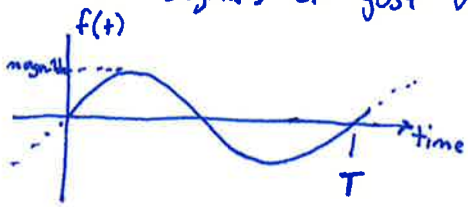
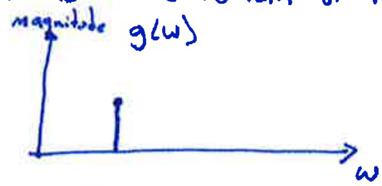
Vibration and Gust Loading

Our flight vehicles operate in an atmosphere that is not uniform or steady.

- gusts, storms, topographical separation, thermals, other aircraft, etc

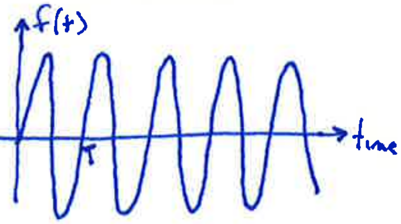
We can view signals or gust velocities in the time domain or frequency domain.





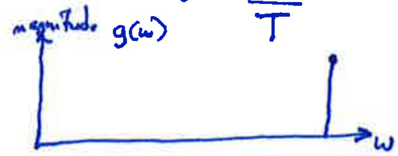
time domain

frequency domain



$\omega = \frac{2\pi}{T}$

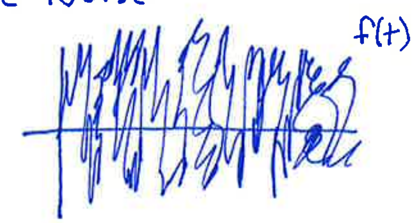
$freq\,[Hz] = \frac{1}{T}$

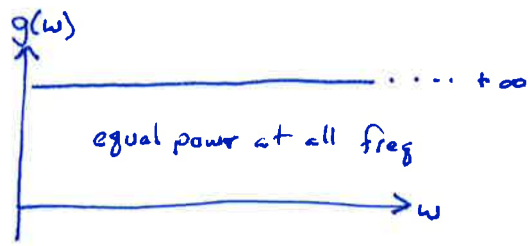


This is the Fourier transform that you studied in math classes.

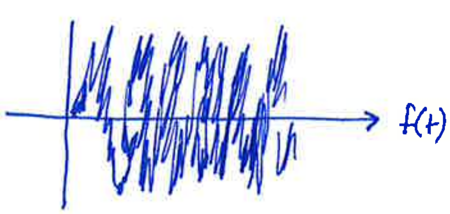
$$G(\omega) = \mathcal{F}(f(t)) = \int_{-\infty}^{\infty} f(\tau)\, e^{-i\omega\tau}\, d\omega$$

