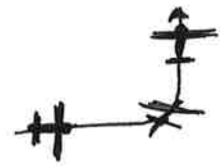
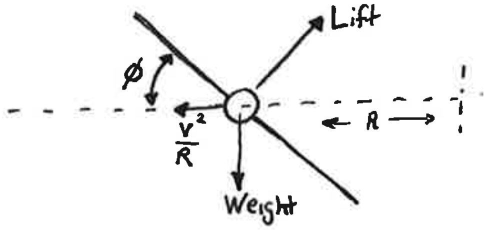
Loads Part 2

Steady state Banked Maneuvers

Aircraft change course with banked turns.





Summation of forces in ~~vertical~~ horizontal direction

$$\Sigma F_y = -\frac{V^2}{R} + Lift \cdot \sin\phi = 0$$

Summation in vertical direction

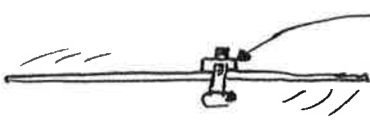
$$\Sigma F_z = -Weight + Lift \cdot \cos(\phi) = 0$$

Solve for g-load $\left(n \equiv \frac{Lift}{Weight}\right)$ from vertical direction.

$$-W + L\cos\phi = 0 \Rightarrow \cos\phi = \frac{W}{L} \Rightarrow n = \frac{L}{W} = \frac{1}{\cos\phi}$$

The g-load of a steady state banked turn depends on ϕ the bank angle

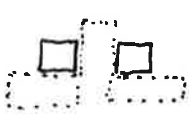
Ex: A 9000^{lb} UH-1 "Huey" helicopter is in a sustained 60° banked turn. Determine the load on the main rotor retaining nut.

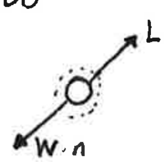


① Bank Angle to g-load

$$\phi = 60° \Rightarrow n = \frac{1}{\cos 60°} = \frac{1}{0.5} = 2.0$$

② Load with FBD





$$n = \frac{L}{W} \Rightarrow L = nW = 2.0 \cdot 9000^{lbf}$$

$$= 18000^{lbf}$$

$$\boxed{Load = 18000^{lbf}}$$

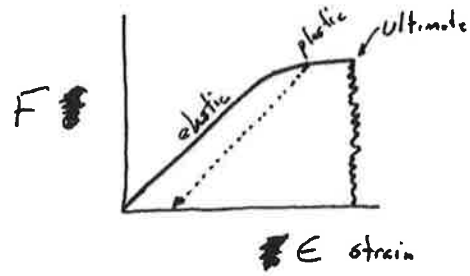
V-n diagram "Flight Envelope"

Where aerodynamics and structures interact.

From structures, you design an aircraft to meet a particular load factor

e.g. $n^{+} = 5$ $n^{-} = 3$ at a particular weight

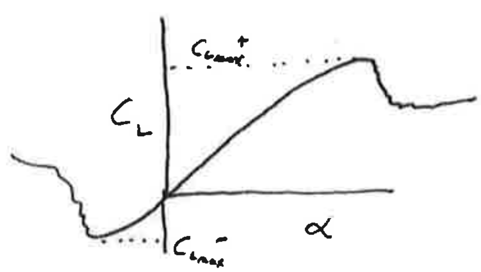
you design a buffer region between the load limit (plastic) and the ultimate load (breaks)



From aerodynamics:

$$L = \tfrac{1}{2}\rho V^2 C_L S \quad \Rightarrow \quad n = \frac{L}{W} = \tfrac{1}{2}\rho V^2 C_L \left(\frac{W}{S}\right)^{-1}$$

