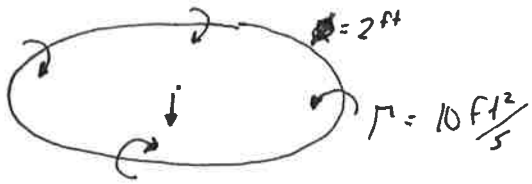
AEM 313 Problem Set #6

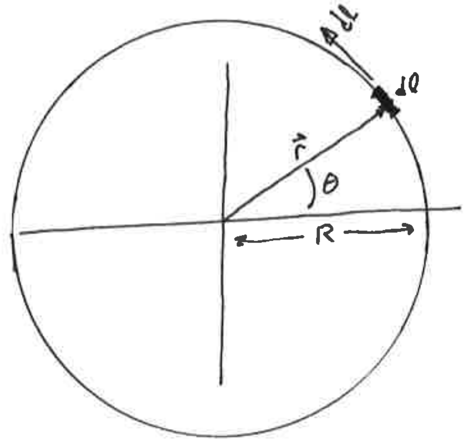
Due: 23rd October 2017

1. Compute the "induced" velocity at the center of a 2 foot circular ring-vortex of strength 10 ft^2/s.
2. Given a wing of b=40 ft with an elliptical lift distribution generating 3000 lbf of lift at SSL, determine the shed vorticity distribution. Assume $V_\infty = 100\ ft/s$
3. For the above wing, determine the downwash velocity along the wing's quarter chord.
4. Compute the induced drag coefficient for an AR=10 elliptical wing.
5. Compute the induced drag coefficient for an AR=10 elliptical wing in ground effect. Plot induced drag as a function of height (h/b).

1) Circular vortex ring velocity.



- Long Way
  - Biot Savart



$$dV = \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3}$$

  - dl segment

$$dl = R\, d\theta \quad \Rightarrow \quad \vec{dl} = 0_r\,,\ R d\theta_\theta$$

  - plug into B-S above

$$dV = \frac{\Gamma}{4\pi} \frac{1}{R^3} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ 0 & Rd\theta & 0 \\ R & 0 & 0 \end{vmatrix} = -\frac{\Gamma}{4\pi} \frac{1}{R^3} R^2 d\theta$$

  - Integrate

$$V = \int dV = \int_0^{2\pi} -\frac{\Gamma}{4\pi} \frac{1}{R^3} R^2 d\theta = -\frac{\Gamma}{4\pi} \frac{1}{R}\, \theta \Big|_0^{2\pi}$$

$$\boxed{V = \frac{\Gamma}{2}}$$

- Short way

Since circle creates a completely arbitrary $\vec{dl}$, $dl \times r$ is constant

$$V = \int dV = \int_0^{2\pi} \frac{\Gamma}{4\pi} \frac{R\,d\theta \cdot R}{R^3} = \frac{\Gamma}{2\pi} \frac{1}{R} \int_0^{2\pi} d\theta$$

$$\boxed{= \frac{\Gamma}{2}}$$

2) Wing (elliptical), $L = 3000^{lbf}$, at SSL, $b = 40^{ft}$

Studio Math

$$G := \frac{4 \cdot L}{\rho \cdot V \cdot b \cdot \pi}$$

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$$\Gamma := G \cdot \sqrt{1 - \left(\frac{2 \cdot y}{b}\right)^2}$$

$$\Gamma = \frac{4 \cdot L \cdot \sqrt{\frac{b^2 - 4 \cdot y^2}{b^2}}}{\rho \cdot V \cdot b \cdot \pi}$$

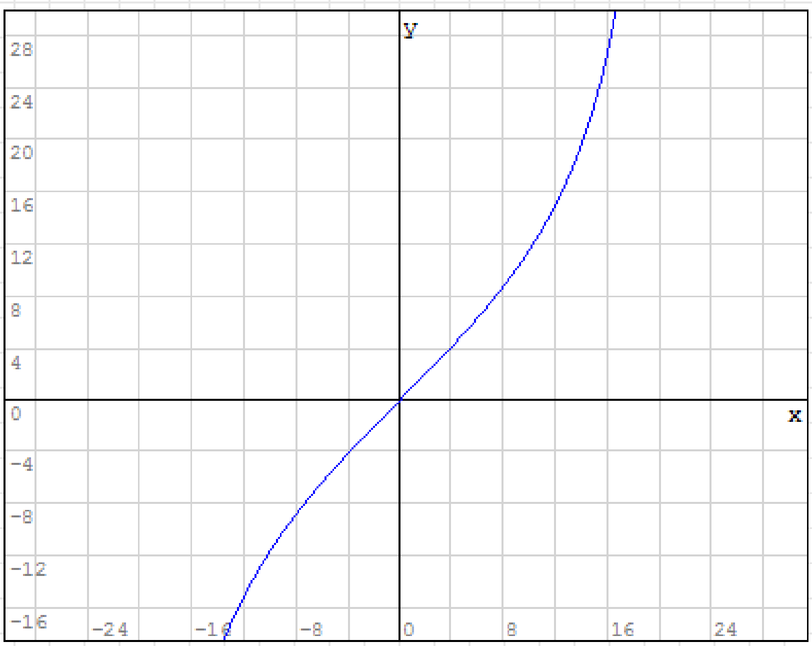
$$\gamma := -\frac{d}{dy}\Gamma$$

$$\gamma = \frac{16 \cdot L \cdot y \cdot \sqrt{\frac{b^2 - 4 \cdot y^2}{b^2}}}{\left(b^2 - 4 \cdot y^2\right) \cdot \rho \cdot V \cdot b \cdot \pi}$$

Simplify

$$\frac{16 \cdot L \cdot y}{\sqrt{b^2 - 4 \cdot y^2} \cdot \rho \cdot V \cdot b^2 \cdot \pi}$$

$L := 3000$ $\rho := 0.00237$ $b := 40$ $V := 100$



$$\frac{16 \cdot L \cdot x}{\sqrt{b^2 - 4 \cdot x^2} \cdot \rho \cdot V \cdot b^2 \cdot \pi}$$

3) Downwash

$$W(y) = -\frac{\Gamma_0}{2b} = \frac{4L}{\rho V_\infty b \pi}\,\frac{1}{2b}$$

$$= \frac{4 \cdot 3000\ \text{lbf}\cdot \text{ft}^3 \cdot \text{s} \cdot \text{slug ft}}{0.00237\ \text{slug} \cdot 100\ \text{ft} \cdot 40\ \text{ft} \cdot \pi \cdot 2 \cdot 40\ \text{ft} \cdot \text{lbf s}^2\ \text{s}}$$

$$\boxed{W = 5\ \text{ft/s}}$$

4) C_{D_i} for elliptical $AR = 10$ wing

$$C_{D_i} = \frac{C_L^2}{\pi AR} = \boxed{\frac{C_L^2}{10\pi} = C_{D_i}}$$

5) C_{D_i} in Ground Effect

$$C_{D_i} = \frac{C_L^2}{\pi AR} = k C_L^2 \Rightarrow k = \frac{1}{\pi AR}$$

$$C_{D_{i_{ge}}} = \frac{K_{eff}}{K} K C_L^2 = \frac{K_{eff}}{K}\frac{C_L^2}{\pi AR} = \frac{33\left(\frac{h}{b}\right)^{1.5}}{1 + 33\left(\frac{h}{b}\right)^{1.5}}\,\frac{C_L^2}{\pi AR}$$