White Noise

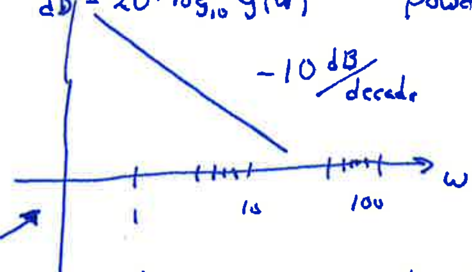




Pink Noise

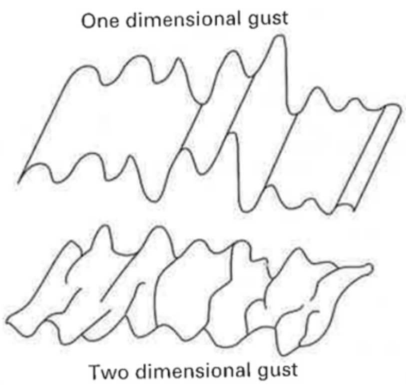


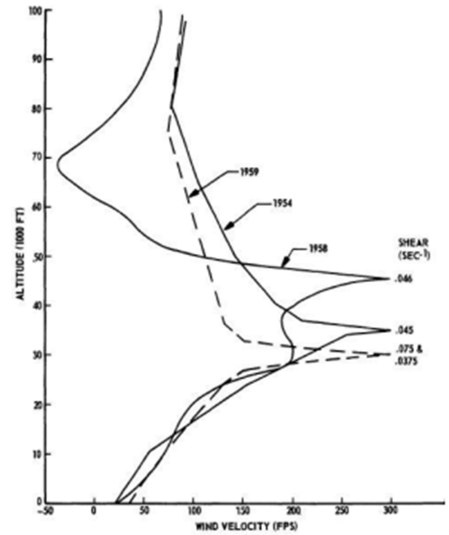
$dB = 20 \cdot \log_{10} g(\omega)$ Power Spectral Density



decreasing power with frequency

Atmospheric Turbulence Model





Sissenwine Wind Profiles For 1% Probability

von Karman model

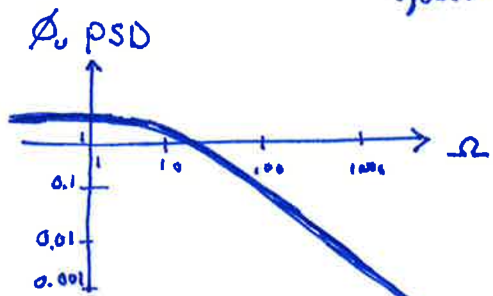
Φ_u = Power Spectral Density of "u" x-axis Velocity

$$= \sigma_u^2 \frac{2L_u}{\pi} \frac{1}{\left(1 + (1.339\, L_u \Omega)^2\right)^{5/6}}$$

Intensity squared

scale/size of turbulence.

$\Omega = \frac{\omega}{U_0}$ a scaling for velocity

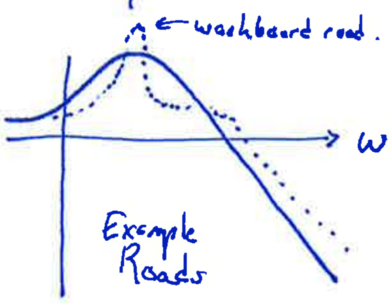


constant power at low frequencie

decreasing power at high freq

More details on vehicle response in AEM 468

This is why flying "feels" different than driving

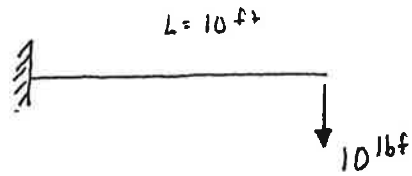


- More low freq power in flying
- Monotonic decrease wrt freq (sometimes not!)

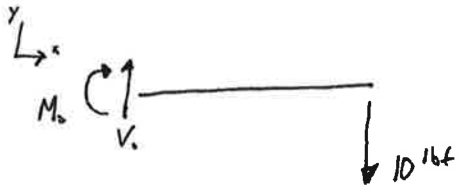
Why? Civil Engineers love moving earth

Review of Shear and Moment

Appendix A of Allen + Haisler book



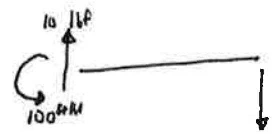
① Reaction at the wall (Free body diagram)



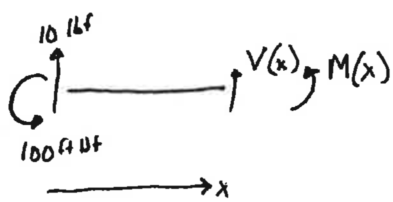
at x = 0

$$\begin{cases} \sum F_y = 0 = V_0 - 10\,\text{lbf} & \Rightarrow \quad V_0 = 10\,\text{lbf} \\ \sum M_{z_0} = 0 = -M_0 - (10\,\text{lbf})(10\,\text{ft}) & \Rightarrow \quad M_0 = -100\,\text{lbf}\cdot\text{ft} \end{cases}$$

$\Rightarrow$



② Shear and Moment along x-axis (cut at specific x point)