And a maximum C_L is determined by aero



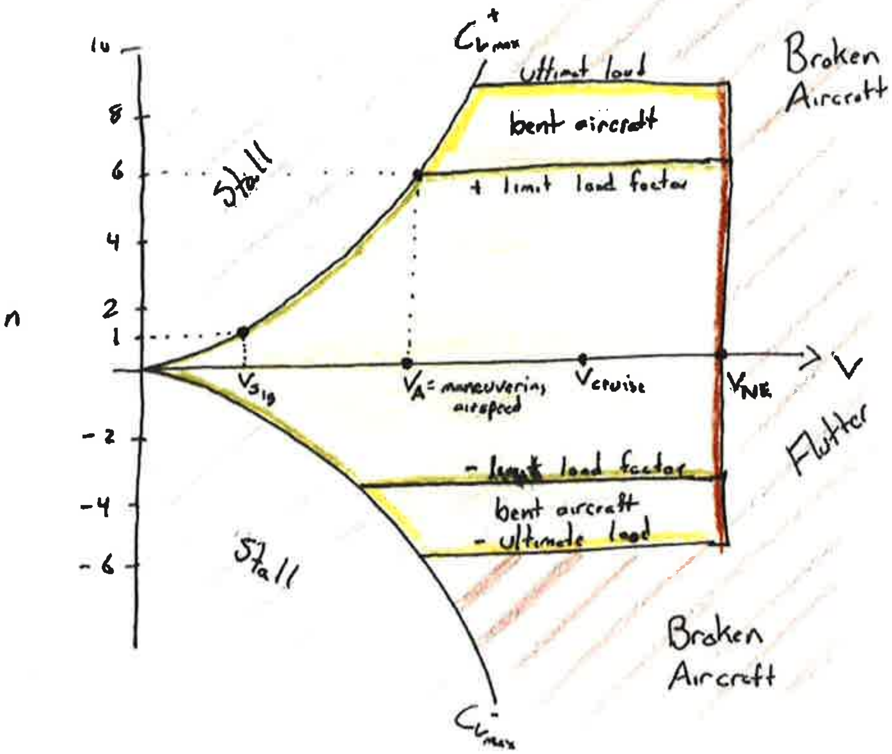
$$\Rightarrow \quad n_{max} = \tfrac{1}{2}\rho V^2 C_{L_{max}} \left(\frac{W}{S}\right)^{-1}$$

From aerodynamics + performance + aero-structural dynamics

- The aircraft has a maximum "red-line" airspeed. V_{NE}
  Above this airspeed, parts may not remain on the aircraft!

- ~~Certain~~ All aircraft will exhibit "flutter" above a certain dynamic pressure at certain flight conditions

V-n



$V_{s_{1g}}$ is the stall speed at $n=1$

V_A = maneuvering speed, the intersection of aero and structural limits.

At V_A, the aircraft can not be broken/bent by "normal" acceleration (e_n).

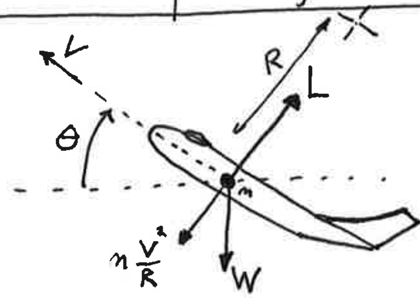
At V_A, the aircraft can be broken/bent by control deflections and non e_n accelerations.

V_C = Cruise speed (FAA certification)

V_{NE} = Never exceed.

The aircraft can not sustain operation outside of the stall $C_{L_{max}}$ curves.

Steady State Pitching Maneuvers



In the aircraft body frames' vertical direction

$$\Sigma F_z = L - \frac{V^2 m}{R} - W\cos\theta = 0$$

In x-dir

$$\Sigma F_x = -W\sin\theta = m \cdot a_x$$

In the x-direction (flight direction),

$$-W\sin\theta = m a_x \Rightarrow -\frac{W}{m}\sin\theta = a_x \Rightarrow \underline{-g\sin\theta = a_x}$$

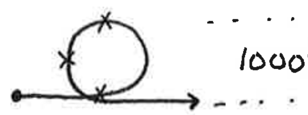
In the z direction

divide by W $\Rightarrow \frac{L}{W} - \frac{V^2 m}{RW} = \cos\theta \Rightarrow \underbrace{\frac{L}{W}}_{n_z} = \cos\theta + \frac{V^2 m}{RW}$

The load factor is

$$\boxed{n_z = \cos\theta + \frac{V^2 m}{RW}} \Rightarrow \boxed{n_z = \cos\theta + \frac{V^2}{R}\frac{1}{g_0}}$$

Ex: An aerobatic Zivko Edge 540 ($W = 1500^{lb}$) performs a perfect 1000^{ft} loop at exactly 180^{kts}. Determine the normal load factor (g-load) at the bottom, top, and sides of the loop.



