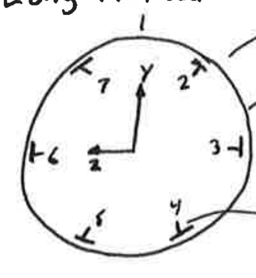
4.10 Long Method



$\varnothing = 100$ in

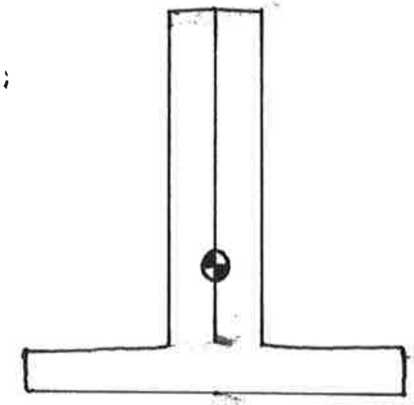
$t = 0.1$ in

0.25 | 0.5 | 0.25

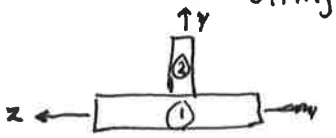
1.75^{in}

0.25^{in}

$E = 10 \times 10^6$ psi

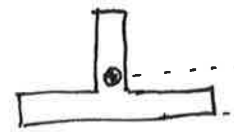


① Find Centroid of stringer.

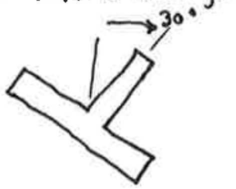


part	A	$\bar{y}^*$	$\bar{y}A$	$\bar{z}_i$	$\bar{z}A$
1	0.5	0	0	0	0
2	0.875	1.0	0.875	0	0
	$A^* = 1.375$ in²		0.875		0

$$\bar{y}^* = \frac{0.875}{1.375} = 0.636 \qquad \bar{z}^* = 0$$



② MoI of 30° rotated stringer



$$I^r_{yy} = \frac{I_{yy} + I_{zz}}{2} + \frac{I_{yy} - I_{zz}}{2} \cos 2\theta$$

Find I_{yy} and I_{zz} in non rotated frame

part	y'	z'	$I_{y_0y_0}$	$I_{z_0z_0}$	A	y'^2A	z'^2A
1	0	0	0.1667	0.002604	0.5	0	0
2	1.0	0	0.018229	0.2233	0.875	0.825	0
			0.184889				

$$I_{yy} = \left(I_{y_0y_0} + z'^2A\right) - \bar{z}^{*2}A^*$$

$$= 0.184889 \text{ in}^4$$

$$I_{zz} = \left(I_{z_0z_0} + y'^2A\right) - \bar{y}^{*2}A^*$$

$$= 0.225904 + 0.825 - 0.636^2 \cdot 1.375 = 0.4947 \text{ in}^4$$

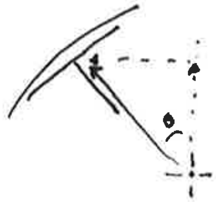
$\theta = 30°$

$$I^r_{yy} = 0.3398 + -0.07745 = 0.262 \text{ in}^4$$

$$I^r_{zz} = \text{''} - \text{''} = 0.417 \text{ in}^4$$

4.10 continued

(3)



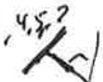
From center of fuselage to stringer centroid

$r = \cancel{100}^{50} - 0.1 - 0.636 = 49.264$ in

$r\cos 30^\circ = 42.664$ in

$r\sin 30^\circ = 24.632$ in

Part 2, 4, 5, 7

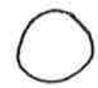


$\hat{I}_{yy} = 0.262$
$\hat{I}_{zz} = 0.417$

part 3, 6

$\hat{I}_{yy} = 0.4947$
$\hat{I}_{zz} = 0.1849$

Part 1



$I = \pi r^3 t$
$= \pi (49.95)^3 (0.1)$
$= 39152$

By inspection, $\bar{y}^* = \bar{z}^* = 0$

Part	$I_{y_0 y_0}$	$I_{z_0 z_0}$	$\bar{y}'$	$\bar{z}'$	$\bar{A}$	$\bar{z}'^2 A$	$\bar{y}'^2 A$
1	39152	39152	0	0	—	0	0
2	0.262	0.417	42.664	-24.632	1.375	834.26	2502.8
3	0.4947	0.1849	0	-49.264		3337.0	0
4	0.262	0.417	-42.664	-24.632	1.375	834.26	2502.8
5	0.262	0.417	-42.684	24.632		834.26	2502.8
6	0.4947	0.1849	0	49.264		3337.0	0
7	0.262	0.417	42664	24.632		834.26	2502.8
	39154	39154				10011.1	10016.2

