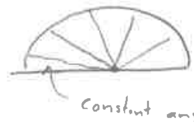
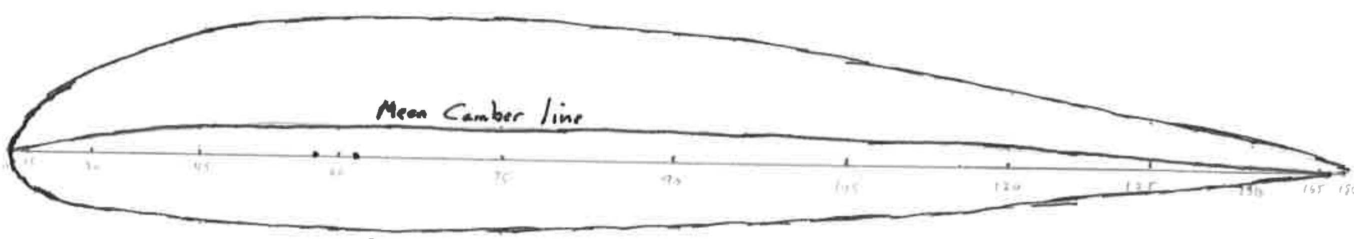
1. Plot an NACA 2416 airfoil exactly to scale with a chord of 7 in. Show mean camber line, the location of maximum thickness and camber.

I chose to use cosine spacing $x = \frac{1+\cos(\theta)}{2}$ Visually 

Attached Excel document contains calculations

Constant angles give better LE and TE spacing! You are not expected to know or use this!



Max camber 40%

Max thickness 30%

angle	x/c	X*7	CamberFor	CamberAft	Aftv2	Camber	Camber*7		Thickness	T/2	T/2*7
			0.02	0.4	0.16						
0	0	0	0	0.011111	0.011111	0	0		0	0	0
15	0.017037	0.11926	0.001667	0.011852	0.011852	0.001667	0.011672		0.058409	0.029205	0.204433
30	0.066987	0.468911	0.006138	0.013839	0.013839	0.013839	0.096873		0.107054	0.053527	0.374688
45	0.146447	1.025126	0.011964	0.016428	0.016428	0.016428	0.114999		0.141555	0.070778	0.495443
60	0.25	1.75	0.017188	0.01875	0.01875	0.01875	0.13125		0.158433	0.079217	0.554516
75	0.37059	2.594133	0.019892	0.019952	0.019952	0.019952	0.139664		0.157303	0.078651	0.55056
90	0.5	3.5	0.01875	0.019444	0.019444	0.019444	0.136111		0.141174	0.070587	0.494109
105	0.62941	4.405867	0.013421	0.017076	0.017076	0.017076	0.119533		0.115059	0.057529	0.402706
120	0.75	5.25	0.004688	0.013194	0.013194	0.013194	0.092361		0.084275	0.042137	0.294962
135	0.853553	5.974874	-0.00571	0.008572	0.008572	0.008572	0.060001		0.053619	0.02681	0.187668
150	0.933013	6.531089	-0.01551	0.004217	0.004217	0.004217	0.029516		0.02743	0.013715	0.096006
165	0.982963	6.88074	-0.02248	0.00112	0.00112	0.00112	0.007838		0.009668	0.004834	0.033839
180	1	7	-0.025	0	0	0	0		0.00336	0.00168	0.01176

2. ASHRAE Psychrometric Chart (attached)

① Temp 90°F

② RH 90%

③ Intersection of ① and ②

④ Track ③ horizontally to find Humidity Mass ratio ④ of $0.028 \frac{lb}{lb}$

⑤ Interpolate Volume curves 14.5 and 14.4 to give $\approx 14.47 \frac{ft^3}{lb}$

Dry Air:

$$\rho = \frac{1}{\text{specific volume}} = \frac{1\ lb}{14.47\ ft^3} \cdot \frac{slug}{32.174\ lb} = 0.002148 \frac{slug}{ft^3}$$ ← Dry Air ONLY

Humidity Ratio:

Mass Ratio $= 0.028 \frac{lb}{lb}$ thus total mass is $(1 + 0.028) \cdot Mass_{dry\ air}$

$$\rho_{wet\ air} = (1 + 0.028)\ 0.002148 \frac{slug}{ft^3} = \boxed{0.00221 \frac{slug}{ft^3} = \rho}$$

This shows how the psychrometric chart is useful for determining Sea Level densities. It is amazing that for every 100 lb of air, there is 2.7 lb of water.

$$\left(= 100 - \frac{100}{1.028}\right)$$

problem Set #1, problem #2

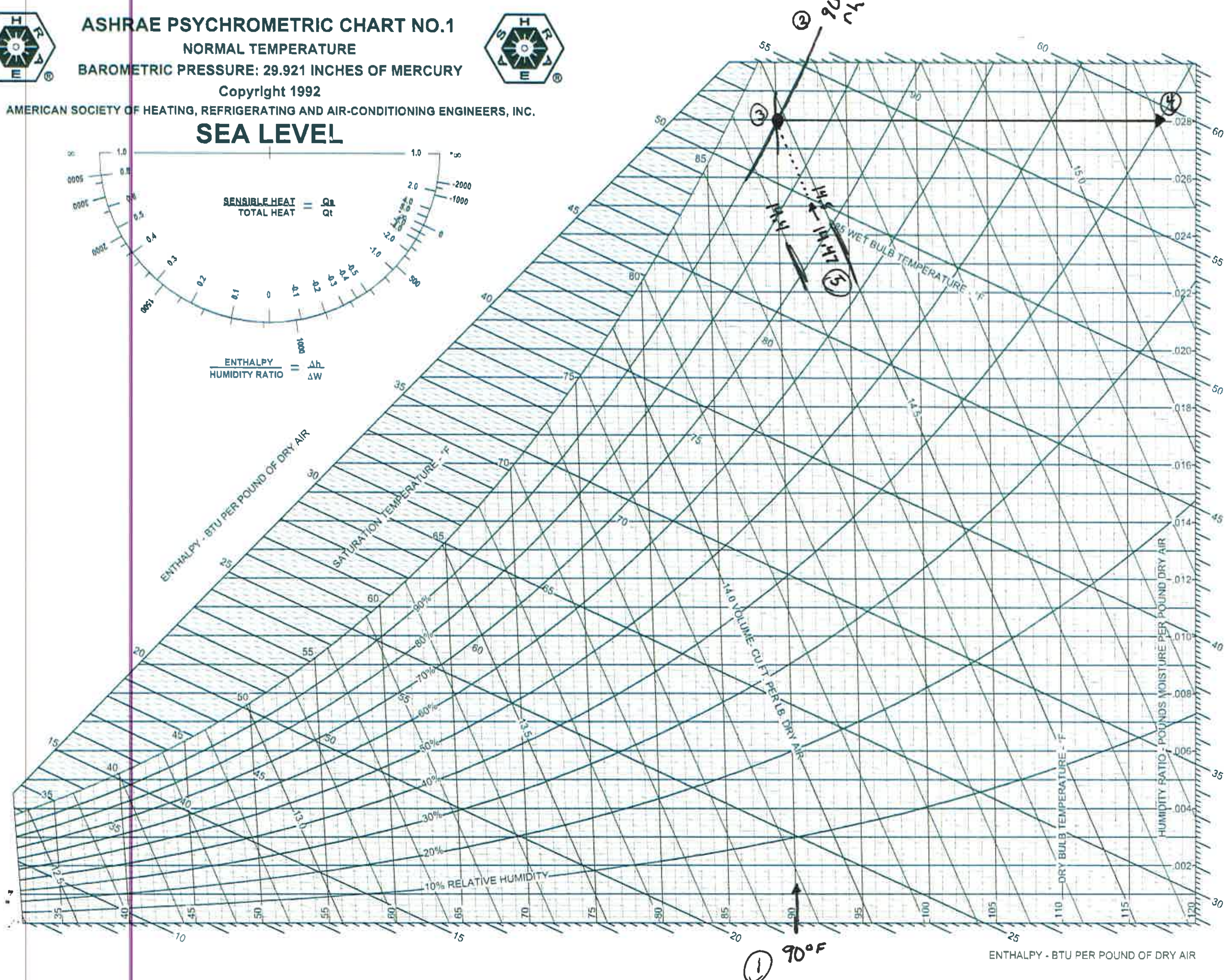
ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

BAROMETRIC PRESSURE: 29.921 INCHES OF MERCURY

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.