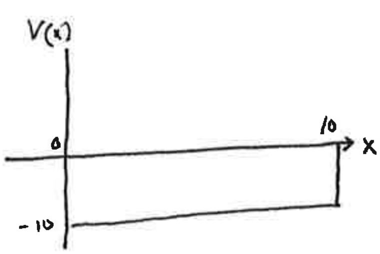


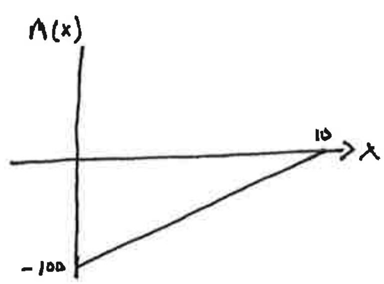
at right end

$$\begin{cases} \sum F_y = 0 = 10\,\text{lbf} + V(x) & \Rightarrow \quad V(x) = -10\,\text{lbf} \\ \sum M_y = 0 = 100\,\text{ft lbf} - (10\,\text{lbf})(x) + M(x) & \end{cases}$$

$$\Rightarrow \quad V(x) = -10\,\text{lbf}, \qquad M(x) = -100 + 10x$$

③ Sketch

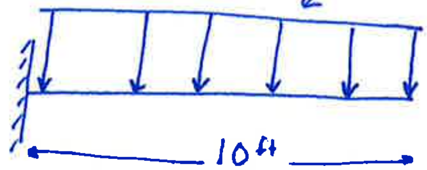




Review of Shear + Moment

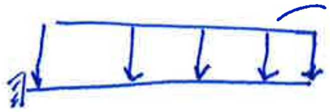
Ex: Uniform load

10000 lbf distributed over 10 ft

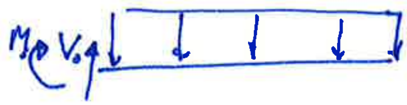


① Canonical loading

1000 lb/ft



② Wall Reaction



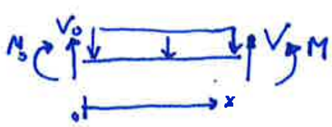
$$\Sigma F_z = V_0 - \underbrace{\int_0^{10\,ft} 1000 \tfrac{lb}{ft} \cdot dx}_{1000 \frac{lb}{ft} x \Big|_0^{10\,ft}} = 0$$

$$= V_0 - 10000\,lbf = 0 \Rightarrow \boxed{V_0 = 10000\,lbf}$$

$$\Sigma M_x = M_0 + \underbrace{\int_0^{10\,ft} 1000 \tfrac{lb}{ft} \cdot x \cdot dx}_{1000 \frac{1}{2} x^2 \Big|_0^{10\,ft}} = 0$$

$$M_0 + \frac{1000 \cdot 100}{2} = 0 \Rightarrow \boxed{M_0 = -50000\,ft\,lbf}$$

③ Shear and Moment along x-axis (cut at x)

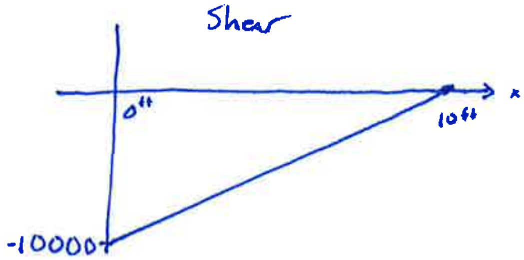


$$\Sigma F_z = V_0 - \int_0^x 1000 \cdot dx + V(x) = 0 \Rightarrow V(x) = -V_0 + 1000x$$
$$= -10000 + 1000x$$

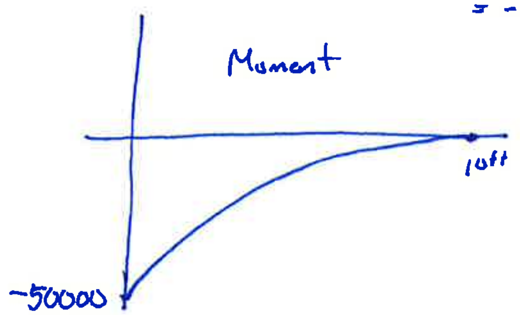
$$\Sigma M_x = M_0 + \int_0^x 1000\, x\, dx - M(x) \Rightarrow M(x) = M_0 + \frac{1000}{2}x^2$$
$$= -50000 + 500x^2$$

