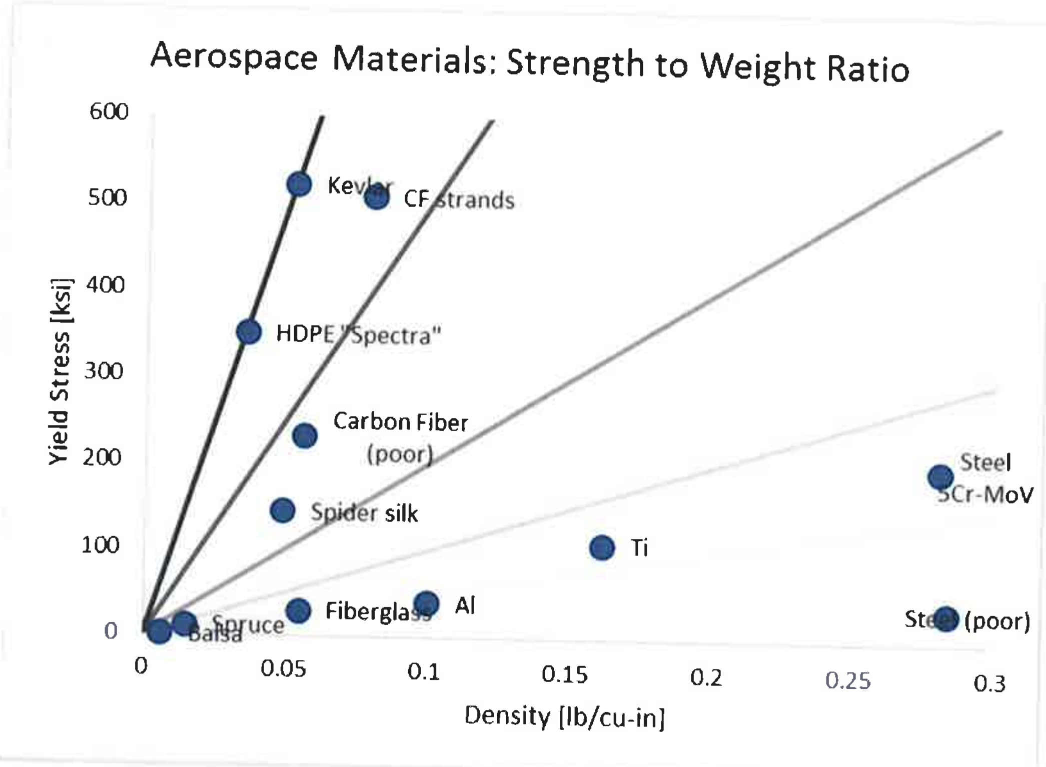
Aerospace Materials

As of yet, we have not considered material properties. We found the equations of equilibrium for a body, the strain displacement relations, and read about the thermodynamic constraints. Our general problem is still underspecified, we have more ~~eqns~~ unknowns than equations. The missing link is

Constitutive Laws/Equations "Material Properties"



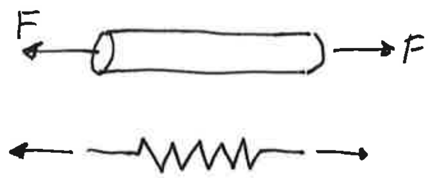
	Strength to Weight [in]	Yield Strength [ksi]	Density [lb/cu-in]
Steel (poor)	127	36	0.28400
Balsa	376	2	0.00579
Al	400	40	0.10000
Fiberglass	545	30	0.05500
Ti	679	110	0.16200
Steel 5Cr-MoV	712	200	0.28100
Spruce	843	12	0.01445
Spider silk	2995	145	0.04841
Carbon Fiber (po	4143	232	0.05600
CF strands	6403	509	0.07948
HDPE "Spectra"	9987	350	0.03505
Kevlar	10033	522	0.05203

Creep and Viscoelastic materials

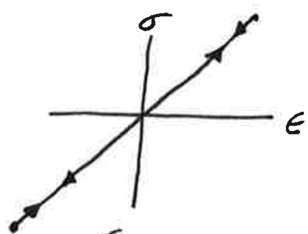
All materials have a time component to the stress/strain relationship. Some materials are **much** more pronounced.