$I^*_{yy} = 39154 + 10011.1 = 49165 \text{ in}^4$

$I^*_{zz} = $ by inspection (!!) $= 49165 \text{ in}^4$

(4) Reviewing Ex 1.7 gives

$V(x)_{max}$ @ 442 $\approx$ 45740 lb

M_{max} @ 442 $\approx -11.7 \times 10^6$

$$\sigma_{xx} = -\frac{My}{I} = -\frac{-11.7\times10^6 \text{ lbf-in} \;|\; 50 \text{ in}}{49165 \text{ in}^4}$$

$= 11.9$ ksi

(5)

Do you have the theory capable of designing a transport jet fuselage or rocket body? Yes!

4.10 Shortcut Method "Lumped"

$r = 50$ in

$r\cos 30° = 25\sqrt{3}$ (43.3)

$r \sin 30° = 25$

Skip steps ① + ②

③ Compute moments of inertia

part	$I_{y_0y_0}$	$I_{z_0z_0}$	$\bar{y}'$	$\bar{z}'$	A	$\bar{z}'^2 A$	$\bar{y}'^2 A$
1	39152	39152	0	0	—	0	0
2	≈ 0	0	43.3	-25	1.375	859	2578
3	0	0	0	-50	|	3437	
4	0	0	-43.3	-25	|	859	2578
5	0	0	-43.3	25	|	859	2578
6	0	0	0	50	|	3437	
7	0	0	43.3	25	1.375	859	2578
	39152	39152				10310	10312

$I^*_{yy} = 39152 + 10310 = 49462 \text{ in}^4$

$I^*_{zz} = \quad " \quad " \quad = 49462 \text{ in}^4$

④

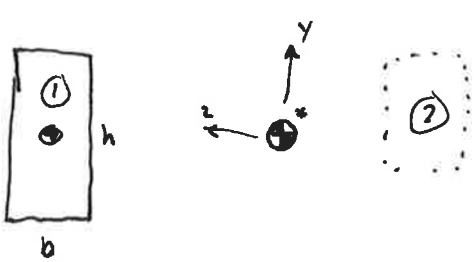
$$\sigma_{xx} = -\frac{My}{I} = -\frac{-11.7\times10^6 \;|\; 50 \text{ in}}{49462 \text{ in}^4}$$

$= 11.83$ ksi

This is 0.5% difference from "exact" methodolgy.

Half the work!

Guideline for when to ignore MoI about centroid of part $I_{y_0y_0}$ etc.



$$I^*_{yy} = \underbrace{I_{y_0y_0}}_{\frac{1}{12}hb^3} + z^2 A$$

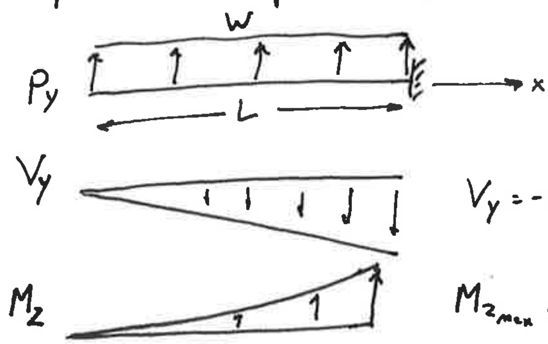
$$\frac{1}{12}hb^3 + z^2 bh$$

When $z > 86\%\, b$, z^2A provides $> 90\%$ of I_{yy}

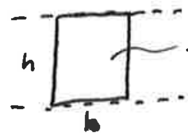
$z > 2.8\, b$, z^2A provides $> 99\%$ of I_{yy}

