AEM 341: Exam 1 B   Name: O'Neill

14th Feb 2018   55 minutes   ~~6~~ 4 Pages   Closed book, Closed notes, Calculator

100 total points   **Read, think, plan, and then write.**

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Signature:____________________

Date:____________________

1. [5 pts] A 1000 lbf aircraft is in a 60 degree bank. What is the root bending moment of the elliptical wing of span 20 ft?

4244 ft lbf

$$M = \frac{L}{2}\frac{b}{2}\cdot 0.4244$$

$$L = nW = \frac{1}{\cos\phi}W = \frac{1}{0.5}\cdot 1000 = 2000\text{ lb}$$

$$M = 1000\text{ lb}\cdot 10\text{ ft}\cdot 0.4244$$

2. [5 pts] What are possible units of a traction vector?

Unitless	Meters^2	N/m	lbf-in^2	$\boxed{\text{None of the above}}$

$T(v)$ is a pressure (psi, N/m^2)

3. [5 pts] You constrain a cube to give zero initial strain and zero initial stress. While constrained, if you heat the cube +100 F, determine the σ_{xx} stress state? The cube has a Poisson's ratio of 0.3, $E = 10\times10^6\ psi$ and $\alpha = 10\times10^{-6}\ {}^1\!/_F$.

-25000 psi

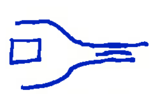
$$(\sigma) = [D]\cancel{(\varepsilon)}^{0} - \frac{\alpha\Delta T E}{1-2\nu}$$

$$= -\frac{10\times10^{-6}}{F}\cdot 100F\cdot 10\times10^{6}\,psi\cdot\frac{1}{1-2\cdot0.3}$$

4. [5 pts] Which is the most isotropic Hookean material below?

Wood grown in a tropic region	Unidirectional Carbon Fiber Tape	Cold rolled Al sheet	Toothpaste	$\boxed{\text{Block of 5Cr-Mo-V steel}}$





orthotropic

Flattened grains

Not hookean even if it is isotropic

You can take the wood out of the tropics, tropics

but you can't take the orthotropics out of the wood!

5. [20 pts] Determine the principal stresses given the following stress state.

$$[\sigma] = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 1 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

① eigenvalues of σ

$$\boxed{\sigma_1 = 7.598 \quad \sigma_2 = -0.844 \, , \quad \sigma_3 = 1.246}$$

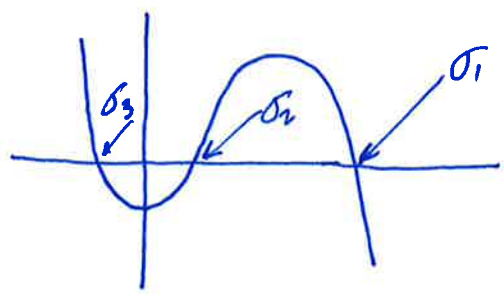
② Solve characteristic eqn.

$I_1 = 3+1+4 = 8$

$I_2 = (3-1) + (4-0) + \overset{(12-16)}{\cancel{}} = 2+4-4 = 2$

$I_3 = \det(\sigma) = -8$

$$-\sigma_p^3 + 8\sigma_p^2 \overset{-2}{\cancel{+4}}\sigma_p - 8 = 0$$



③ By hand.

I'll pass on that!

6. [10 pts] Determine the direction (eigenvector) associated with the $\sigma_p = 1.0$ eigenvalue of the following stress state.

$$[\sigma] = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$Av = \lambda v$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 1\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \Rightarrow 2V_1 + V_2 = V_1$$

or $V_1 + V_2 = 0$

$V_1 = 1$

$V_2 = -1$

$$\boxed{V = \begin{matrix} 1 \\ -1 \end{matrix}}$$ or any multiple of this

7. [20 pts] Given a ductile metal with a yield stress of 60 ksi and a factor of safety of 1.5, is the following stress state acceptable?

$$[\sigma]=\begin{bmatrix}50 & 0 & 10\\ 0 & 10 & 0\\ 10 & 0 & 40\end{bmatrix} ksi$$

Von - Mises since ductile metal

$\sigma_y = 60^{ksi} \Rightarrow \sigma_{accept} = \frac{60}{1.5} = 40^{ksi}$

- $\frac{1}{6}\left[\underbrace{(50-10)^2}_{40^2} + \underbrace{(10-40)^2}_{30^2} + \underbrace{(50-40)^2}_{10^2} + 6(10)^2 + 6(0)^2 + 6(0)^2\right]$

