\cancel{s}} \Big| 4\ \cancel{ft} \Big| \frac{\cancel{ft}\ \cancel{s}}{3.7383 \times 10^{-7}\ \cancel{slug}}$$

$$= 2.5 \times 10^6$$

$$Re \approx 6350 \cdot V^{[ft/s]} \cdot L^{[ft]}$$

$$= 6350 \cdot 100 \cdot 4 = 2.54 \times 10^6$$

$$\boxed{Re = 2.5 \times 10^6}$$

$$q \equiv \tfrac{1}{2} \rho V^2 = \frac{1}{2} \Big| \frac{0.00237\ \cancel{slug}}{ft^2\ \cancel{ft^2}} \Big| \frac{100^2\ \cancel{ft^2}}{\cancel{s^2}} \Big| \frac{lbf\ \cancel{s^2}}{\cancel{slug\,ft}} = 11.8\ psf$$

$$\boxed{q = 11.8\ psf}$$

4) $b = 5$ in , $\Lambda_{c/4} = 30°$, $AR = 5$, $\lambda = 0.5$

Calculate:

$$AR \equiv \frac{b^2}{S} \Rightarrow S = \frac{b^2}{AR} = \frac{5^2}{5} = 5 \text{ in}^2$$

Average wing chord $\equiv \bar{c}$ and $S = \bar{c}\, b \Rightarrow \bar{c} = \frac{S}{b} = \frac{5 \text{ in}^2}{5 \text{ in}} = 1$ in

Taper ratio $\equiv \frac{C_t}{C_r} = 0.5$ and averge chord $\equiv \underbrace{\frac{C_t + C_r}{2}}_{\frac{0.5 C_r + C_r}{2} = 1^{\text{in}}} = 1^{\text{in}}$

Sweep: $30° \Rightarrow \tan(30°) = 0.577$ aft per inch outboard

$\tan(30°) \cdot \frac{b}{2} = 1.44$ in aft at tip

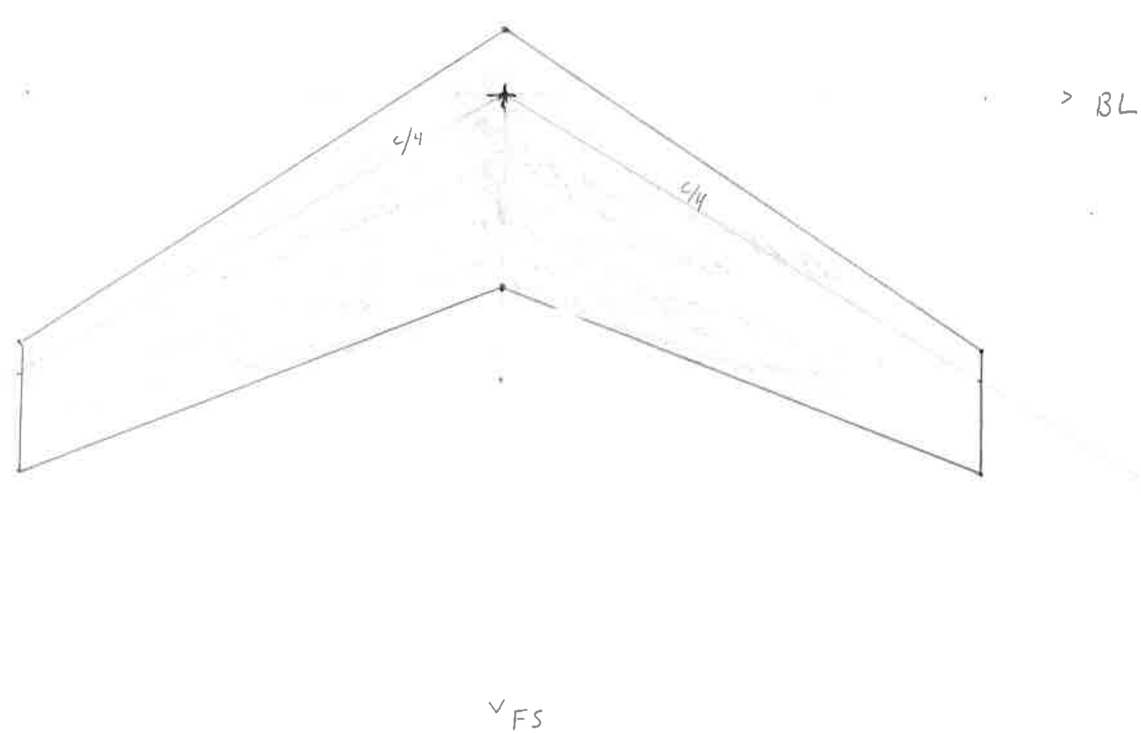
Notice that $\Lambda_{LE} \neq \Lambda_{c/4}$

Solve for $1.5 C_r = 2^{\text{in}}$

$C_r = 1.333^{\text{in}}$

$C_t = 0.667^{\text{in}}$

Draw



5) Determine density at 15 kft on a non standard but dry day

$p = 29.80$ in-Hg

$T_{SL} = 80°F = 539.67\ R$

$$\lambda = \begin{cases} -0.005\ \frac{R}{ft} & h < 10000\ ft \\ 0 & h > 10000\ ft \end{cases}$$

From Lesson 3

For linear lapse rates,

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{-g_0}{R\lambda}}$$

For isothermal lapse rates

$$\frac{p}{p_0} = \exp\left(\frac{-g_0 (h_1 - h_0)}{RT_0}\right)$$

Since our lapse rate has two parts (below 10 kft and above 10 kft), use the linear lapse up to 10 kft, record the pressure and then transition to the isothermal for the portion up to 15 kft.