SEA LEVEL



1.18.

a) Compute the Reynolds number based on chord for a DC 3 at 200 mph. ($c_r = 14.25$ ft)

$$Re \equiv \frac{\rho V L}{\mu}$$

$$\rho_{SSL} = 0.00237 \ \frac{\text{slug}}{\text{ft}^3}$$

$$V = 200 \text{ mph}$$

$$L = 14.25 \text{ ft}$$

$$\mu = 3.7373 \times 10^{-7} \ \frac{\text{slug}}{\text{ft s}} \quad (\text{page } 72)$$

$$Re = \frac{0.00237 \ \cancel{\text{slug}}}{\cancel{\text{ft}^3}} \left| \frac{200 \ \cancel{\text{mile}}}{\cancel{\text{hr}}} \right| \frac{14.25 \ \cancel{\text{ft}}}{} \left| \frac{\cancel{\text{ft}} \ \cancel{\text{s}}}{3.7373 \times 10^{-7} \ \cancel{\text{slug}}} \right| \frac{5280 \ \cancel{\text{ft}}}{1 \ \cancel{\text{mile}}} \left| \frac{\cancel{\text{hr}}}{3600 \ \cancel{\text{s}}} \right.$$

$$\boxed{Re_c = 26 \times 10^6}$$

b) Compute Re for an F-22 at $1320 \frac{\text{ft}}{\text{s}}$ with $c_r = 21.5$ ft

$$Re_c = \frac{0.00237 \ \cancel{\text{slug}}}{\cancel{\text{ft}^3}} \left| \frac{1320 \ \cancel{\text{ft}}}{\cancel{\text{s}}} \right| \frac{21.5 \ \cancel{\text{ft}}}{} \left| \frac{\cancel{\text{ft}} \ \cancel{\text{s}}}{3.7373 \times 10^{-7} \ \cancel{\text{slug}}} \right. = \boxed{180 \times 10^6 = Re_c}$$

Quick rule of thumb:

$$Re \approx 6350 \cdot V\left[\tfrac{\text{ft}}{\text{s}}\right] \cdot L\,[\text{ft}]$$