④ plot

Shear



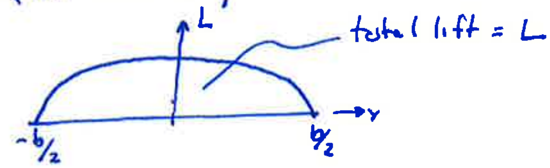
Moment



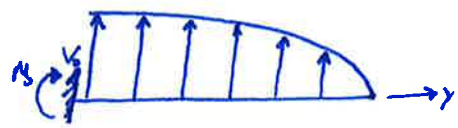
Shear and Moment of an elliptical lift distribution (cantilever wing)

The lift per unit span of an elliptical distribution is (from lesson 2)

$$L' = \frac{L}{b}\frac{4}{\pi}\sqrt{1-\left(\frac{2y}{b}\right)^2}$$



Let us look at only the positive x-axis.



① Wall reaction

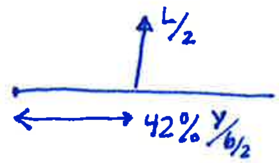
$$\Sigma F_z = 0 = V_0 + \int_0^{b/2} L'\,dy = 0 \qquad \text{but } \int_0^{b/2} L'\,dy \text{ is by inspection } \underline{\text{half}} \text{ of the total Lift}$$

$$\underline{V_0 = -L}$$

$$\Sigma M_z = 0 = M_0 - \int_0^{b/2} L'\,y\,dy = 0 = M - \underbrace{\int_0^{b/2} \frac{L}{b}\frac{4}{\pi}\sqrt{1-\left(\frac{2y}{b}\right)^2}\,y\,dy}_{}$$ Math elves....

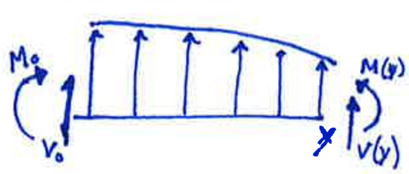
$$M = \frac{Lb}{\pi 3} = \frac{4}{3\pi}\cdot\frac{b}{2}\cdot\frac{L}{2} \approx 0.42\cdot\frac{b}{2}\cdot\frac{L}{2}$$

The moment at the root is as if the load were at 42% span



slightly inboard of 50% (uniform loading)

② Shear and moment



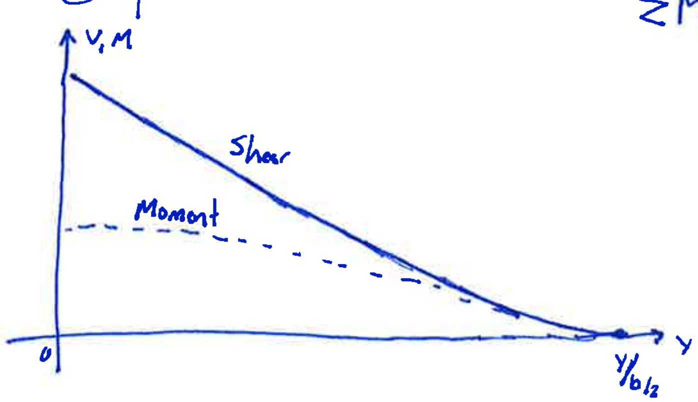
$$\Sigma F_z = +V_0 + \int_0^{y} L'\,dy + V(y) \Rightarrow V(y) = -\cancel{V_0}^{\,L} - \int_0^{y} L'\,dy$$

$$V(y) = L - \frac{1}{\pi}\,\mathrm{asin}\left(\frac{2y}{b}\right) - \frac{y}{2\pi}\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