Viscoelastic $\equiv$ Stress is dependent on the history of the strain

Ex: Take a sample, load it, and measure the strain



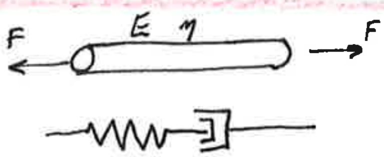
system model
$F = kx$



Hookean Material
"linear $\sigma = E\epsilon$"

$$\boxed{\epsilon = \frac{\sigma}{E}}$$

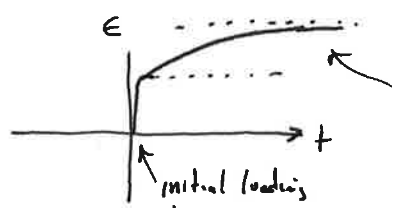
Ex:



Spring damper
$f = kx$ $\quad F = kv$

$$\boxed{\frac{d\epsilon}{dt} = \frac{1}{E_M}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \quad \text{Maxwell model}}$$

HDPE!



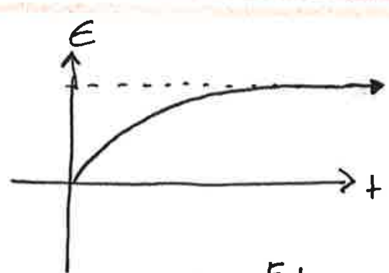
Steady state

$$\cancel{\frac{d\epsilon}{dt}}^{0} = \frac{1}{E_M}\cancel{\frac{d\sigma}{dt}}^{0} + \frac{\sigma}{\eta} = 0?$$

No S.State unless no loading

Ex:

$$\boxed{\frac{d\epsilon}{dt} = \frac{1}{\eta}\left(\sigma - E_v\epsilon\right) \quad \text{Voigt Model}}$$



$$\varepsilon = -\frac{\sigma_0}{E}e^{-\frac{Et}{\eta}} + \frac{\sigma_0}{E}$$

Steady state

$$\cancel{\frac{d\epsilon}{dt}}^{0} = \frac{1}{\eta}\left(\sigma - E_v\epsilon\right) \Rightarrow \quad \sigma = E\epsilon$$

von Mises Yield Criteria

A yield criteria that estimates when a material exits the linear "Hookian" region of stress. Best for ductile materials (e.g. steel, Al, metals...)

$$\boxed{J_2 \geq \frac{Y^2}{3}}$$

which is

$$\boxed{J_2 = \frac{3}{2}\tau_o^2 = \frac{1}{6}\Big[(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{xx}-\sigma_{zz})^2 + 6\sigma_{yz}^2 + 6\sigma_{xz}^2 + 6\sigma_{xy}\Big] \geq \frac{Y^2}{3}}$$

If we take the σ stress state and determine the principal stresses.

$$\sigma_{P1}, \sigma_{P2}, \sigma_{P3} \Rightarrow \frac{\sigma_{P_1} - \sigma_{P2}}{2} = \sigma_{S1}$$

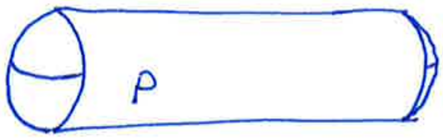
So von-Mises is

$$\frac{1}{6}\underbrace{\Big[4\sigma_{S_1} + 4\sigma_{S2} + 4\sigma_{S3}\Big]}_{\text{a measure of the shear, which is a fundamental failure mode for metals (at molecular level)}} \geq \frac{Y^2}{3}$$

