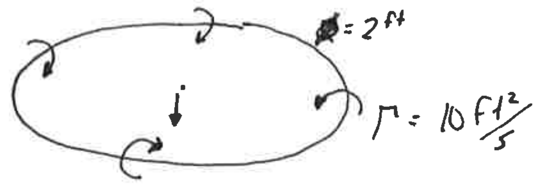
AEM 313 Problem Set #6

Due: 23rd October 2017

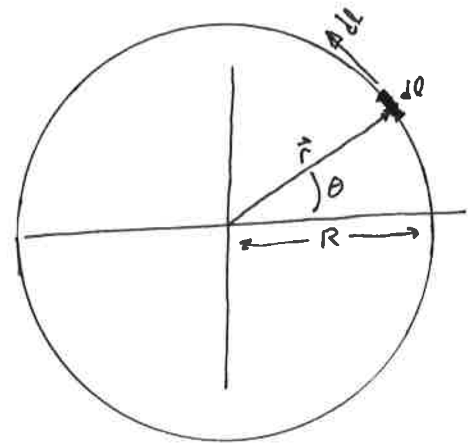
1. Compute the "induced" velocity at the center of a 2 foot circular ring-vortex of strength 10 ft^2/s.
2. Given a wing of b=40 ft with an elliptical lift distribution generating 3000 lbf of lift at SSL, determine the shed vorticity distribution. Assume $V_\infty = 100$ ft/s
3. For the above wing, determine the downwash velocity along the wing's quarter chord.
4. Compute the induced drag coefficient for an AR=10 elliptical wing.
5. Compute the induced drag coefficient for an AR=10 elliptical wing in ground effect. Plot induced drag as a function of height (h/b).

1) Circular vortex ring velocity.



- Long Way
  - Biot Savart

$$dV = \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3}$$



  - dl segment

$$dl = R\,d\theta \Rightarrow \vec{dl} = 0_r \,,\; R d\theta_\theta$$

  - plug into B-S above

$$dV = \frac{\Gamma}{4\pi} \frac{1}{R^3} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ 0 & Rd\theta & 0 \\ R & 0 & 0 \end{vmatrix} = -\frac{\Gamma}{4\pi} \frac{1}{R^3} R^2 d\theta$$

  - Integrate

$$V = \int dV = \int_0^{2\pi} -\frac{\Gamma}{4\pi} \frac{1}{R^3} R^2 d\theta = -\frac{\Gamma}{4\pi} \frac{1}{R} \theta \Big|_0^{2\pi}$$

$$\boxed{V = \frac{\Gamma}{2}}$$

- Short way

Since circle creates a completely arbitrary $\vec{dl}$, $dl \times r$ is constant

$$V = \int dV = \int_0^{2\pi} \frac{\Gamma}{4\pi} \frac{R\,d\theta \cdot R}{R^3} = \frac{\Gamma}{2\pi} \frac{1}{R} \int_0^{2\pi} d\theta$$

$$\boxed{= \frac{\Gamma}{2}}$$

2) Wing (elliptical), $L = 3000^{lbf}$, at SSL, $b = 40^{ft}$

$$\Gamma(y) = \Gamma_0 \left(1 - \left(\frac{2y}{b}\right)^2\right)$$

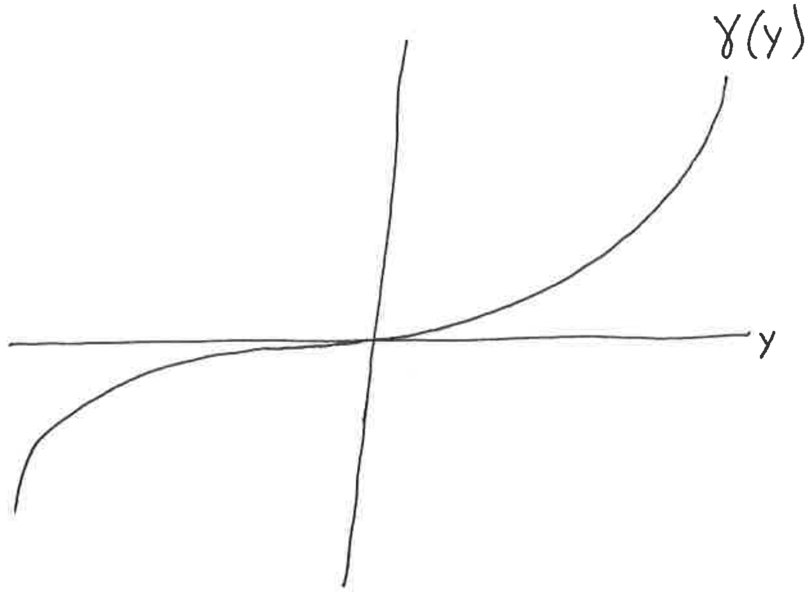
$$L = \frac{1}{2}\rho V_\infty \Gamma_0 \frac{b}{2}\pi \Rightarrow \Gamma_0 = \frac{2L}{\rho V_\infty \pi}\frac{2}{b} = \frac{4L}{\rho V_\infty b \pi}$$

Shed vorticity

$$\gamma = -\frac{d\Gamma}{dy} = \frac{4\frac{y}{b}\Gamma_0}{\sqrt{b^2 - 4y^2}} = \frac{4\frac{y}{b}}{\sqrt{b^2-4y^2}} \frac{4L}{\rho V_\infty b\pi}$$

Assume $V_\infty = 100$ ft/s

plot



3) Downwash

$$W(y) = \frac{-\Gamma_0}{2b} = \frac{4L}{\rho V_\infty b \pi} \frac{1}{2b}$$

$$= \frac{4 \cdot 3000 \text{ lbf} \cdot ft^3 \cdot s \cdot \text{slug ft}}{0.00237 \text{ slug} \cdot 100 \text{ ft} \cdot 40 \text{ ft} \cdot \pi \cdot 2 \cdot 40 \text{ ft} \cdot \text{lbf } s^2} \; s$$

$$\boxed{W = 5 \, ft/s}$$

4) C_{D_i} for elliptical AR = 10 wing

$$C_{D_i} = \frac{C_L^2}{\pi AR} = \boxed{\frac{C_L^2}{10\pi} = C_{D_i}}$$

5) C_{D_i} in Ground Effect

$$C_{D_i} = \frac{C_L^2}{\pi AR} = k C_L^2 \Rightarrow k = \frac{1}{\pi AR}$$

$$C_{D_{i\,ge}} = \frac{K_{eff}}{K} K C_L^2 = \frac{K_{eff}}{K} \frac{C_L^2}{\pi AR} = \frac{33\left(\frac{h}{b}\right)^{1.5}}{1+33\left(\frac{h}{b}\right)^{1.5}} \frac{C_L^2}{\pi AR}$$