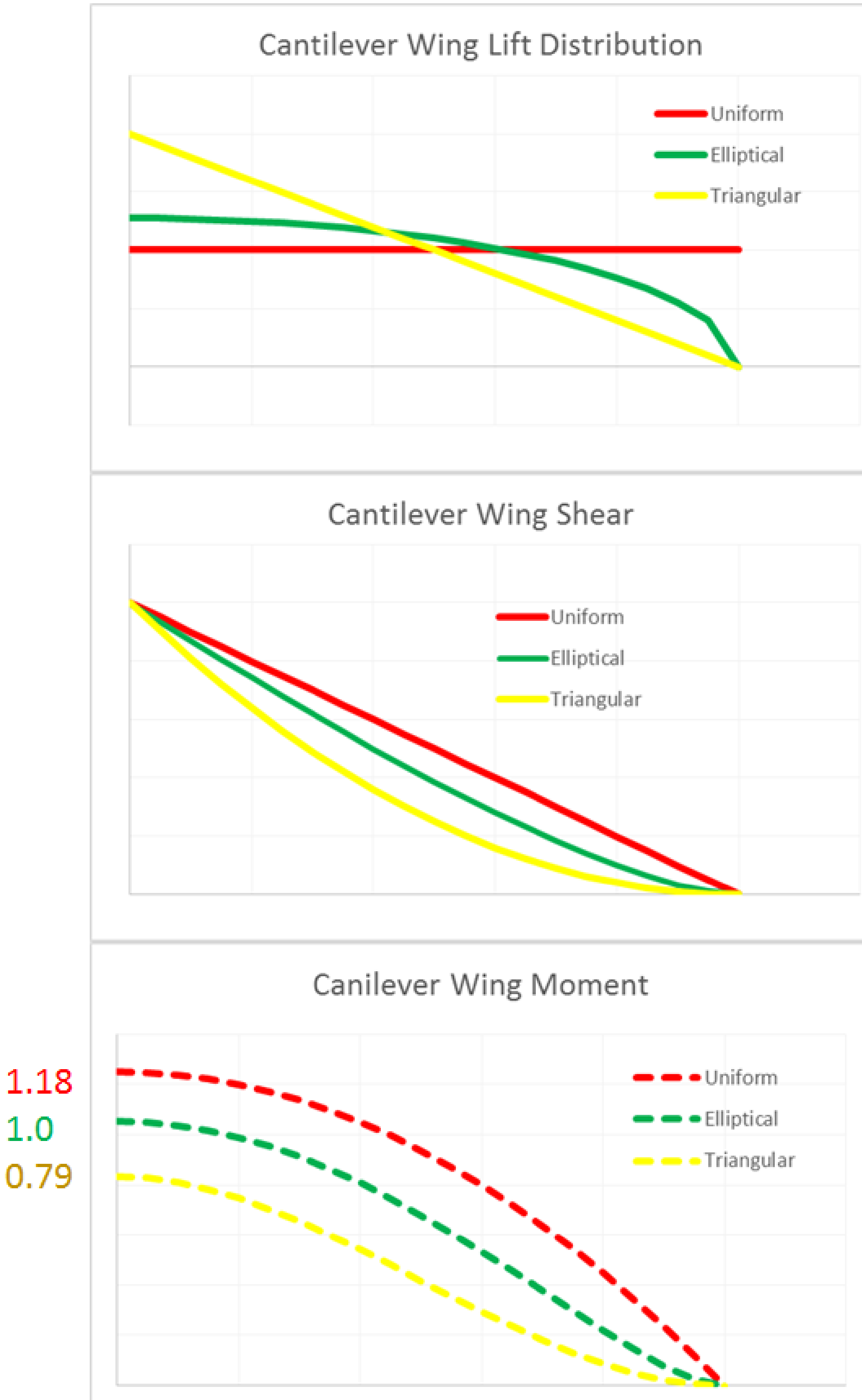
$$\Sigma M_x = M_0 - \int_0^{y} L'\,y\,dy - M(y)$$

$$M(y) = -M_0 + \frac{Lb}{3\pi} + \frac{L}{\pi b}\left(-\frac{b^2}{3}\sqrt{1-\frac{4y^2}{b^2}} + \frac{4y^2}{b}\sqrt{1-\frac{4y^2}{b^2}}\right)$$

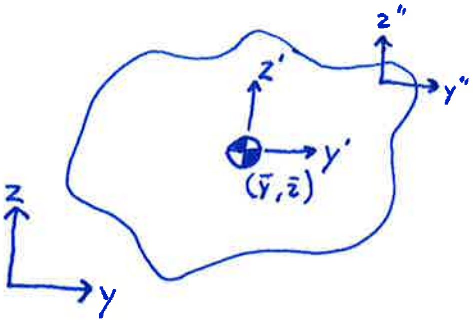
③ plot



A bit of complexity.... is it useful?


