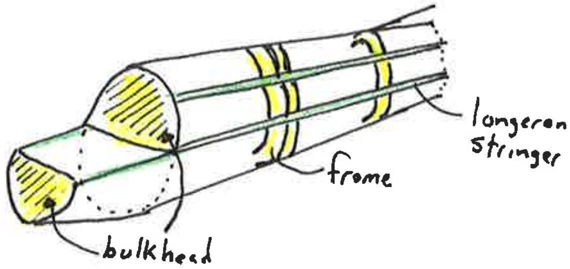
Review for Exam 1

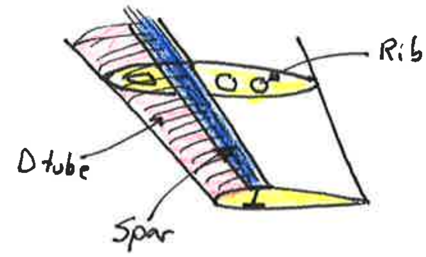
W 14th
St. Valentines Day

Do you really ♥ aero structures?!

1 Motivation

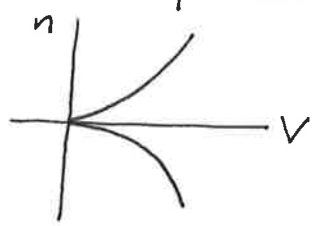
- Strong, lightweight, robust volumes and areas with inertial and aerodynamic loads.
- Terminology





2 Loads

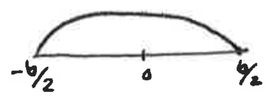
- Loads are the $\underbrace{\text{forces and pressures}}_{\text{inertial, aero, etc}}$ applied to the structure during operation
- Aero loads scale with dynamic pressure $\equiv \frac{1}{2}\rho V^2$ at low speeds



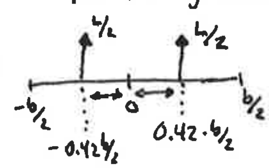
This is why the stall line of a V-n diagram is a parabola: $n \propto V^2$

- The lift distribution is a measure of how the lift forces/pressure are spanwise loaded
  Elliptical is a classic distribution, but is not alway true… depends on geometry

$$L' = \frac{L}{2}\frac{2}{b}\frac{4}{\pi}\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

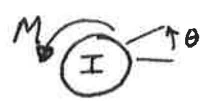


=



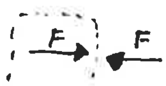
3 Inertial Loading

$\vec{F} = m\vec{a}$



$I\ddot{\theta} = M$

Equal and opposite reaction (on either side of control volume) $I = \int r^2 dm$



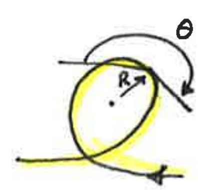
Free body diagram

Dynamics:

$$M = r \times F \qquad a_{pt} = a_{xyz} + \ddot{\theta} \times \vec{r} + \dot{\theta} \times (\dot{\theta} \times \vec{r}) + 2\dot{\theta} \times V$$

A level banked turn gives a g-load of $n = \frac{L}{W} = \frac{1}{\cos\phi}$

A steady state pitching maneuvers $n_z = \cos\theta + \frac{V^2}{R}\frac{1}{g_0}$



Static column of fluid $\frac{dp}{dz} = -\rho g h$

4 Gust Loading

Concepts, not much calculation in this class.

5 Shear and Moment Diagrams

- Find wall reaction shear and Moment
- Cut at desired point

If in doubt, integrate

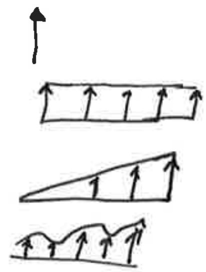
$$V = \int \left(F\delta(x-x_i) + F' \right) dy \qquad M = \int \left(F\delta(x-x_i) + F' \right) y \, dy$$

pt loads

uniform loads — acts at ½ span

linear dist — acts at ⅓ from major end

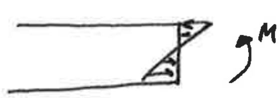
function



6 Beam properties

$$\text{Area} = \int dA \qquad \bar{y} \text{ Centroid} = \frac{1}{A}\int y \, dA \qquad I_{yy} = \int (z')^2 dA$$