① Loop radius

$D = 1000^{ft}$ so $R = 500^{ft}$

② Velocity

$$180^{kts} \left| \frac{1.68\ ft}{kt\ s} \right| = 302\ \frac{ft}{s}$$

③ Generic load vs θ

$$n_z = \cos\theta° + \frac{302^2\ ft^2}{s^2} \left| \frac{1}{500\ ft} \right| \left| \frac{s^2}{32.174\ ft} \right| =$$

$$= \cos\theta + 5.67$$

④ Bottom

$\theta = 0 \Rightarrow n_z = \cos\overset{1}{\cancel{0}} + 5.67 = \boxed{6.67\ g = n_z}$

⑤ Top

$\theta = 180°$ $\quad n_z = \cos\overset{-1}{\cancel{180}} + 5.67$

$$\boxed{n_z = +4.67_g}$$

⑥ Side

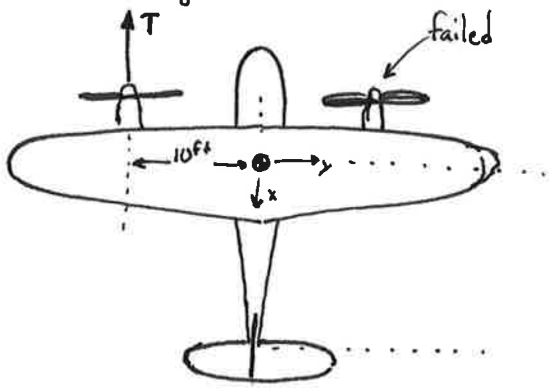
$\theta = 90°$ $\quad n_z = \cos\overset{0}{\cancel{90}} + 5.67 = \boxed{5.67_g = n_z}$

Steady State Moments

For an aircraft in steady state, the total moment about the Center of Gravity, CG, must be zero

$$\sum M_{cg} = 0$$

Ex: A twin engined aircraft has an engine that fails. Qualitatively determine the loading on the fuselage.



Thrust offset = $d = 10^{ft}$

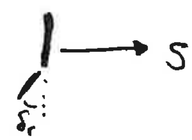
Vertical tail length = $L_v = 25\,ft$

Thrust = 1000^{lbf}

Tail height $\approx 8\,ft$

① Moment about z-axis

$\sum M_{cg_z} = -T \cdot d = 0$ (!) Not possible? Add a rudder deflection



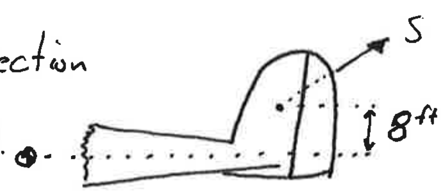
So now, we have a side force from the vertical

$$\sum M_{cg_z} = -T \cdot d + S \cdot L_v = 0 \checkmark \Rightarrow S = \frac{Td}{L_v} = \frac{1000^{lbf} \cdot 10\,ft}{25\,ft} = 400^{lbf}$$

② Moment about x-axis

The vertical is not symmetrical about the tail section

Perhaps the center of force is 8 feet above the CG.

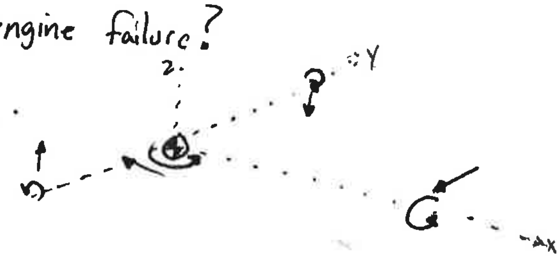


$\sum M_{cg_x} = -S \cdot h = 0$ (!) Again not possible?

Add an aileron deflection

$= -S \cdot h + M_a = 0 \checkmark$

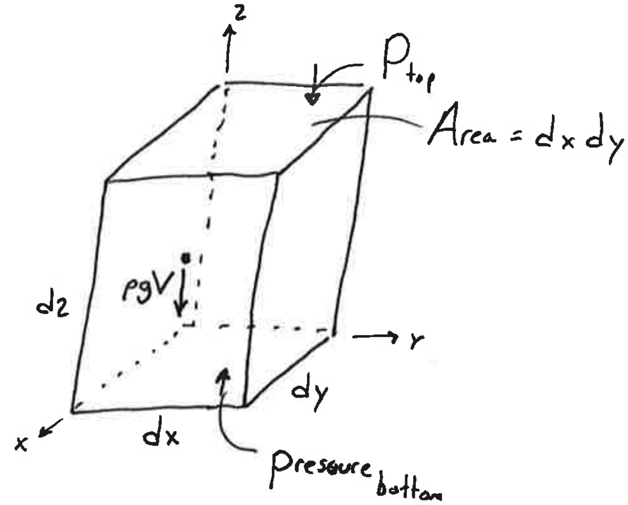
③ What are the loads associated with this engine failure?