$= \frac{3200}{6} = \frac{1600}{3}$

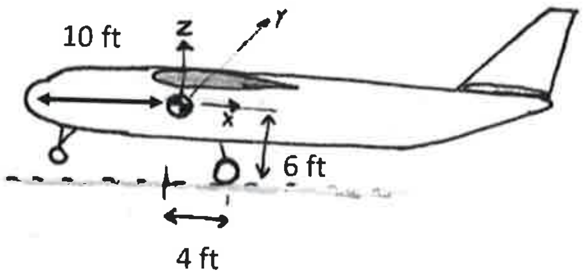
- $\frac{Y^2}{3} = \frac{40^2}{3} = \frac{1600}{3}$

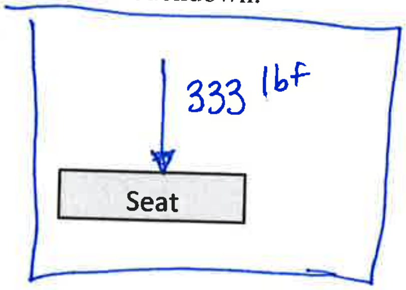
Equal

$\boxed{\text{SAFE, but just barely}}$

8. [30 pts] A small 15000 lbf transport aircraft lands firmly (+3g) on the Greenland icesheet. The rolling coefficient on ice is nearly zero. The inertia about the y axis is 90000 slug-ft^2. The landing gear is located 6 feet below and 4 feet aft of the CG.

Determine the total load of a 140 lbf pilot **on a seat** located 10 feet forward of the CG at touchdown. Clearly indicate the **direction** and **magnitude** of the resulting loan **on** the seat.





① Mass

$$m = \frac{15000\ \text{lbf}}{32.174} = 466\ \text{slug}$$

$$m_{pilot} = \frac{140}{32.174} = 4.3\ \text{slug}$$

0) 3g impact

$$a = 3 \cdot 32.174 = 96.5\ \hat{z} \uparrow$$

② 3g impact

$$F = m \cdot a = 466\ \text{slug} \cdot 3g \cdot 32.174\ \frac{\text{ft}}{\text{s}^2} \left| \frac{1\ \text{lbf s}^2}{\text{slug ft}} \right. = 45000\ \text{lbf}\ \hat{z} \uparrow$$ 5

2a) or just

$$3g \cdot 15000\ \text{lbf} = 45000\ \text{lbf}\ \hat{z} \uparrow$$

3) $$M = r \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 4 & 0 & -6 \\ 0 & 0 & 45000 \end{vmatrix} = -4 \cdot 45000\ \hat{y} = -180000\ \text{ft lbf}\ \hat{y}$$ 5

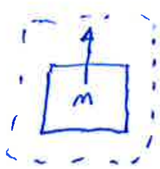
nose down

4) $$\ddot{\theta} = \frac{M}{I} = \frac{-180000\ \text{ft lbf}}{90000\ \text{slug ft}^2} \left| \frac{\text{slug ft}}{\text{lbf s}^2} \right. = -2\ \frac{\text{rad}}{\text{s}^2}\ \hat{y}$$ nose down 5

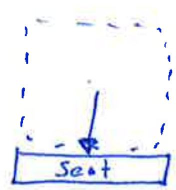
5) $$a = \ddot{\theta} \times r = \text{by inspection} = \ddot{\theta} r \quad \text{or} \quad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -2\ \frac{1}{s^2} & 0 \\ -10\ \text{ft} & 0 & 0 \end{vmatrix} = -(-10)(-2)\ \hat{z} = -20\ \frac{\text{ft}}{\text{s}^2}\ \hat{z}$$ 5

6) $$a_{total} = 96.5\ \hat{z} - 20\ \hat{z} = 76.5\ \hat{z}\ \frac{\text{ft}}{\text{s}^2} \uparrow$$ 5