Cantilever Wing Lift Distribution
Uniform
Elliptical
Triangular
Cantilever Wing Shear
Uniform
Elliptical
Triangular
Canilever Wing Moment
1.18
1.0
0.79
Uniform
Elliptical
Triangular

Properties of Beams



Centroid ≡ Area weighted center

$$\bar{y} = \frac{1}{A}\int y\,dA \qquad \bar{z} = \frac{1}{A}\int z\,dA$$

Moment of Inertia

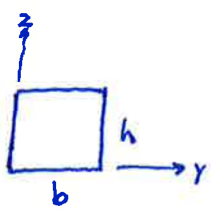
$$I_{yy} \equiv \int_A (z')^2\,dA \qquad I_{yz} = \int y'z'\,dA \qquad I_{zz} = \int (y')^2\,dA$$

Coordinate Transforms

$$I_{y''y''} = I_{yy} + z^2 A \qquad I_{y''z''} = I_{yz} + \bar{y}\bar{z}A$$

$$I_{z''z''} = I_{zz} + y^2 A$$

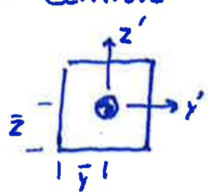
Ex:



$A = bh$ $\qquad dA = b\,dy$

① Find Centroid

$$\bar{y} = \frac{1}{A}\int y\,dA = \frac{1}{A}\int_0^b y\,h\,dy = \frac{1}{bh} h \frac{y^2}{2}\Big|_0^b = \frac{1}{bh} h \frac{b^2}{2} = \frac{b}{2}$$

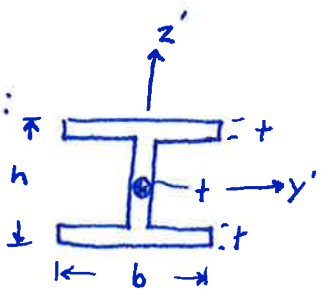


$\bar{z}$ = by inspection with $\bar{y}$ = $h/2$

② Moment of Inertia

$$I_{yy} = \int_A z'^2\,dA = \int_{-h/2}^{h/2} z'^2 b\,dz' = \frac{bz'^3}{3}\Big|_{-h/2}^{h/2} = \frac{bh^3}{3\cdot 8} - \frac{-bh^3}{3\cdot 8} = \frac{bh^3}{12}$$

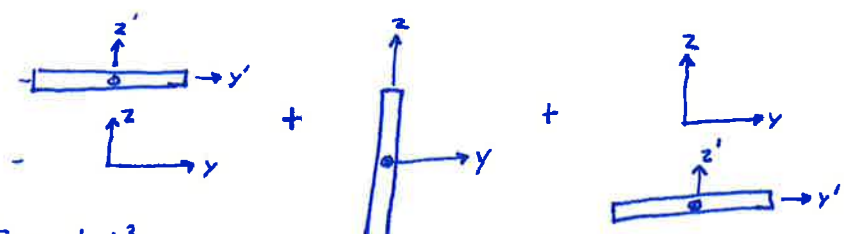
Ex:



① Centroid by inspection $\qquad$ Area =

② Moment of Inertia

Split into 3 parts:



Shift coordinates

$I_{y'y'} = \dfrac{bt^3}{12}$ $\qquad$ $I_{yy} = \dfrac{t(h-2t)^3}{12}$ $\qquad$ $I_{yy} = \dfrac{bt^3}{12}$ $\qquad$ $A = bt$