- Thrust of engine
- Moment of T at wing.
- Sideforce from rudder deflection
- Fuselage Torque/Moment from rudder deflection
- Wing forces and Moments from aileron.
- Control System Forces from Pilot.
- Moisture load from Pilot ?!?

Columns of Fluid

Tall containers of fluid in an accelerating environment lead to static columns where the pressure increases with depth. An example is the atmosphere where the pressure is high at the surface and low in the upper atmosphere.



Summation of forces in the z-direction

$$\underbrace{P_{bottom} \cdot A}_{\text{Force on bottom}} - \underbrace{P_{top} \cdot A}_{\text{force on top}} \underbrace{- \rho g V}_{\text{gravity force on dense fluid}} = 0$$

Assume the pressure varies.

small # squared → 0

$$P_{top} = P_{bottom} + \frac{dP}{dz} dz + \frac{1}{2} \frac{d^2P}{dz^2} dz^2 + \frac{1}{6} \frac{d^3P}{dz^3} \cancel{dz^3} \to 0$$

Substitute

$$\underbrace{P_{bottom} \cdot A - P_{bottom} \cdot A}_{\to 0} - \frac{dP}{dz} dz \cdot A - \rho g V = 0$$

Substitute Area = dx · dy and Volume = dx · dy · dz

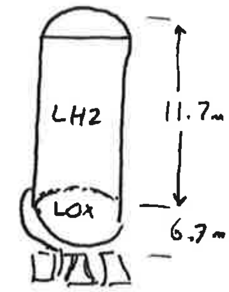
$$-\frac{dP}{dz} dz \cdot dx \cdot dy = +\rho g \, dx \cdot dy \cdot dz$$

$$\boxed{+\frac{dP}{dz} = -\rho g}$$

the slope of pressure is the negative of the fluid density and gravity

Ex:

The 2nd Stage of Apollo 17th Saturn 5 is a LOX + LH2 rocket. Interestingly, the tank uses common bulkheads for the tank contains both LOX and LH2.



Just prior to 1st stage shutdown, Apollo 17 showed 3.87 g acceleration. Compute the pressure spike in the LH2 line in the 3 seconds between S1 and S2.

① Geometry

The LH2 tank is about 11.7 m high with a line of about 6.7 m to the motor valve (approx only...)

$$H = 11.7 + 6.7 \approx 18.4 \text{ m}$$

② Static column

$$\frac{dp}{dz} = -\rho g \Rightarrow dp = -\rho g dz \Rightarrow \overbrace{P_{h_2} - P_{h_1}}^{\Delta p} = \overbrace{-\rho \overset{a}{g} (h_2 - h_1)}^{+\rho a \Delta h}$$

- Liquid hydrogen has a density of $\approx \rho \approx 70.8 \frac{gr}{L}$ ← gram
- acceleration

$$a = 3.87 g = 3.87 \cdot 9.8 \frac{m}{s^2} = 37.9 \, m/s^2$$

③ pressure Surge

$$\Delta p = +\rho a \Delta h = 70.8 \frac{gr}{L} \cdot 37.9 \frac{m}{s^2} \cdot 18.4 \, m \cdot \frac{1000 L}{m^3} \cdot \frac{kg}{1000 gr} \cdot \frac{N s^2}{kg \, m} \cdot \frac{Pa \, m^2}{N}$$