$$\boxed{Re = 180 \times 10^6}$$

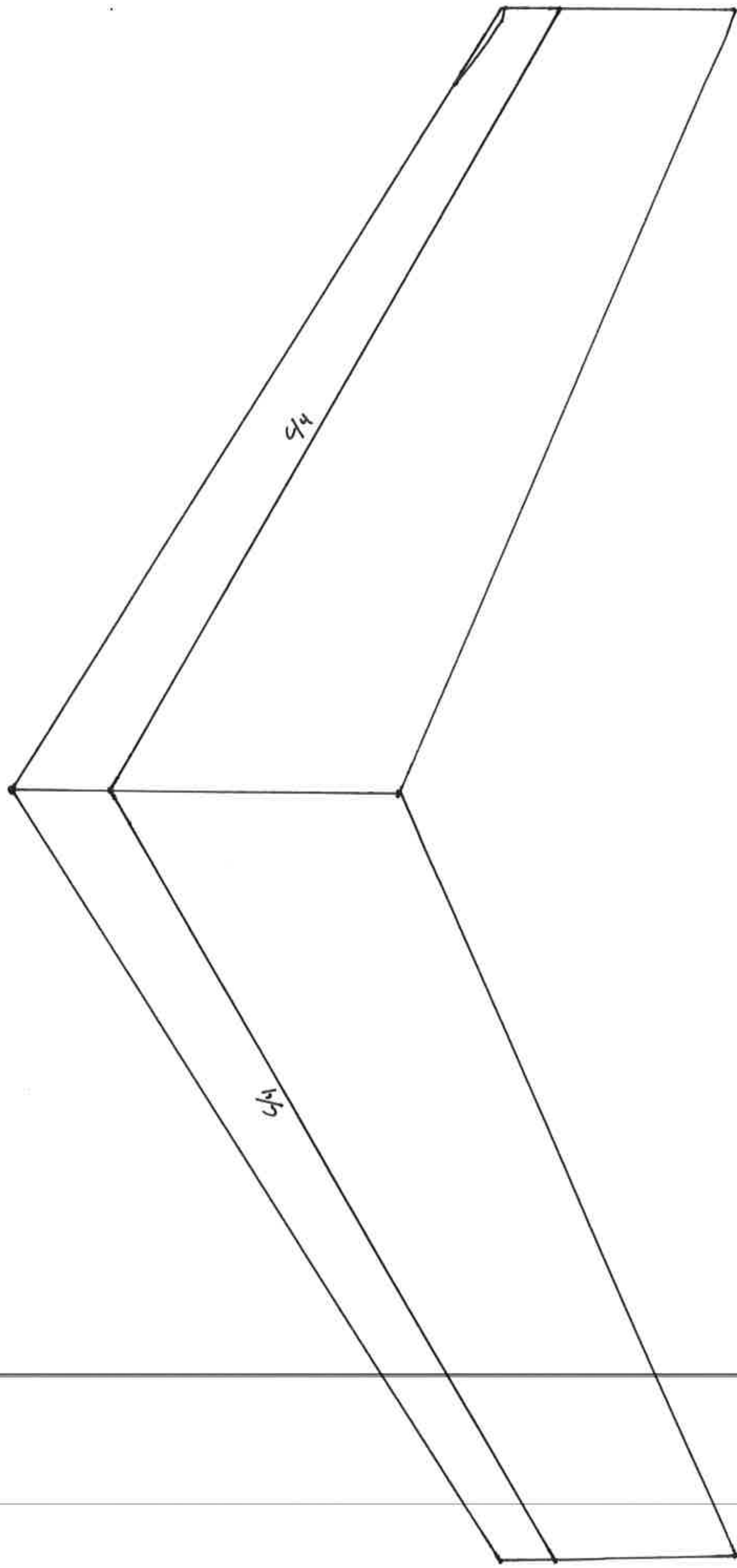
4. Draw a wing

Computations:

1) Root chord is 2.5^{in}, so $c/4$ is 0.625^{in}

2) $c/4$ sweep is $30°$. $\tan(30°) = 0.57735$, sweep back is $b/2 \cdot 0.57735 = 2.886^{in}$

3) Tip chord is 1.5^{in}, so $c/4$ is 0.375^{in} $\frac{3}{4}c_t$ is 1.125

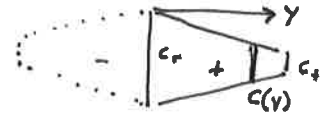


5. $b = 10$ in, $\Lambda_{c/4} = 30°$, $AR = 5$, $S = 20$ in², linear taper, $C_r = 2.5$

a) Average Chord

Since the wing is linearly tapered, write a function for chord

$$C(y) = Ay + B \quad \text{with} \quad C(y=0) = C_r \ \text{root chord}, \quad C(y=b/2) = C_t \ \text{tip chord}$$



$$C^+(y) = C_r + \left(\frac{C_t - C_r}{b/2}\right) y^+ \quad \text{and} \quad C^-(y) = C_r - \left(\frac{C_t - C_r}{b/2}\right) y^-$$

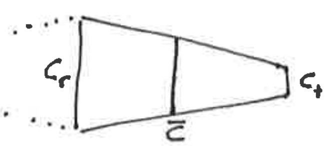
Average chord

$$\bar{c} = \frac{1}{b}\int_{-b/2}^{b/2} C^{+-}(y)\,dy = \frac{2}{b}\int_0^{b/2} C^+(y)\,dy = \frac{2}{b}\int_0^{b/2}\left(C_r + \left(\frac{C_t - C_r}{b/2}\right)y\right)dy$$

$$= \frac{2}{b}\left(C_r y + \left(\frac{C_t - C_r}{b/2}\right)\frac{y^2}{2}\right)\Bigg|_0^{b/2} = \frac{2}{b}\left(C_r \frac{b}{2} + \left(\frac{C_t - C_r}{b/2}\right)\frac{b^2}{4 \cdot 2}\right)$$

$$= C_r + \frac{C_t - C_r}{2} = \frac{C_t + C_r}{2} = \text{Average of } C_r \text{ and } C_t\,!$$

