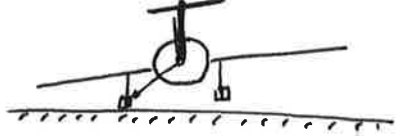
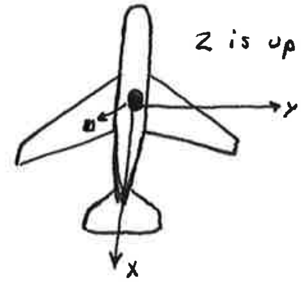
HW#2



$W = 500000\ lbf$

$V_v = 1000\ fpm$

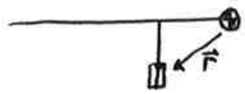


1

① Aircraft Mass

$$m = \frac{F}{g} = \frac{500000\ lbf \cdot s^2}{32.174\ ft} = 15540\ slug$$

② Vector from CG to port landing gear



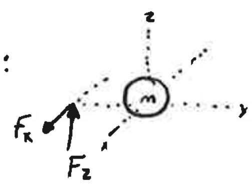
$$\vec{F} = (10^{ft}, -20^{ft}, -10^{ft})$$

③ Determine initial/impact g-load/acceleration

Assuming a constant acceration over the impact (lesson 2, p9): $S = \frac{1}{2}\frac{V^2}{a}$

Solve for "a": $a = \frac{1}{2}\frac{V^2}{S} = \frac{1}{2} \cdot \frac{1000^2\ ft^2}{min^2} \cdot \frac{1}{2\ ft} \cdot \frac{min^2}{60^2 s^2} = 69.4\ ft/s^2$

FBD:



$$\Sigma F_z = F_z = ma = 15540\ slug \cdot \frac{69.4\ ft}{s^2} \cdot \frac{lbf\ s^2}{slug\ ft}$$

$$= 1.08 \times 10^6\ lbf$$

④ Wheel scrub/rolling force

$\mu_r = 0.1$ Thus $F_x = \mu_r F_z = (0.1)(1.08 \times 10^6\ lbf) = 1.08 \times 10^5\ lbf$

⑤ Calculate accelerations (translational)

$$a_x = \frac{F_x}{m} = \frac{1.08 \times 10^5\ lbf}{15540\ slug} \cdot \frac{slug\ ft}{lbf\ s^2} = \boxed{6.94\ \frac{ft}{s^2} = a_x}$$

Short cut: $a_x = \mu_r a_z$

$\boxed{a_y = 0}$ $\boxed{a_z = 69.4\ \frac{ft}{s^2}}$

⑥ Calculate moment about CG

$$M = r \times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 10 & -20 & -10 \\ \mu F_z & 0 & F_z \end{vmatrix} = (-20F_z)\hat{\imath} - \underbrace{(10F_z + 10\mu F_z)}_{11 F_z}\hat{\jmath} + (20\mu F_z)\hat{k}$$

$$M = -21.6 \times 10^6\ ft\text{-}lbf\ \hat{\imath} - 11.88 \times 10^6\ ft\text{-}lbf\ \hat{\jmath} + 21.6 \times 10^5\ ft\text{-}lbf\ \hat{k}$$

⑦ Calculate angular accelerations

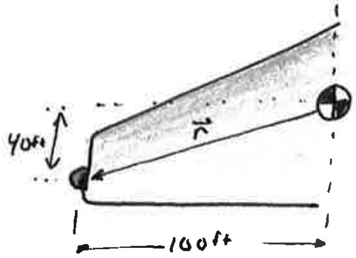
$$I\ddot{\theta} = M$$

$$\ddot{\theta}_{xx} = \frac{M_x}{I_{xx}} = \frac{-21.6 \times 10^6\ ft\text{-}lbf}{14.0 \times 10^6\ slug\ ft^2} \cdot \frac{slug\ ft}{lbf\ s^2} = \boxed{-1.5\ \frac{rad}{s^2} = \ddot{\theta}_x}$$ "right roll"

$\theta_y = \frac{M_y}{I_{yy}} = \frac{-11.88 \times 10^6}{32 \times 10^6} = \boxed{-0.37\ \frac{rad}{s^2} = \ddot{\theta}_y}$ "nose down"

$\theta_z = \frac{M_z}{I_{zz}} = \frac{2.16 \times 10^6}{45 \times 10^6} = \boxed{0.048\ \frac{rad}{s^2} = \ddot{\theta}_z}$ "yaw left"

2



$\vec{r} = (-40, -100, 0)$

① Mass of light

$$m = \frac{W}{g} = \frac{10\ \text{lbf}\ s^2}{32.174\ \text{ft}} = 0.31\ \text{slug}$$

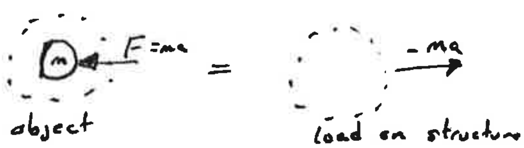
② Acceleration at light (use vector form of acceleration in rotating frame ... & see dynamics book)

$$a = a_{xyz} + \ddot{\theta} \times \vec{r} + \cancel{\dot{\theta}}^{0} \times (\cancel{\dot{\theta}}^{0} \times \vec{r}) + 2\omega \times \cancel{V_{relative}}^{0}$$

$$= a_{xyz} + \ddot{\theta} \times r$$

$$= \begin{pmatrix} 6.94 \\ 0 \\ 69.4 \end{pmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1.5 & -0.37 & 0.048 \\ -40 & -100 & 0 \end{vmatrix} = \begin{pmatrix} 11.74 \\ -1.92 \\ 204.6 \end{pmatrix} \frac{ft}{s^2}$$

③ FBD



④ Loads

$$F = ma = -0.31 \cdot \begin{pmatrix} 11.74 \\ -1.92 \\ 204.6 \end{pmatrix} = \boxed{\begin{pmatrix} -3.64 \\ +0.60 \\ -63.4 \end{pmatrix} \text{lb} = \begin{matrix} F_x \\ F_y \\ F_z \end{matrix}}$$

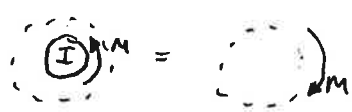
fore (light wants to continue forward)
inboard (slight aft mounting)
down (impact with gnd)

⑤ Rotational?

Worst case is the radius of gyration = size of light ≈ 12in ≈ 1ft

$$I \leq I_{est} = R_g^2 \cdot m = 10\ ft^2 \cdot 0.31\ \text{slug}$$

FBD



$$M = -I\ddot{\theta}$$

$$M_x = -10\ ft^2 \cdot \underset{0.31}{\text{slug}} \cdot -1.5\ \frac{rad}{s^2} = \boxed{4.65\ \text{ft lbf}}$$

$$M_y = -(-0.37) \cdot (10 \cdot 0.31) = \boxed{1.15\ \text{ft lbf}}$$

$$M_z = -(0.048)(10 \cdot 0.31) = \boxed{-0.148\ \text{ft lbf}}$$