7) $$F = ma = 333\ \text{lbf}$$ 5

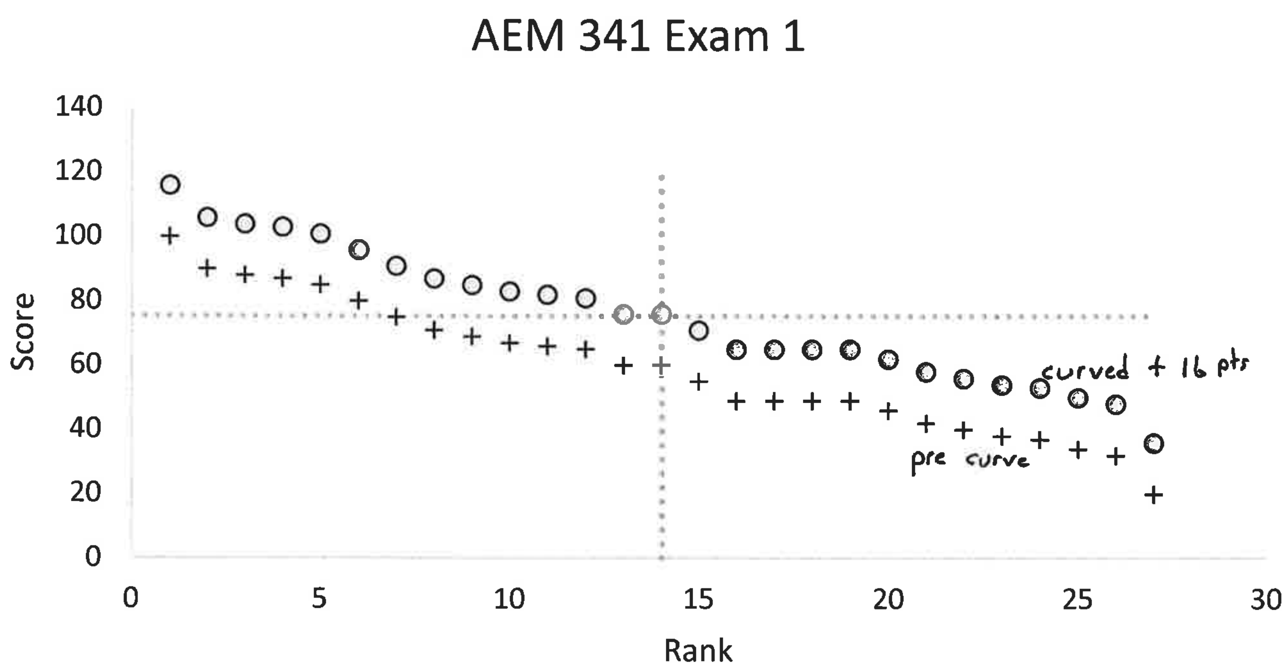


8) Seat

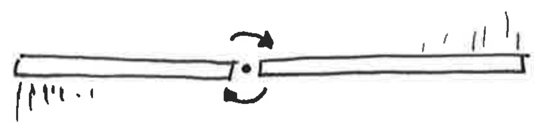


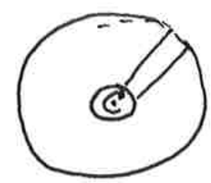
75 average

Std dev 20 pts

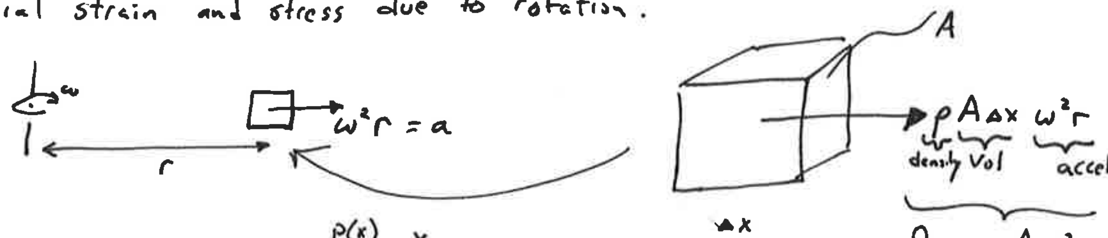


Spinning Beam (e.g. helicopter blade) (or turbine blade)





Determine the axial strain and stress due to rotation.



$p_x = \rho A \omega^2 r = \rho A \omega^2 x$

① $\dfrac{dp}{dx} = -p_x = -\rho A \omega^2 x$ integrate $\displaystyle\int_{p(L)}^{p(x)} dp = \int_L^x -\rho A \omega^2 x \, dx$

$$p(x) - p(L) = -\rho A \omega^2 \left(\frac{x^2}{2} - \frac{L^2}{2}\right)$$ ← Assumes A is constant!