$z = h/2 - t/2$ $\qquad$ $A = t(h-2t)$ $\qquad$ Same as #1, but inverted

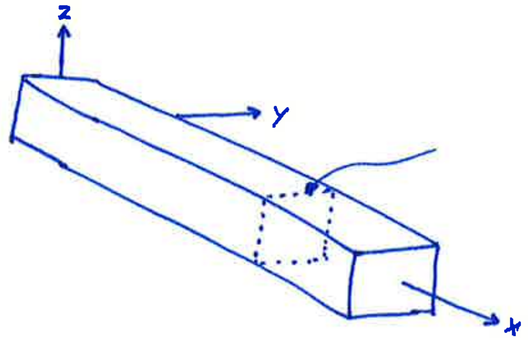
$A = bt$

$$I_{yy} = \left(\frac{bt^3}{12} + \left(\frac{h}{2} - \frac{t}{2}\right)^2 bt\right)2 + \frac{t(h-2t)^3}{12}$$

for thin beams ($t \ll h$)

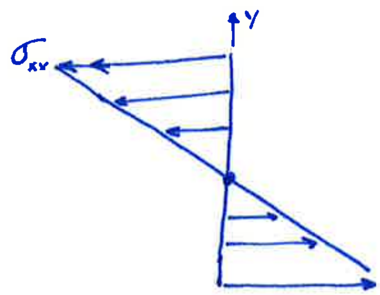
$$I_{yy} = \frac{1}{2}bth^2 + \frac{th^3}{12}$$

Simple Beam in Pure Bending



Stress

$$\sigma_{xx} = -\frac{M_z y}{I_{zz}}$$



$$\sigma_{xx} = \frac{-M_y z}{I_{yy}}$$

Deflection

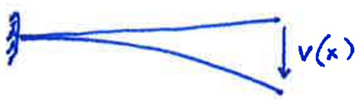
$$\frac{d^2 v}{dx^2} = \frac{M}{EI}$$

- if M is constant
- Boundary conditions

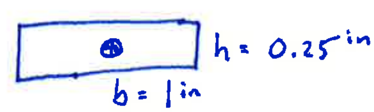
$$v = \frac{M}{EI}\frac{x^2}{2} + Bx + C$$

$v(0) = 0$ and $\frac{dv}{dx}(0) = 0$

$$v = \frac{M}{EI}\frac{x^2}{2}$$

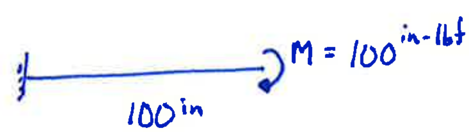


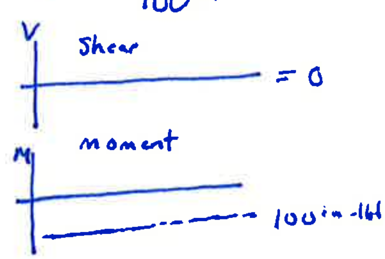
Ex: Find the maximum stress and deflection in a simple steel beam



$I_{yy} = \frac{bh^3}{12} = 0.001302 \text{ in}^4$ $\quad I_{zz} = \frac{1}{12} h b^3 = 0.020833 \text{ in}^4$

$E_{steel} \approx 30 \cdot 10^6$ psi





$$\sigma_{xx} = -\frac{My}{I} = -\frac{100 \text{ in lbf} \cdot 0.125 \text{ in}}{0.001302 \text{ in}^4}$$

$= \boxed{9.6 \text{ ksi on upper and lower surface}}$

$$v = \frac{M}{EI}\frac{x^2}{2} = \frac{100 \text{ in lbf} \cdot 100^2 \text{ in}^2 \cdot \text{psi in}^2}{30 \cdot 10^6 \text{ psi} \cdot 0.001302 \text{ in}^4 \cdot 2 \cdot \text{lbf}}$$

$= \boxed{12.8 \text{ in}}$

Ex: Rotate to ▯ to give $\sigma_{xx} = 2.4$ ksi and $v(100 \text{ in}) = 0.8$ in

Which is better? 1/4 less ~~deflection~~ stress $\frac{1}{16^{th}}$ less deflection