- $0 \longrightarrow 10^{kft}$

$p = 29.80$ in-Hg and 14.7 psi $= 29.92$ in-Hg $\Rightarrow P_{SL} = \frac{29.80\ \cancel{in\text{-}Hg}}{} \Big| \frac{14.7\ lbf}{in^2} \Big| \frac{}{29.92\ \cancel{in\text{-}Hg}} = 14.64$ psi

$T_{15ft} = T_{SL} + \lambda\,(10000\,ft - 0\,ft) = 539^R - 0.005\,\frac{R}{ft}\,(10000\,ft) = 489$

$$\frac{p}{p_0} = \left(\frac{489^R}{539^R}\right)^{\frac{-32.174\ ft}{s^2}\Big|\frac{R\ slug}{1716.5\ ft\ lbf}\Big|\frac{ft}{-0.005\ R}\Big|\frac{lbf\ s^2}{slug\ ft}} = \left(\frac{489}{539}\right)^{3.749} = 0.694$$

$P_{10kft} = \delta P_{SL} = 14.64\ psi \cdot 0.694 = 10.163$

- $10^{kft} \longrightarrow 15^{kft}$

$P_{10kft} = 10.163\,psi$ $\quad T_{10\,kft} = 489\ R$

$\lambda = 0 \quad \Rightarrow \quad T_{15kft} = 489\ R$

$$\frac{P_{15}}{P_{10}} = \exp\left(\frac{-g_0(h_1 - h_0)}{RT_0}\right) = \exp\left(-\frac{32.174\ \cancel{ft}}{s^2}\Big|(15000 - 10000)\,ft\Big|\frac{\cancel{R}\ slug}{1716.5\ ft\ lbf}\Big|\frac{}{489\ \cancel{R}}\Big|\frac{lbf\ s^2}{slug\ ft}\right)$$

$= 0.826 \quad \Rightarrow \quad P_{15} = P_{10} \cdot 0.826 = 8.39\ psi$

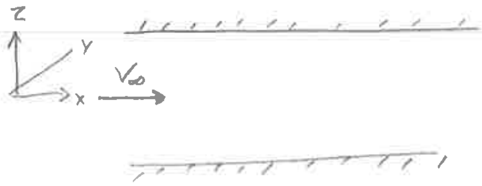
- Density

$$\rho = \frac{P}{RT} = \frac{8.39\ lbf}{\cancel{in^2}}\Big|\frac{\cancel{R}\ slug}{1716.5\ ft\ lbf}\Big|\frac{}{489\ \cancel{R}}\Big|\frac{144\ \cancel{in^2}}{ft^2} = \boxed{0.00144\ \frac{slug}{ft^3} = \rho}$$

6) Wind tunnel Sting : Determine α and β

$\phi = 80^\circ$, $\theta = 30^\circ$, $\psi = -45^\circ$

Wind tunnel geometry



$$\vec{V}_{\infty_{\text{global frame}}} = \begin{pmatrix} V_\infty \\ 0 \\ 0 \end{pmatrix} = V_\infty \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Transformation

The given sting connects from the floor to the model as: yaw, pitch, roll. This is exactly the canonical aero Euler angles. We need to transform $\vec{V_\infty}$ from global to local

$$T^{gl} = \left(T^{lg}\right)^T \quad \Rightarrow \quad \begin{pmatrix} u \\ v \\ w \end{pmatrix}_{local} = T^{gl} \begin{pmatrix} V_\infty \\ 0 \\ 0 \end{pmatrix}$$

Also, V_∞ only has a non zero term in the 1st row, so the matrix multiplication only catches the 1st column of T^{gl} or the 1st row of T^{lg}

Local frame velocities

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}_{local} = T^{gl} \begin{pmatrix} V_\infty \\ 0 \\ 0 \end{pmatrix} = \left(T^{lg}\right)^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} V_\infty \Rightarrow \frac{1}{V_\infty} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix}$$

$$\begin{pmatrix} u/V_\infty \\ v/V_\infty \\ w/V_\infty \end{pmatrix} = \begin{pmatrix} 0.612 \\ 0.471 \\ -0.635 \end{pmatrix}$$

verify that $u^2 + v^2 + w^2 = V_\infty^2$ ✓

Compute α + β

$$\alpha \equiv \operatorname{atan}\left(\frac{w}{u}\right) = \operatorname{atan}\left(\frac{-0.635}{0.612}\right) = -0.804 \text{ rad} = -46^\circ$$

$$\beta = \operatorname{asin}\left(\frac{v}{|V|}\right) = \operatorname{asin}(0.471) = 0.490 \text{ rad} = 28^\circ$$

$$\boxed{\alpha = -46^\circ, \quad \beta = 28^\circ}$$