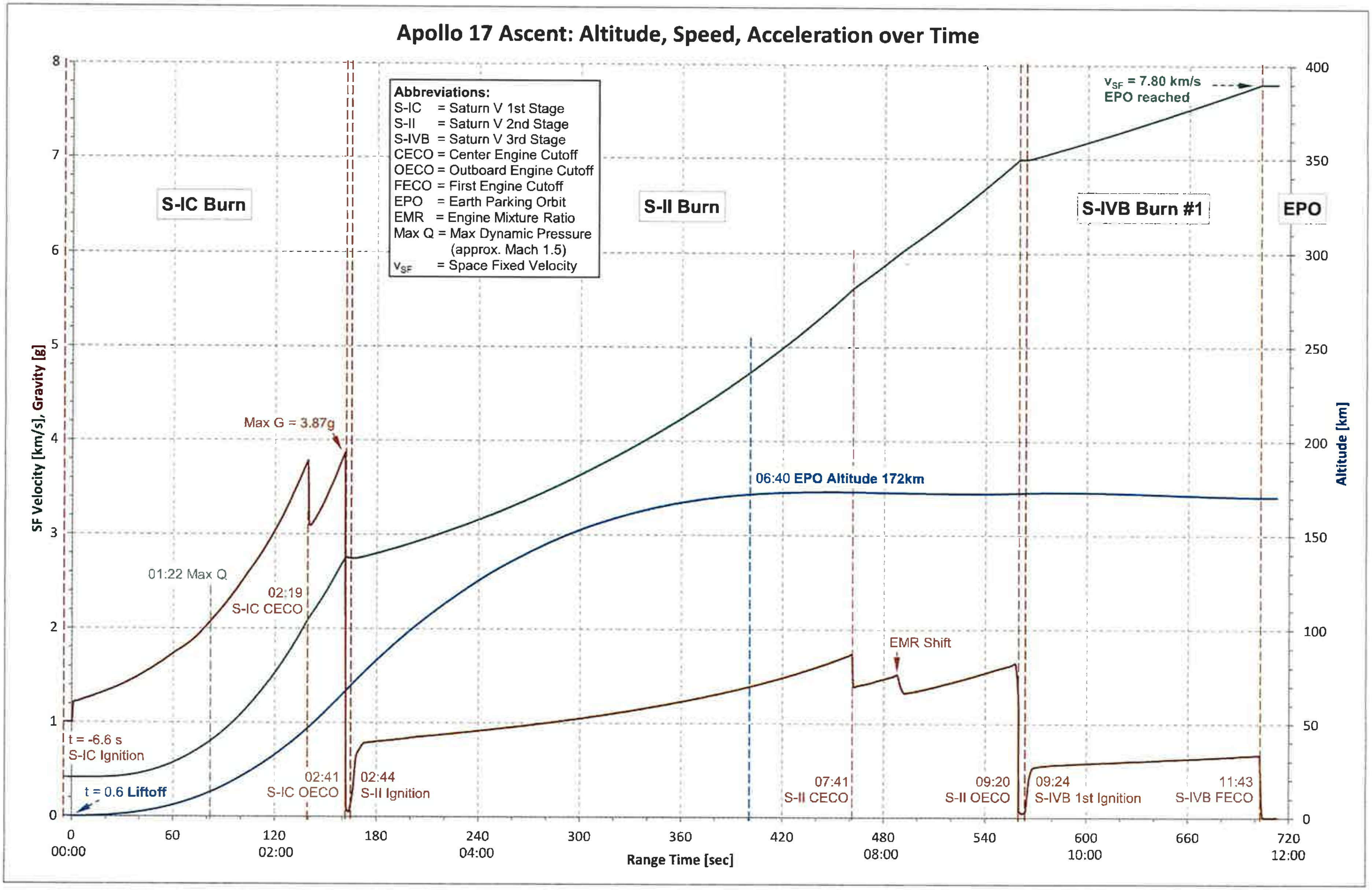
$$\boxed{\Delta p = 50 \text{ kPa}} \approx 7 \text{ psi} \approx \tfrac{1}{2} \text{ atmosphere}$$

④ What happens next?

Actually, the surge can cause the engine pump to deliver more fuel. This creates more thrust.....

The S-V 1st stage had a problem with this

- Kerosene ≈ 10x denser
- 1st stage is <u>much</u> taller



Apollo 17 Ascent: Altitude, Speed, Acceleration over Time
Abbreviations:
S-IC = Saturn V 1st Stage
S-II = Saturn V 2nd Stage
S-IVB = Saturn V 3rd Stage
CECO = Center Engine Cutoff
OECO = Outboard Engine Cutoff
FECO = First Engine Cutoff
EPO = Earth Parking Orbit
EMR = Engine Mixture Ratio
Max Q = Max Dynamic Pressure (approx. Mach 1.5)
v_{SF} = Space Fixed Velocity
S-IC Burn
S-II Burn
S-IVB Burn #1
EPO
v_{SF} = 7.80 km/s
EPO reached
Max G = 3.87g
06:40 EPO Altitude 172km
01:22 Max Q
02:19 S-IC CECO
EMR Shift
t = -6.6 s S-IC Ignition
t = 0.6 Liftoff
02:41 S-IC OECO
02:44 S-II Ignition
07:41 S-II CECO
09:20 S-II OECO
09:24 S-IVB 1st Ignition
11:43 S-IVB FECO
SF Velocity [km/s], Gravity [g]
Altitude [km]
Range Time [sec]
0 00:00
60
120 02:00
180
240 04:00
300
360
420
480 08:00
540
600 10:00
660
720 12:00