7 Beam Bending



$$M \qquad \sigma_{xx} = -\frac{My}{I}$$

$$v \qquad \frac{d^2 v}{dx^2} = \frac{M}{EI}$$

8 Traction Vector

$$T(v) = \frac{dF}{dA} \text{ in a direction } v \qquad \sigma_n = T \cdot v \qquad \sigma_s = |T - (T \cdot v)v|$$

Cauchy Formula relates surface tractions to stresses

$$T_x = \sigma_{xx} v_x + \sigma_{yx} v_y + \sigma_{zx} v_z$$

Can be used to find normal + shear in direction

9 Stress Transformation

$$\sigma' = A \sigma A^T \quad \text{with} \quad A_{ij} = e_i' \cdot e_j$$

10 Principal Stresses

Eigenvalues of $[\sigma]$ give σ_{p_1} σ_{p_2} σ_{p_3}

Eigenvectors of $[\sigma]$ give directions

shear is ½ difference of σ_ps

11 von Mises

Yield criterion

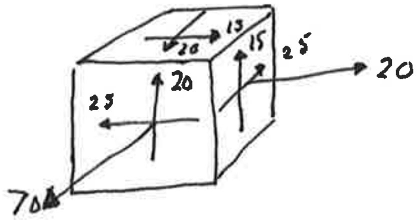
12 Aero materials $(\sigma) = [D](\varepsilon) - [T]\alpha\Delta T$

2D loading is a special case of 3D

Non-isotropic materials

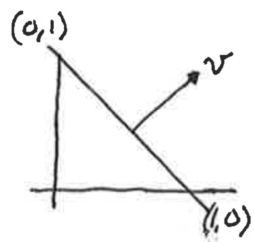
Quiz

Determine if the following stress state is safe. The material fails at $\sigma_n = 80$ ksi



Quiz

Given the stress state $\sigma = \begin{bmatrix} 10 & 8 \\ 8 & 20 \end{bmatrix}$ determine the traction vector on the plane A-B



Some Useful Equations + Properties

$F = ma$ $\quad I\ddot{\theta} = M$ $\quad I = \int r^2 \rho \, dx\, dy\, dz = m R_g^2$

$I_o = I_{cg} + M\bar{x}^2$

$a_{pt} = a_{xyz} + \ddot{\theta} \times r + \dot{\theta} \times (\dot{\theta} \times r) + 2\dot{\theta} \times V_{relative}$ $\qquad g_o = 32.174 \, ft/s^2$

Constant acceleration: $V = at \quad s = \frac{1}{2}at^2 = \frac{1}{2}\frac{V^2}{a}$

$n = \frac{1}{\cos\phi} \qquad n_z = \cos\theta + \frac{V^2}{R}\frac{1}{g_o}$

Power $= F \cdot V$

JetA $\rho \approx 6.7 \frac{lbf}{gal}$ $\qquad 1\,gal = 0.133\,ft^3$

$7.4\,gal = 1\,ft^3$

$\frac{dP}{dz} = -\rho g$ $\qquad \rho_{H_2O} = 8.34 \frac{lbf}{gal}$

$\bar{y} = \frac{1}{A}\int y\,dA \qquad \bar{z} = \frac{1}{A}\int z\,dA$

$\sigma_{xx} = -\frac{My}{I} \qquad \frac{d^2v}{dx^2} = \frac{M}{EI}$

$I_{yy} = \int (z')^2 dA \qquad I_{yz} = \int y'z' \, dA$

$I_{y'y'} = I_{yy} + z^2 A$

$T(v) = \frac{dF}{dA} \qquad \sigma_N = T \cdot v$

$\sigma_s = |T - (T \cdot v)v|$

$\sigma' = A \sigma A^T$ with $A_{ij} = e'_i \cdot e_j$

$T_x = \sigma_{xx}v_x + \sigma_{yx}v_y + \sigma_{zx}v_z$

$T_y = \sigma_{xy}v_x + \sigma_{yy}v_y + \sigma_{zy}v_z$

$T_z = \sigma_{xz}v_x + \sigma_{yz}v_y + \sigma_{zz}v_z$

$-\sigma_p^3 + I_1\sigma_p^2 - I_2\sigma_p + I_3 = 0$

$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$

$I_2 = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{zz} & \sigma_{xz} \\ \sigma_{xz} & \sigma_{xx} \end{vmatrix}$