Ex Compare a composite and solid spar types.

$$f = \frac{\text{Str to Weight Ratio}}{A\ \sigma_{xx_{max}}}$$



$$V_y = -\frac{W}{L}x$$



feasible area of spar limited by thickness mainly

$$M_{z_{max}} = \frac{W}{L}\frac{x^2}{2}\Big|^L = \frac{WL^2}{L\,2} = \frac{WL}{2} \quad \text{or by inspection.}$$

① Solid Al spar 

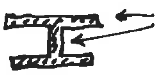
$$I_{zz} = \frac{1}{12}bh^3 \qquad \sigma_{xx} = -\frac{My}{I} = -\frac{WL}{2}\frac{h}{2}\frac{12}{bh^3} = \frac{3WL}{bh^2}$$

Strength to weight ratio is (lecture 7): 400 in

$$f = \frac{400}{A\,\sigma_{xx}} = \frac{400}{3}\frac{bh^2}{WL\,bh} = 133.3\left(\frac{bh^2}{WLbh}\right) = \boxed{133\ \frac{h}{WL}}$$

② Spruce solid spar

$$\sigma_{xx} = \frac{3WL}{bh^2} \qquad StW = 843 \qquad f = \frac{843}{3} = \boxed{281\ \frac{h}{WL}}$$

③ "I" beam Al spar  t = thickness

$$I_{zz} = \frac{1}{12}bh^3 - 2\left(\frac{1}{12}\frac{b-t}{2}(h-2t)^3\right) \quad \text{if } t < \min(b,h) \qquad I_{zz} \approx \frac{h^3t}{12} + \frac{bh^2t}{2} - \cancel{\frac{h^2t^2}{2}} - \cancel{bht^2}$$

if $t = 0.1\,h$

$$\sigma_{xx} = \frac{WL}{2}\frac{h}{2}\cdot\frac{12}{h^3t + 6bh^2t}$$

$$f = \frac{400}{A\,\sigma_{xx}} = \frac{400}{A\,3}\frac{h^3t + 6bh^2t}{WLh} = \frac{400}{3}\frac{h^2t + 6bht}{WL}\cdot\frac{1}{2bt + th}$$

$$= \frac{400}{3}\underbrace{\frac{h + 6b}{2b + h}}_{\approx 6/2 = 3}\frac{h}{WL} \approx \boxed{400\ \frac{h}{WL}}$$

④ Spruce I beam

$$\boxed{f = 843\ \frac{h}{WL}} \quad \text{by inspection}$$

Warning: This is NOT the whole story for spars!