② Stress

$\sigma = \dfrac{p}{A} = -\rho \omega^2 \left(\dfrac{x^2}{2} - \dfrac{L^2}{2}\right)$ at the hub, $\sigma = \dfrac{\rho \omega^2 L^2}{2}$

③ Strain

$\varepsilon = \dfrac{\sigma}{E} = \dfrac{\rho \omega^2 (L^2 - x^2)}{2E}$

④ Deflection (x direction)

$$\frac{d}{dx}\left(AE \frac{du}{dx}\right) = -p_x = -\rho A \omega^2 x$$

$$\int_{\varepsilon(L)}^{\varepsilon(x)} d\varepsilon = \int_L^x -\frac{\rho \omega^2 x}{E} dx \Rightarrow \varepsilon(x) - \cancel{\varepsilon(L)} = -\frac{\rho \omega^2}{E}\left(\frac{x^2}{2} - \frac{L^2}{2}\right)$$

0 at tip!

4.5) Integrate again

$$\varepsilon(x) = \frac{du}{dx} = -\frac{\rho \omega^2}{E}\left(\frac{x^2}{2} - \frac{L^2}{2}\right) \Rightarrow \int_{u_0}^{u(x)} du = \int_0^x -\frac{\rho \omega^2}{2E}(x^2 - L^2)\, dx$$

$$U(x) - \cancel{u_0}^{\,0} = \frac{\rho \omega^2}{2E}\left(L^2 x - \frac{x^3}{3}\right)$$

5) Summary

$$\boxed{\sigma = +\frac{\rho \omega}{2}(L^2 - x^2) \qquad \varepsilon = \frac{\sigma}{E} \qquad U(x) = \frac{\rho \omega^2}{2E}\left(L^2 x - \frac{x^3}{3}\right)}$$

Ex: Apache AH 64

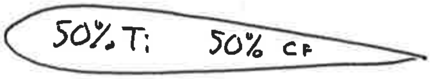
Rotor span = 48 ft

Rotor rpm ≈ 300

Material is actually a composite + metal.

Assume 50% Ti and 50% CF

Stress is independent of blade area and material stiffness!



$$\rho \approx 0.5\,(0.162 + 0.056)\ \frac{\text{lb}}{\text{in}^3}\Big|\ 32.174$$

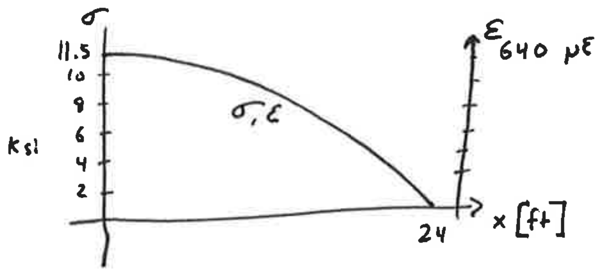
$$= 0.00338\ \frac{\text{slug}}{\text{in}^3}$$

$$E \approx 0.5\,(15\times10^6 + 21\times10^6)\ \text{psi}$$

$$= 18\times10^6\ \text{psi}$$

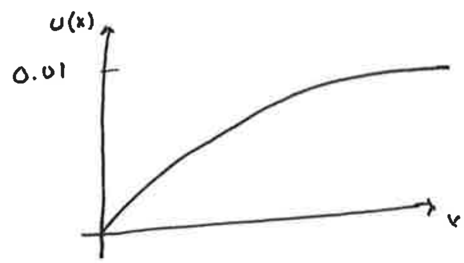
$$\sigma = \frac{0.00338\ \text{slug}}{\text{in}^2\ \cancel{\text{ft}}}\,\Big|\,\frac{300^2\ \cancel{\text{rev}^2}}{\cancel{\text{min}^2}}\,\Big|\,\frac{(2\pi)^2\ \cancel{\text{rad}^2}}{\cancel{\text{rev}^2}}\,\Big|\,\frac{\cancel{\text{min}^2}}{60^2\ \cancel{\text{s}^2}}\,\Big|\,\frac{1}{2}\,\Big|\,(24^2\ \cancel{\text{ft}^2} - x^2)\,\Big|\,\frac{\text{lbf}\ \cancel{\text{s}^2}}{\cancel{\text{slug ft}}}\,\Big|\,\frac{12\ \cancel{\text{in}}}{\cancel{\text{ft}}}$$

