AEM 313 Problem Set #3

Due: 15th September 2017

1. Read AfE chapter 3 up to 3.12.3.
2. Watch "Vorticity Part 1" and "Vorticity Part 2" at http://web.mit.edu/hml/ncfmf.html.
   - Provide a **short** summary/abstract of these films. Concise, technical, and succulent.
3. Determine the vorticity field $\omega(x,y)$ of the following flow:

$$u(x,y) = y^2 + \cos(x)$$
$$v(x,y) = x + \sin(\mathrm{y})$$

4. For the following flow:

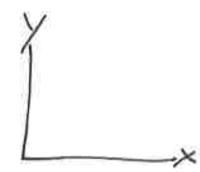
$$u_r(\mathrm{r},\theta) = \frac{\Lambda}{2\pi r}$$
$$\mathrm{u}_\theta(\mathrm{r},\theta) = \frac{\Gamma}{2\pi r}$$

   - Compute the circulation about a unit circle (i.e. circle of radius=1)
   - Compute the divergence about a unit circle
   - Where is this flow rotational? Where is this flow irrotational?

3) $\omega \equiv \nabla \times V$ $\qquad V = (\underbrace{y^2 + \cos(x)}_{V_x}, \underbrace{x + \sin y}_{V_y}, \underbrace{0}_{V_z})^T$

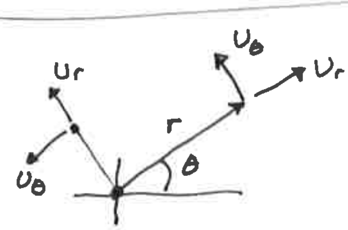
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \cancel{\frac{d}{dz}}^{0} \\ y^2+\cos(x) & x+\sin(y) & 0 \end{vmatrix} = \left[\frac{d}{dx}\left(x+\sin(y)\right) - \frac{d}{dy}\left(y^2+\cos(x)\right)\right]\hat{k}$$

$$\boxed{\omega = (1 - 2y)\,\hat{k}}$$



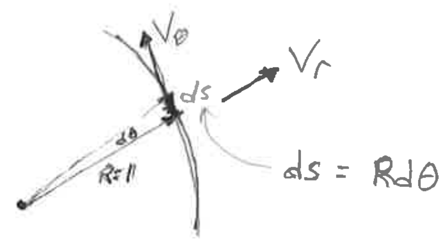
4) polar coordinate system

$U_r = \frac{\Lambda}{2\pi r}$ $\qquad U_\theta = \frac{\cancel{\Gamma_0}^{C_0 \; A_1}}{2\pi r}$



- Circulation

$$\Gamma = -\oint V \cdot ds$$



$$V \cdot ds = V_\theta ds = V_\theta R\,d\theta$$

$$\Gamma = -\oint_{R=1,\,0}^{2\pi} \frac{\Gamma_0}{2\pi}\cancel{R}^{1}\,d\theta = -\frac{\Gamma_0}{2\pi}\,\theta\Big|_0^{2\pi} = -\frac{\Gamma_0}{2\pi}(2\pi - 0)$$

$$\boxed{\Gamma = -\Gamma_0}$$

- Divergence

$$\sigma = \underbrace{\nabla \cdot V}_{div\, V}$$

~~Don't do this~~ $\cancel{\frac{dU_x}{dx} + \frac{dU_y}{dy} = ?}$ ~~Not 3D cartesian~~

in a polar coordinate, look up $div\, V = \frac{1}{r}\frac{d}{dr}(r F_r) + \frac{1}{r}\frac{dF_\theta}{d\theta}$

$$\sigma = \frac{1}{r}\frac{d}{dr}\cancel{\left(r\,\frac{\Lambda}{2\pi r}\right)}^{0} + \frac{1}{r}\frac{d}{d\theta}\cancel{\left(\frac{\Gamma}{2\pi r}\right)}^{0}$$

$\boxed{\sigma = 0}$??

Except that we canceled $\frac{1}{r} \cdot r$ and $\frac{0}{r}$ when $r \neq 0$

So at $r = 0$, $\neq 0$