$I_3 = \det[\sigma]$

Equilibrium

$\frac{d\sigma_{xx}}{dx} + \frac{d\sigma_{xy}}{dy} + \frac{d\sigma_{xz}}{dz} + X = 0$

$\sigma_{ij} = \sigma_{ji}$

Von-Mises

$$\frac{1}{6}\left[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6\sigma_{xy}^2 + 6\sigma_{xz} + 6\sigma_{xy}\right] \leq \frac{Y^2}{3}$$

Isotropic $\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$;

iso 3D

$$(\sigma) = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & v & & & \\ v & 1-v & v & & & \\ v & v & 1-v & & & \\ & & & \frac{1-2v}{2} & & \\ & & & & \ddots & \end{bmatrix}(\varepsilon)$$

2D iso

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & \\ v & 1 & \\ & & \frac{1-v}{2} \end{bmatrix}\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}$$

iso 3D

$$(\varepsilon) = \frac{1}{E}\begin{bmatrix} 1 & -v & -v & & \\ -v & 1 & -v & & \\ -v & -v & 1 & & \\ & & & 2(1+v) & \\ & & & & \ddots \end{bmatrix}(\sigma)$$

2D iso

$$(\varepsilon) = \frac{1+v}{E}\begin{bmatrix} 1-v & -v & \\ -v & 1-v & \\ & & \frac{2}{1-2v} \end{bmatrix}(\sigma)$$

$(\varepsilon) = +\alpha\Delta T$ and $(\sigma) = -\frac{\alpha \Delta T E}{1-2v}$

Ex: A fairing has an aerodynamic load of 0.5 psi at 200 mph. What is the velocity of failure if the fairing has an ultimate load of 3 psi?

$$\underbrace{\Delta P}_{\text{aero load}} = P - P_\infty = \tfrac{1}{2}\rho V^2 C_p \Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{\tfrac{1}{2}\rho V_1^2 C_p}{\tfrac{1}{2}\rho V_2^2 C_p} = \frac{V_1^2}{V_2^2}$$

- 0.5 psi at 200 mph

$$V_2^2 = V_1^2 \frac{\Delta P_2}{\Delta P_1} = \cancel{200\text{ mph}} \Rightarrow V_2 = V_1 \sqrt{\frac{\Delta P_2}{\Delta P_1}}$$

$$V_2 = 200\text{ mph}\left(\frac{3.0\text{ psi}}{0.5\text{ psi}}\right)^{1/2} = \boxed{490\text{ mph} = V_2}$$

6X higher ~~speed~~ pressure with only 2.4 X higher speed

Ex: Determine the root moment of an elliptical wing of span 34 ft generating 6000 lbf of lift.

Hard way: Integrate the lift distribution

$$M = \int dM\,dy = \int L' y\,dy = \int_0^{b/2} \frac{L}{2}\frac{2}{b}\frac{4}{\pi}\sqrt{1-\left(\frac{2y}{b}\right)^2} = \ldots\ldots\ldots$$

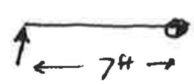
Easy: Remember Elliptical acts at 42% semispan (exactly 42.44%)

$$M = F \cdot \text{distance} = \left(\frac{L}{2}\right)\left(\frac{b}{2}\right)(0.42)$$

$$= 3000\text{ lbf} \,|\, 17\text{ ft} \,|\, 0.4244 = \boxed{21644 \text{ ft-lbf}}$$

Ex: A student pilot lands poorly on the nosegear of ~~an~~ a Piper Cherokee. The nose wheel is 7 ft forward of the CG. The aircraft has a weight of 1500 lb and a radius of gyration of 9 ft. A g meter reads an instantaneous +3 g. Determine the pitch acceleration and the time to reach 15° = θ if the aircraft "lands" at θ = -5°

① FBD



$$I = m R_g^2 = \frac{1500\text{ lbf}}{32.174} \left|\frac{s^2}{ft}\right| 9^2 ft^2 \left|\frac{\text{slug ft}}{\text{lbf } s^2}\right| = 3776\text{ slug-ft}^2$$