⑤ Carbon Fiber I beam

StW = 4100 $\qquad \boxed{f = 4100}$

⑥ Unidirection Carbon Fiber Spar $\boxed{f = 6400}$

⑦ Fiberglass I beam $\boxed{f = 545}$

Different Boundary Conditions

$$P = t\,b\,F_{c_{cr}} = t\,b\,K\,E\left(\frac{t}{b}\right)^2$$

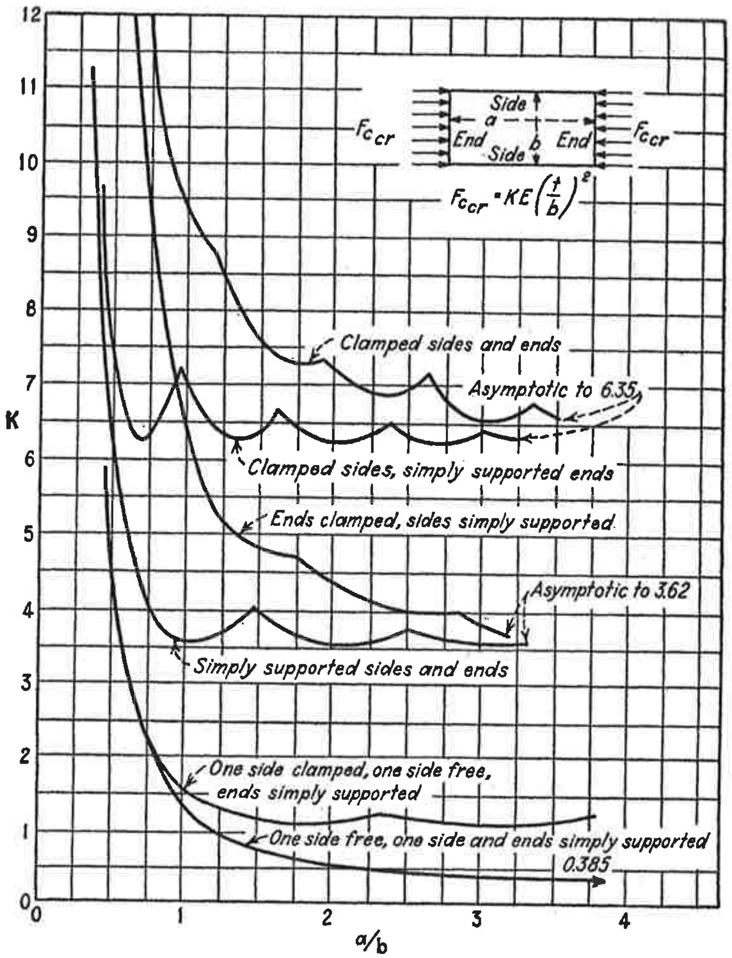
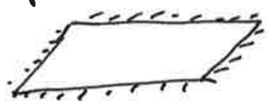
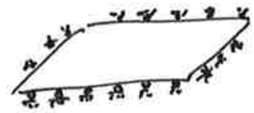


FIG. 14.25.

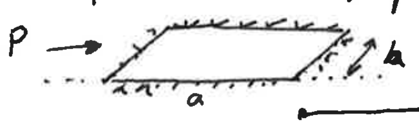
Clamped



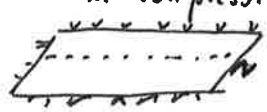
Simply Supported



Ex: Compare the buckling of an upper skin in compression w/wo stringers



vs



acts as simple support

4 x 8' Al 0.040 thick

Simply supported sides + ends

~~Km~~ $a/b = 2 \Rightarrow K = 3.5$

$a/b = 4 \Rightarrow K = 3.5$

$$P = t \cdot b \cdot K \cdot E\left(\frac{t}{b}\right)^2$$

$$= \frac{0.040\,\text{in} \;|\; 48\,\text{in} \;|\; 3.5 \;|\; 10\times10^6\,\text{psi} \;|\; (0.040)^2\,\text{in}}{48^2\,\text{in}^2}$$

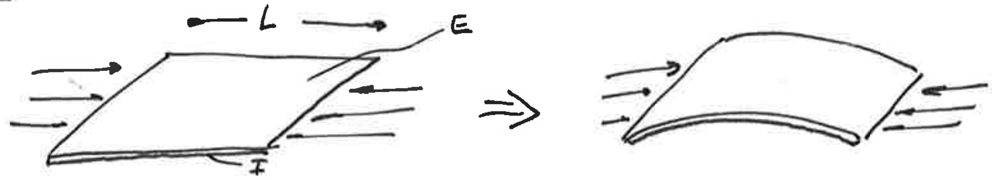
$= 46.6$ lbf

$$P = \frac{0.040 \;|\; 24 \;|\; 3.5 \;|\; 10\times10^6 \;|\; 0.040^2}{24^2}$$

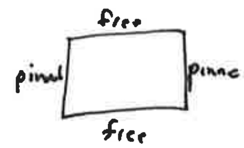
$= 93$ lbf

Buckling of flat plates

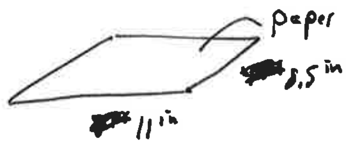
Similar to Euler beam buckling



$$P_{cr} = \frac{\pi^2 E I}{(1-\nu^2) L^2}$$



Ex:



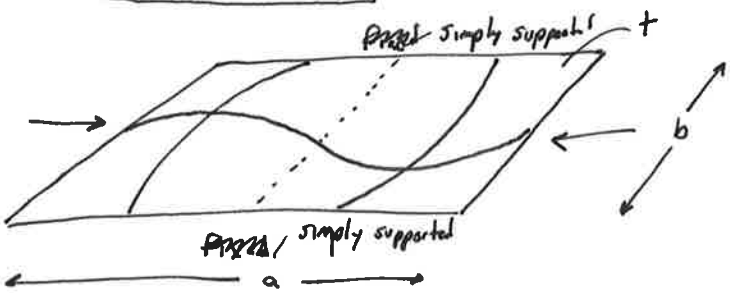
$E = 300\,000$ psi

$t = 0.004$ (I measured with calipers)

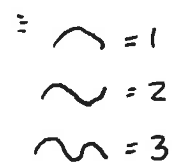
$$I = \frac{1}{12}\, 8.5\, (0.004)^3 = 4.5 \times 10^{-8}$$

$$P_{cr} = \pi^2 \left| \frac{300\,000\ \text{lbf}}{\text{in}^2} \right| 4.5 \times 10^{-8}\ \text{in}^4 \Big/ \left| (1-0.3^2)^2 \right| 11^2\ \text{in}^2 = 0.013\ \text{lb} \ !!$$

Flat plate fixed on all 4 edges



$m \equiv$ # waves in the sheet



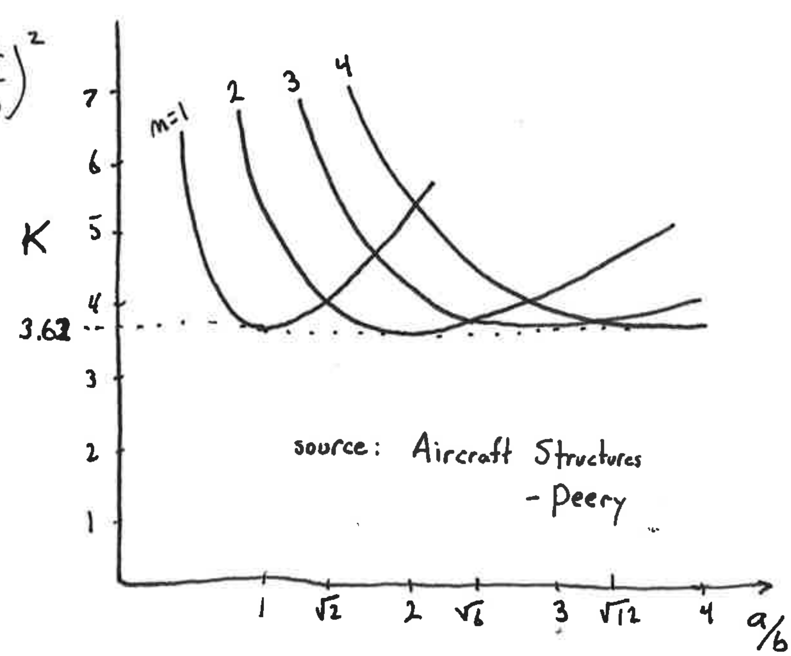
$$P_{cr} = tb \cdot \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{bm}{a} + \frac{a}{bm} \right)^2 \left(\frac{t}{b} \right)^2$$

$$= tb \cdot K \cdot E \cdot \left(\frac{t}{b} \right)^2$$

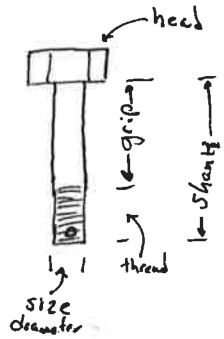
The minimum value of K is 3.62

The plate will buckle at the lowest of all K_m values.

So the buckling shape (m) depends on the a/b ratio.



Bolts



Aircraft bolts are usually identified by

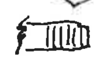
- AN (Army - Navy)
- NAS (National Aircraft Standard
- MS (Military Standard)

Usually a Nickel Chrome Moly Steel (8740)
or a Manganese Silicon Moly Steel (4037)
or an Al
or Stainless steel

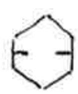
A standard aircraft bolt is sized by its identification #.