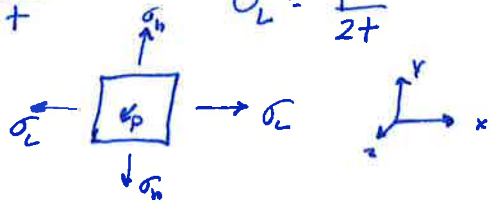
von Mises stress is a <u>common</u> output in numerical simulations + FEA tools

You can also view the von Mises criteria as a cylinder of radius about the hydrostatic axis

Ex: Determine the pressure required to yield an oxygen cylinder of radius 12 in with 0.125 in walls. The material is 6061-T6 Al.



$$\sigma_{hoop} = \frac{pr}{t} \qquad \sigma_L = \frac{pr}{2t} \qquad \frac{r}{t} = \frac{12}{0.125} = 96$$



① Stress State

$$\sigma = \begin{bmatrix} \frac{pr}{2t} & 0 & 0 \\ 0 & \frac{pr}{t} & 0 \\ 0 & 0 & p \end{bmatrix} = \begin{bmatrix} 48 & 0 & 0 \\ 0 & 96 & 0 \\ 0 & 0 & 1 \end{bmatrix} p$$

② material props

$Y = 36$ ksi

Table A.1

③ Von Mises criteria

$$\underbrace{\frac{1}{6}\left[(48-96)^2 + (96-1)^2 + (1-48)^2\right]}p^2 \geq \frac{Y}{3}^2$$

$$2256.3\,p^2 \geq \frac{36000^2}{3}$$

$$p \geq \sqrt{\frac{36000^2}{3} \cdot \frac{1}{2256.3}} = 437 \text{ psi}$$

$$\boxed{p = 437 \text{ psi}}$$

④ Comments

An actual cylinder design would need to account for

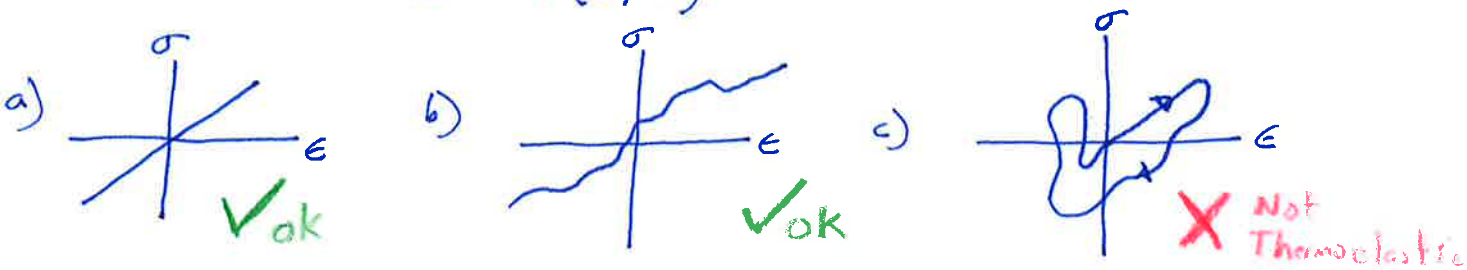
- Joining stresses (welding, riveting, etc)
- Factor of safety
- Damage + Loads

Notice that shear is 6x more than normal stresses!

Thermoelastic material

Stress is a function of strain and temperature.

$$\sigma = \sigma(\varepsilon, T)$$



Another way to consider this is that a thermoelastic material is loaded in stress, and strain ~~and temperature~~ in the same path as it is unloaded.

A material is a Hookean material when the stress is a linear function of ϵ and T

$$\sigma = E\left(\varepsilon - \alpha (T - T_o)\right)$$

Ex: Determine the stress if strain is 0.001 and $\Delta T = 100^F$ for 2024-T4 Al

① Material properties

$$E = 10.5 \times 10^6 \text{ psi} \qquad \alpha = 12.9 \times 10^{-6} /{}^\circ F$$

② Stress

$$\sigma = 10.5 \times 10^6 \text{ psi}\left(0.001 - 12.9 \times 10^{-6} /{}^\circ F\,(100\,F)\right)$$

$$\boxed{\sigma = -13.5 \text{ ksi}}$$

Ex: A 200 ft wingspan aircraft at SSL flies to Antarctica at -60^F. Determine the new wingspan. The wing is 6061-T6 Al.