② g-load

$$F = ma = \frac{1500\text{ lbf}}{\cancel{32.174}} \left| \frac{3g \,|\, \cancel{32.174}}{1s} \right. = 4500\text{ lbf} \qquad M = r \times F = 7\text{ ft} \cdot 4500\text{ lbf} = 31500\text{ ft lbf}$$

③ Acceleration

$$\ddot{\theta} = \frac{M}{I} = \frac{31500\text{ ft lbf}}{3776\text{ slug ft}^2} \left| \frac{\text{slug ft}}{\text{lbf } s^2} \right. = \boxed{8.3 \frac{rad}{s^2}}$$

④ Time $\theta = \frac{1}{2}\ddot{\theta}t^2 \Rightarrow t = \sqrt{\frac{2\theta}{\ddot{\theta}}} = \sqrt{\frac{2 \mid 20° \mid s^2 \mid rad\ \pi}{\mid \mid 8.3\ rad \mid 180°}} = \underline{0.3\ s}$

Could the pilot react?

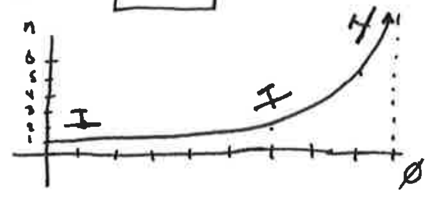
⑤ Reality

The front gear will collapse.

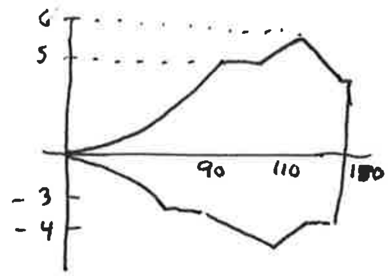
tiny.cc/NMMHYWjEmKY

Ex: What is the g-load of an aircraft in a steady 45° bank?

$n = \frac{1}{\cos 45°} = \boxed{1.41}$ Acceleration $= ng = 1.41 \cdot 32.174 = 45.5\ \frac{ft}{s^2}$

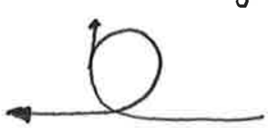


Ex: Determine the limit load g load and V_{NE} from the V-n diagram.

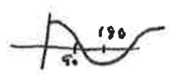


g-load	+6
V_{NE}	180

Ex: A 140 lb pilot performs a 1000 ft radius loop at 200 ft/s. What is the g-load when going vertical up?



$\theta = 90°$



$n_z = \cancel{\cos 90°}^{0} + \frac{200^2\ ft^2 \mid \mid s^2}{s^2 \mid 1000\ ft \mid 32.174\ ft}$

$\boxed{n = 1.24}$

Ex: ~~Hmm~~ A rocket accelerates at 5g. If a 100 ft tank is full of JetA, what is the pressure at the tank's bottom?

$\frac{dp}{dz} = -\rho g \Rightarrow dp = -\rho g dz \Rightarrow \Delta p = -\rho g \Delta z$

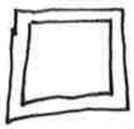
$\Delta p = -\underbrace{\frac{6.7\ lbf}{gal}}_{\text{Need mass not weight!}} \frac{32.174\ ft}{g\ s^2} \Big| 100\ ft \Big| \frac{7.4\ gal}{ft^3} \Big| 5g \Big| \frac{slug\ ft}{lbf} \Big| \frac{s^2}{32.174\ ft} \Big| \frac{ft^2}{144\ in^2}$

$= \boxed{172\ psi}$

2024 AI

Ex: Given a square tube spar, determine the deflection due to a uniform loading of 1000 lbs over 24 ft with a 2 inch OD and 1/8 in wall thickness

① Beam

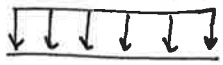


$$I_{yy_{outer}} = \frac{1}{12} b h^3 \qquad I_{yy_{inner}} = \frac{1}{12} b' h'^3$$

$$I_{yy} = \frac{1}{12}(2)(2)^3 - \frac{1}{12}(1.75)(1.75)^3 = 0.55 \text{ in}^4$$