Visually



$$\boxed{\bar{c} = \frac{C_t + C_r}{2}}$$

$\bar{c} = ?$

b) Root chord

Adjusted problem statement to $C_r = 2.5$ in

c) tip chord

$$\text{Area} = S = b \cdot \bar{c} = 10\text{ in} \cdot \frac{C_t + C_r}{2} = 20 \Rightarrow 20 = 10 \cdot \frac{2.5 + C_t}{2}$$

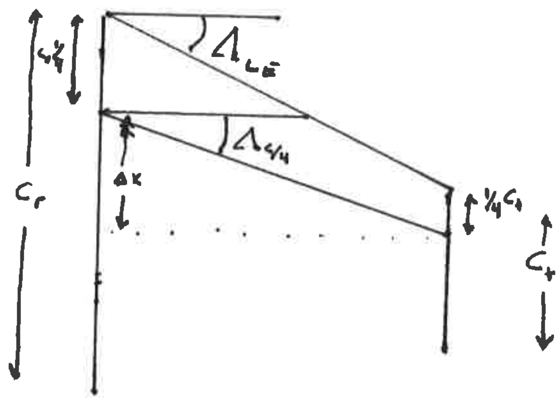
$$\boxed{C_t = \frac{2 \cdot S}{b} - C_r} \Rightarrow \boxed{C_t = 1.5\text{ in}}$$

d) Taper ratio

$$\lambda = \frac{C_t}{C_r} = \frac{1.5}{2.5} = \boxed{0.6 = \lambda}$$

e) LE sweep angle

A bit of geometry



- Sweep moves the tip $\frac{1}{4}c$ back by $\frac{b}{2}\tan(\Lambda_{c/4}) = \Delta x_s$
- Taper moves the LE back by $\Delta x_t = \frac{1}{4}c_r - \frac{1}{4}c_t = \frac{1}{4}(c_r - c_t)$

Add together

$$\Delta x = \frac{1}{4}(c_r - c_t) + \frac{b}{2}\tan(\Lambda_{c/4})$$

$$= \frac{1}{4}(c_r - \lambda c_r) + \frac{b}{2}\tan(\Lambda_{c/4})$$

$$= \frac{c_r}{4}(1-\lambda) + \frac{b}{2}\tan(\Lambda_{c/4})$$

$$\Lambda_{LE} = \operatorname{asin}\left(\frac{2\Delta x}{b}\right) = \boxed{\operatorname{atan}\left(\frac{c_r}{2b}(1-\lambda) + \tan(\Lambda_{c/4})\right) = \Lambda_{LE}}$$

$$\Lambda_{LE} \approx 32^\circ$$

f) Trailing Edge Angle

Similar to e) except $\Delta X_t = \frac{3}{4}C_t - \frac{3}{4}C_r$

$$\boxed{\Lambda_{TE} = \operatorname{atan}\left(\frac{3}{4}(\lambda-1) + \tan(\Lambda_{c/4})\right)}$$

g) MAC

$$MAC \equiv \text{Mean Aerodynamic Chord} \equiv \frac{1}{S}\int_{-b/2}^{b/2} c(y)^2\,dy$$

$$= \frac{2}{S}\int_0^{b/2}\left(C_r + \left(\frac{C_t - C_r}{b/2}\right)y\right)^2 dy$$

$$= \frac{2}{S}\int_0^{b/2}\left(C_r^2 + 2C_r\frac{(C_t - C_r)}{b/2}y + \left(\frac{C_t - C_r}{b/2}\right)^2 y^2\right) dy$$

$$= \frac{2}{S}\int_0^{b/2} \cdots = \frac{2}{S}\left(C_r^2 y + 2C_r\left(\frac{C_t - C_r}{b/2}\right)\frac{y^2}{2} + \left(\frac{C_t - C_r}{b/2}\right)^2\frac{y^3}{3}\right)\Bigg|_0^{b/2}$$