① Stress = 0 ② Material $\alpha = 13.0 \times 10^{-6} /{}^\circ F$

③ $\sigma = 0 = E\left(\epsilon - 13.0 \times 10^{-6} \cdot (-60 - 59)\right) \Rightarrow \epsilon = 13.0 \times 10^{6} (-119)$
$= 1.55 \times 10^{-3}$

④ $\Delta L = L\epsilon = 200^{ft} \cdot 1.55 \times 10^{-3} = 0.31^{ft}$

$$\boxed{b' = 199.69^{ft}}$$

General anisotropic Hookean Stress / Strain

Given our $\left\{\begin{matrix}\text{stress}\\ \text{strain}\end{matrix}\right\}$ states as a vector $(\sigma) = \begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{pmatrix}$ and $\epsilon = \begin{pmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{zz}\\ \epsilon_{yz}\\ \epsilon_{xz}\\ \epsilon_{xy}\end{pmatrix}$

The generic Hookes law is

$$(\sigma) = [D](\epsilon)$$

Modulus matrix (symmetric)

$$(\epsilon) = [C](\sigma)$$

Compliance matrix $\quad C = D^{-1}$

In general

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{pmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16}\\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26}\\ D_{31} & \text{Symmetric} & D_{33} & D_{34} & D_{35} & D_{36}\\ D_{41} & & & D_{44} & D_{45} & D_{46}\\ D_{51} & & & & D_{55} & D_{56}\\ D_{61} & & & & & D_{66}\end{bmatrix}\begin{pmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{zz}\\ \epsilon_{yz}\\ \epsilon_{xz}\\ \epsilon_{xy}\end{pmatrix}$$

This represents a material where every direction has a different behavior and normal stresses can create shear strains.

1.7

Draw Shear and Moment diagrams

$\frac{10}{10}$

Weight

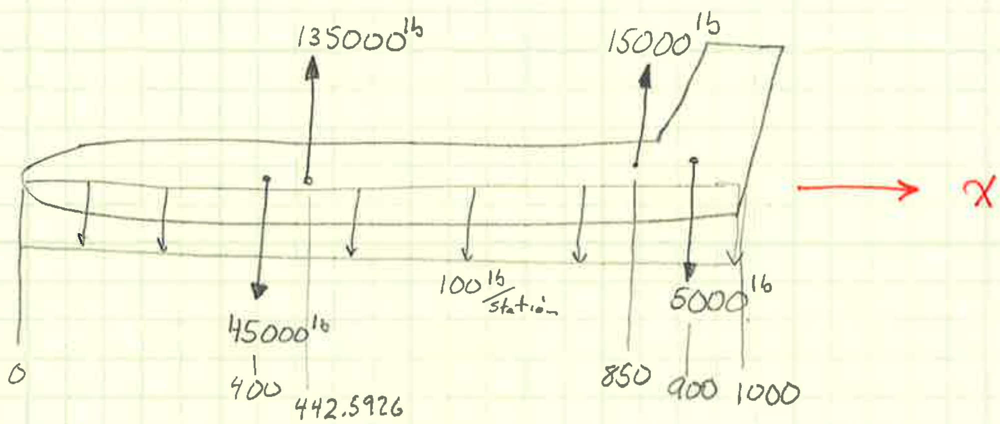
Wing	45000 lb
tail	5000 lb
fus	100000 lb
total	150000

$\frac{100000\text{ lb}}{1000\text{ station}} = 100\ \frac{\text{lb}}{\text{station}}$