② loading

$-1000/24 = -41.67$ lb/ft



③ Wall loading

$$V_0 + \int_0^{24} (-41.67)\, dx = 0 \qquad V_0 = 1000 \text{ lbf}$$

$$M_0 + \int_0^{24} (+41.67)\, x\, dx = 0 \qquad M_0 = -12000 \text{ ft}\cdot\text{lbf}$$

④ Shear + Moment

$$V_0 - \int_0^x -41.67\, dy + V = 0 \qquad V(x) = -V_0 - 41.67\, x = -1000 - 41.67$$

$$M_0 + \int_0^x (41.67)\, x\, dx - M(x) = 0 \qquad M(x) = M_0 - 41.67 \frac{x^2}{2}$$

$$= -12000 \text{ ft lbf} - 20.83\, x^2$$

⑤ deflection

$$\frac{d^2 V}{dx^2} = \frac{M}{EI} = \frac{-12000 - 20.83\, x^2}{EI}$$

$$V = \frac{1}{EI}\left(-12000 \frac{x^2}{2} - \frac{20.83}{3 \cdot 4} x^4 \right)$$

$$= \frac{1}{10.5\times 10^6 \text{ psi} \mid 0.55 \text{ in}^4} \left(-12000 \frac{24^2}{2} - \frac{20.83}{3 \cdot 4} 24^4 \right) \frac{\text{ft-lbf}\cdot\text{ft}^2}{} \Big| \frac{\text{psi in}^2}{\text{lbf}} \Big| \frac{144 \text{ in}^2}{\text{ft}^2}$$

100 ft   Why does this not make sense?

⑥ Stress?

$$\sigma_{xx} = -\frac{My}{I} = \frac{-12000 \text{ ft lb} \mid 1 \text{ in}}{\mid 0.55 \text{ in}^4} \Big| \frac{12 \text{ in}}{\text{ft}} = 261 \text{ ksi}$$

Ex: A 3D orthotropic material is loaded as

$$[\sigma] = \begin{bmatrix} 10^{ksi} & 2^{ksi} & 0 \\ 2^{ksi} & 5^{ksi} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Determine ε_{xx} if $\Delta T = 200^F$ and

$E_x = 25 \times 10^6$ psi

$E_y = 2 \times 10^6$ psi

$\nu = 0.3$

$\alpha = 10 \times 10^{-6}\ 1/F$

1. $$\varepsilon_{xx} = \frac{1}{E_x}\sigma_{xx} - \frac{\nu_{xy}\sigma_{yy}}{E_x} - \frac{\nu_{xy}\sigma_{zz}}{E_x} + \alpha \Delta T$$

$$= \frac{1}{25 \times 10^6 \text{ psi}}\left(10 \times 10^3 - 0.3 \cdot 5 \times 10^3 - 0\right) + 10 \times 10^{-6} \cdot 200$$

$$\boxed{\varepsilon_{xx} = 0.00234}$$

Ex: Sunken Ship

$$p = \rho g h = \frac{8.34 \text{ lbf}}{\cancel{gal}} \left| \frac{\text{slugs}}{32.174\ \cancel{lbf}} \right| \frac{\cancel{gal}}{0.133\ \cancel{ft^3}} \left| \frac{32.174\ \cancel{ft}}{\cancel{s^2}} \right| 12500\ \cancel{ft} \left| \frac{\cancel{lbf\ s^2}}{\cancel{slug\ ft}} \right| \frac{\text{lb f } s^2}{\text{slug ft}} \left| \frac{ft^2}{144 \text{ in}^2} \right.$$

$$= 5400 \text{ psi}$$

$$\boxed{\sigma = \begin{bmatrix} 5.4 & & \\ & 5.4 & \\ & & 5.4 \end{bmatrix} \times 10^3 \text{ psi}}$$

Von Mises

$$\frac{1}{6}\left(\cancel{(5.4 - 5.4)^2}^{\,0} + \cancel{(\cdots)^2}^{\,0} + \cancel{(\cdots)^2}^{\,0} + \cancel{0}^{\,0} \right) \leq \frac{Y^2}{3}$$

yes, always

$\boxed{\text{Not yielded}}$

Solids deflect but do not yield or fail due to the ocean's pressure.

$\boxed{\text{Ships are intact}}$ ... getting the ship out of the ocean floor might be difficult