AN - (diam) - H - <length> - A

- diameter in 1/16 in
- head drilled?
- thread NOT drilled

e.g. AN-3-16A or AN3-16A is an aircraft bolt of diameter 3/16 in 
without a drilled head 
without a drilled shank 
of length 1 25/32 in with a 1 3/8 in grip.

Materials

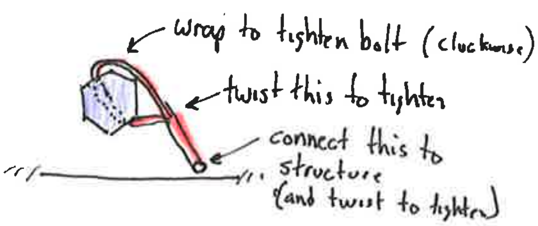
AN3 $\underbrace{\text{DD}}$ 16A 
↖ 2024 Al Alloy

AN3-5
↖ No letter indicates Cd plated steel

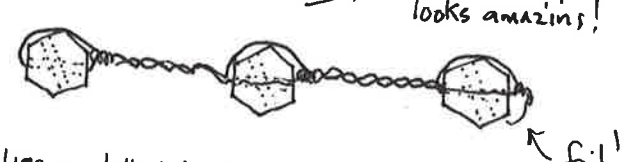
AN4 $\underbrace{\text{C}}$ 5 
↖ Corrosion resistant steel

Drilled Head.

Safety wire: A positive and visible method of preventing bolt rotation.



This is almost an art! A good safety wire job looks amazing!



1) line up bolt holes for clockwise tightening

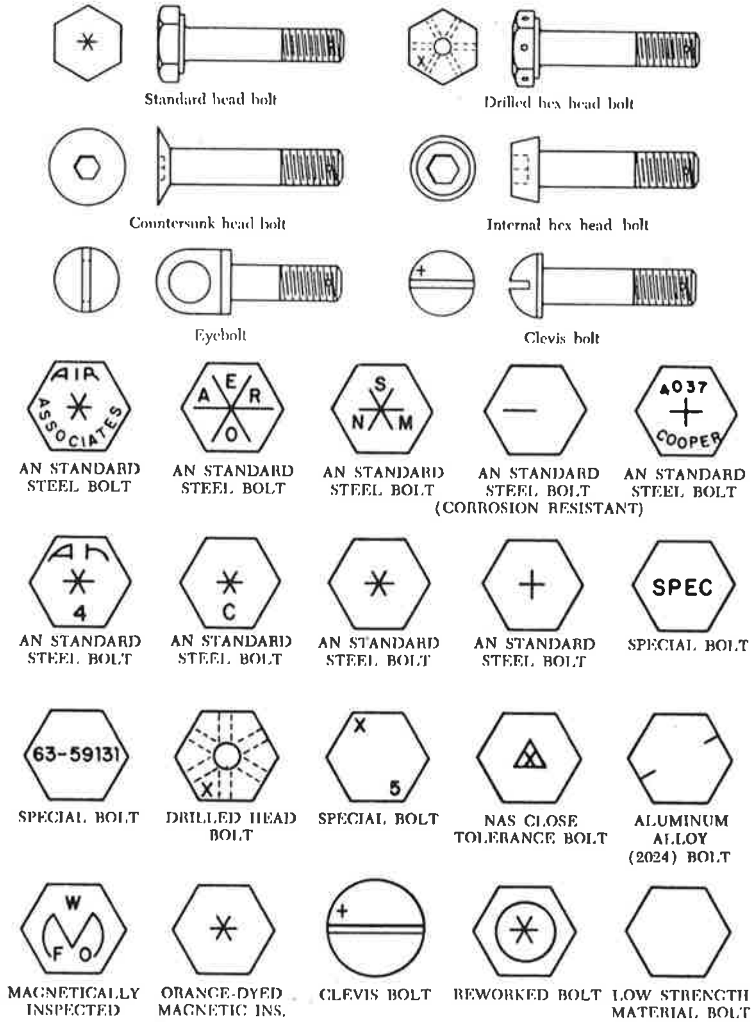


FIGURE 6–1. Aircraft bolt identification.