Lift

Wing 90% $\Rightarrow$ 135000 lb

tail 10% $\Rightarrow$ 15000 lb



$$q = -100^{lb}\langle x\rangle^0 - 45000\langle x-400\rangle^{-1} + 135000\langle x-442.5926\rangle^{-1} + 15000\langle x-850\rangle^{-1} - 5000\langle x-900\rangle^{-1}$$

$$V = -100\langle x\rangle^1 - 45000\langle x-400\rangle^0 + 135000\langle x-442.5926\rangle^0 + 15000\langle x-850\rangle^0 - 5000\langle x-900\rangle^0 + \cancel{C_1} \to 0$$

no constant is needed in V.

$$M = -\frac{100}{2}\langle x\rangle^2 - 45000\langle x-400\rangle^1 + 135000\langle x-442.5926\rangle^1 + 15000\langle x-850\rangle^1 - 5000\langle x-900\rangle^1 + \cancel{C_1}^{\,0} x + \cancel{C_2} \to 0$$

no constant is needed in M.

$$V(0) = 0 = -100(0) + C_1 \Rightarrow C_1 = 0$$

$$M(1000) = 0 = -\frac{100}{2}\langle 1000\rangle^2 - 45000\langle 600\rangle^1 + 135000\langle 1000-442.5926\rangle^1 + 15000\langle 150\rangle^1 - 5000\langle 100\rangle^1 + 0\,(1000) + C_2$$

$$\Rightarrow C_2 \approx 0$$

1.7 continued

$V(0) = 0^{lb}$

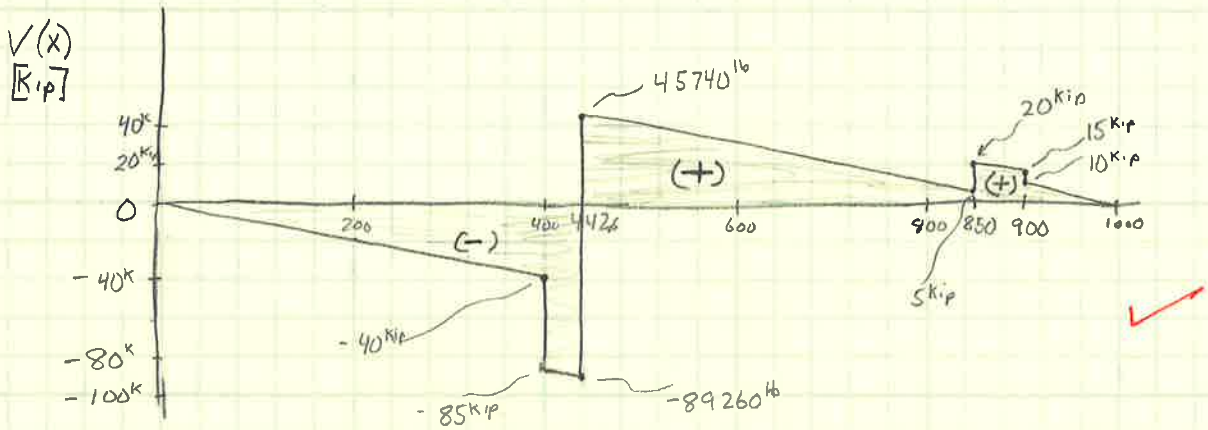
$V(400) = -100(400) = -40000^{lb}$

$V(442.5926) = -100(442.5926) - 45000 = -89260^{lb}$

$V(850)^{+} = -100(850) - 45000 + 135000 = 5000^{lb}$

$V(900) = -100(900) - 45000 + 135000 + 15000 = 15000^{lb}$

$V(1000) = 0$



$M(0) = 0$

$M(400) = -\frac{100}{2}(400)^2 = -8 \times 10^6$ lbstat

$M(442.5926) = -11.7 \times 10^6$ lbstat

$M(850) = -1.38 \times 10^6$ lbstat

$M(900) = -0.5 \times 10^6$ lb stat

$M(1000) = 0$