$= 11.5$ ksi at root



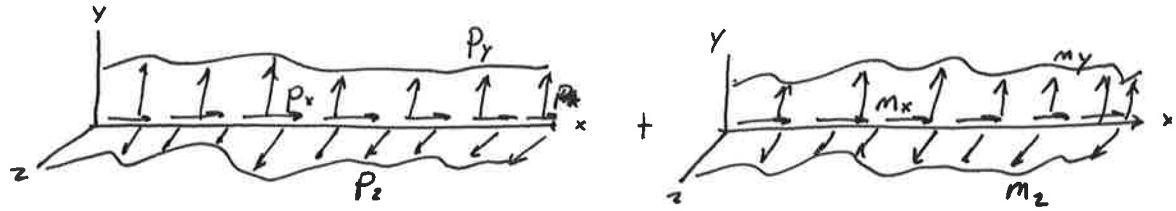
$$\varepsilon = \frac{\sigma}{E} \approx \frac{\sigma}{18\times10^6\ \text{psi}} = 640\ \mu\varepsilon \text{ at root}$$

$$u(x) = \frac{\rho\omega^2}{2E}\left(L^2 - \frac{x^2}{3}\right)x \approx 0.01 \text{ in at tip}$$



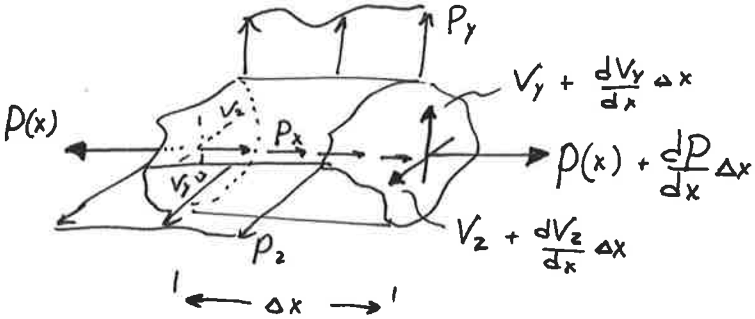
Notice that this loading is relatively mild.

Differential Equations of Equilibrium for bars



3 forces per unit length   3 moments per unit length

For equilibrium $\Sigma F = 0$ and $\Sigma M = 0$ on any and all segments



$$\Sigma F_x = 0 = \underbrace{-P(x)}_{\text{left}} + \underbrace{P(x) + \frac{dP}{dx}\Delta x}_{\text{right}} + \underbrace{p_x \Delta x}_{\text{external force}} \Rightarrow \frac{dP}{dx} + p_x = 0$$

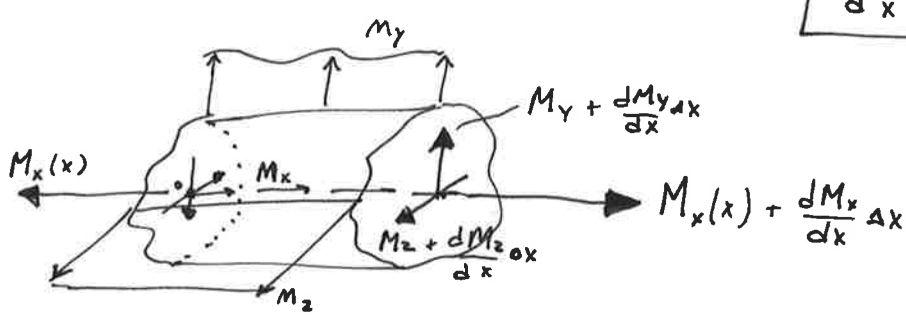
$$\boxed{\frac{dP}{dx} = -p_x}$$

Already derived in prev. lecture

$$\Sigma F_y = 0 = -V_y(x) + V_y(x) + \frac{dV_y}{dx}\Delta x + p_y \Delta x$$

$\Sigma F_z = 0$ Same sort of result

$$\boxed{\frac{dV_y}{dx} = -p_y} \quad \text{and} \quad \boxed{\frac{dV_z}{dx} = -p_z}$$



$$\Sigma M_x = 0 = -M_x + M_x + \frac{dM_x}{dx}\Delta x + m_x \Delta x \Rightarrow \boxed{\frac{dM_x}{dx} = -m_x}$$

$$\Sigma M_y = 0 = \underbrace{-M_y}_{\text{left}} + \underbrace{M_y + \frac{dM_y}{dx}\Delta x}_{\text{right}} + \underbrace{m_y \Delta x}_{\text{applied load}} - \underbrace{\left(V_z + \frac{dV_z}{dx}\Delta x\right)}_{\text{Force}}\underbrace{\Delta x}_{\text{dist}} - \underbrace{p_z \frac{\Delta x^2}{2}}_{\text{force}\cdot\text{dist}}$$

$$\boxed{\frac{dM_y}{dx} = -m_y + V_z}$$

and likewise

$$\boxed{\frac{dM_z}{dx} = -m_z - V_y}$$