$$= \frac{2}{S}\left(C_r^2\frac{b}{2} + C_r\frac{(C_t - C_r)}{b/2}\frac{b^2}{4} + \frac{1}{3}\left(\frac{C_t - C_r}{b/2}\right)^2\frac{b\,b\,b}{2\cdot 2\cdot 2}\right)$$

$$= \frac{C_r^2 b}{S} + \frac{C_r b(C_t - C_r)}{S} + \frac{1}{3}(C_t - C_r)^2\frac{b}{S}$$

$$= \frac{C_r^2 b}{S} + \frac{C_r b}{S}(\lambda-1)C_r + \frac{1}{3}(\lambda-1)^2\frac{C_r b}{S}$$

$$\boxed{MAC = \frac{C_r b}{S}\left(C_r + (\lambda-1)C_r + \frac{1}{3}(\lambda-1)^2 C_r\right)}$$

$$\boxed{MAC \approx 2.05\text{ in}}$$

h) port LE tip location

From e)

$$\Delta X = \frac{c_r}{4}(1-\lambda) + \frac{b}{2}\tan(\Lambda_{c/4})$$

$$= \frac{2.5^{in}}{4}(1-0.6) + \frac{10}{2}\tan(30^\circ)$$

$FS = 3.16^{in}$

$BL = -5^{in}$

$WL = 0$

$$\boxed{FS3.16 \ BL{-5} \ WL0}$$

6. Determine α and β for a wind tunnel model rotated by $\phi = 90^\circ$, $\theta = 20^\circ$, $\psi = -45^\circ$

Freestream in tunnel $\vec{V} = \begin{pmatrix} V_\infty \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} V_\infty$

Transform from global to local is ordered as ψ, θ, ϕ ✓

$$\underbrace{\begin{pmatrix} U \\ V \\ W \end{pmatrix}}_{\text{local body frame}} = T^{gl}\vec{V} = {T^{lg}}^{-1}\vec{V} = {T^{lg}}^{T}\vec{V} = {T^{lg}}^{T}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} V_\infty$$

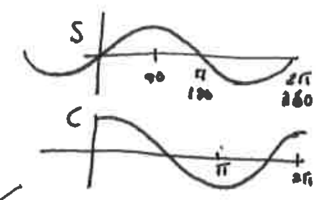
Notice that $C_\theta = \cos(\theta)$, just shorthand notation for busy students and professors! :)

$$= \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} V_\infty$$

Notice that only the upper row of T^{gl} is needed since $\vec{V}$ only has one non-zero term!!

Divide by V_∞

$$\begin{pmatrix} U/V_\infty \\ V/V_\infty \\ W/V_\infty \end{pmatrix} = \begin{pmatrix} C_\theta C_\psi \\ S_\phi S_\theta C_\psi - C_\phi S_\psi \\ C_\phi S_\theta C_\psi + S_\phi S_\psi \end{pmatrix} = \begin{matrix} C(20^\circ)C(-45^\circ) \\ \cancel{S(90^\circ)}^{1}\, S(20^\circ)\, C(-45^\circ) - \cancel{C(90^\circ)}^{0}\, S(-45^\circ) \\ \cancel{C(90^\circ)}^{0}\, S(20^\circ)\, C(-45^\circ) + \cancel{S(90^\circ)}^{1}\, S(-45^\circ) \end{matrix}$$



$$\begin{pmatrix} U' \\ V' \\ W' \end{pmatrix} = \begin{pmatrix} 0.664 \\ 0.241 \\ -0.707 \end{pmatrix} \quad \text{and} \quad U'^2 + V'^2 + W'^2 = 1 \checkmark$$

MUST be 1 or the transform is wrong!

Angles

$$\alpha \equiv \operatorname{atan}\left(\frac{W}{U}\right) = \operatorname{atan}\left(\frac{-0.707}{0.664}\right) = -46.8^\circ$$

$$\beta \equiv \operatorname{asin}\left(\frac{V}{\vec{V}_\infty}\right) = \operatorname{asin}\left(\frac{0.241}{1}\right) = 13.9^\circ$$

$$\boxed{\begin{matrix} \alpha = -46.8^\circ \\ \beta = 13.9^\circ \end{matrix}}$$

Visually verify from photo ✓