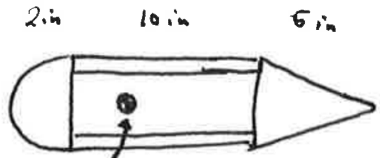
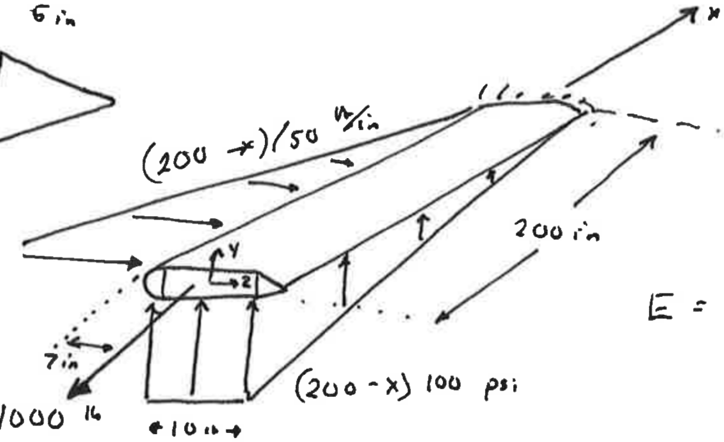
Ex 4.5





① Geometry

From Ex 4.1

$\bar{z}' = 8.14$ in $A = 26.28$

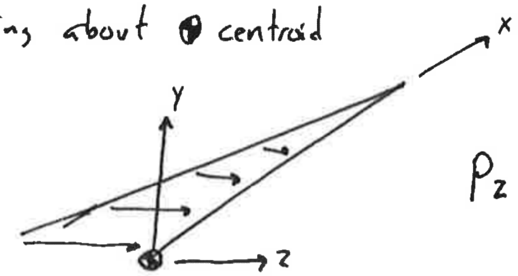
$\bar{y}' = 0$ in

$I_{yy} = 724.69 \text{ in}^4$

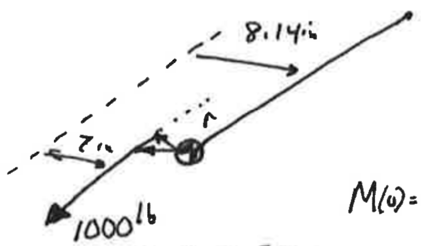
$I_{zz} = 43.77 \text{ in}^4$

$E = 10 \times 10^6$ psi

② Loading about ⊕ centroid

- 

$$p_z = (200 - x) \cdot 50$$

- 

$r_z = -8.14 \text{ in} + 7.0 \text{ in} = -1.14$ in in z direction

$$M(0) = r \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & -1.14 \\ -1000 & 0 & 0 \end{vmatrix} = +1140 \text{ lb in } \hat{y}$$

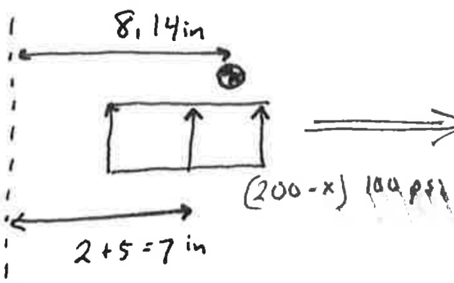
watch out for the sign!

why positive?

negative dir of negative face!!

point load at $x = 0$ (tip) of $+1000$ lb $\hat{x}$

$+1140$ lb in $\hat{y}$

- 

$$p_y = (200 - x) \cdot 100 \text{ psi} \cdot 10 \text{ in} = (200 - x)\, 1000 \ \frac{\text{lb}}{\text{in}}$$

$$M_x = r \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & -1.14 \\ 0 & p_y & 0 \end{vmatrix} = +1.14\, p_y\, \hat{x}$$

Thus $m_x = 1.14\, p_y$

In total, sum all loads..

$$\boxed{\begin{array}{ll} p_x = 0 & P(0) = +1000 \text{ lbf} \\ m_x = 1.14\, p_y & M_y(0) = +1140 \text{ lbf in} \\ p_y = (200 - x)\, 1000 \ \frac{\text{lbf}}{\text{in}} & \end{array}}$$

③ Loading Bar Equ.

- $$\frac{dP}{dx} = -p_x(x) = 0 \quad \text{Integrate} \int_{P(0)}^{P(x)} dP = \int_0^x dx \cdot 0$$

$$P(x) - P(0) = 0 \Rightarrow P(x) = \cancel{P(0)}^{+1000\ lbf} \qquad \boxed{P(x) = +1000\ lbf}$$

- $$\frac{dV_y}{dx} = -p_y(x) = (200-x)/100 \quad \text{Integrate} \int_{V_y(0)}^{V_y(x)} dV_y = \int_0^x \left((200-x)/\tfrac{1}{100}\right) dx$$

$$V_y(x) - \cancel{V_y(0)}^{0} = \frac{1}{100}\left(200x - \frac{x^2}{2}\right)\Big|_0^x$$

$$\boxed{V_y(x) = \frac{1}{100} \cdot \left(200x - \frac{x^2}{2}\right)}$$

- $$\frac{dV_z}{dx} = -p_z(x) = (200-x)/50 \quad \text{Integrate} \int_{V_z(0)}^{V_z(x)} dV_z = \int_0^x (200-x)/50\, dx$$

$$V_z(x) - \cancel{V_z(0)}^{0} = \frac{1}{50}\left(200x - \frac{x^2}{2}\right)$$

$$\boxed{V_z(x) = \frac{1}{50}\left(200x - \frac{x^2}{2}\right)}$$

- $$\frac{dM_x}{dx} = -m_x(x) = 1.14\, p_y = 1.14 \cdot 1000 \cdot (200-x)$$

$$\int_{M_x(0)}^{M_x(x)} dM_x = \int_0^x 1140(200-x)\, dx \qquad M_x(x) - \cancel{M_x(0)}^{0} = 1140(200-x)$$

$$\boxed{M_x(x) = 1140(200-x)}$$

- $$\frac{dM_y}{dx} = -\cancel{m_y(x)}^{0} + \cancel{V_z(x)}^{\frac{1}{50}\left(200x - \frac{x^2}{2}\right)} = \qquad M_y(x) - \cancel{M_y(0)}^{1140\ lbf\,in} = 50\left(200x - \frac{x^2}{2}\right)$$

$$\boxed{M_y(x) = 1140\ lbf\,in + \frac{1}{50}\left(200x - \frac{x^2}{2}\right)}$$

- $$\frac{dM_z}{dx} = -\cancel{m_z(x)}^{0} - V_y(x) = -\frac{1}{100}\left(200x - \frac{x^2}{2}\right)$$

$$M_z(x) - \cancel{M_z(0)}^{0} = -\frac{1}{100}\left(\frac{200x^2}{2} - \frac{x^3}{6}\right) \qquad \boxed{M_z(x) = -\frac{1}{100}\left(\frac{200x^2}{2} - \frac{x^3}{6}\right)}$$

④ Stress Functions + Strain Functions

$$\sigma_{xx} = \frac{P}{A} - \left(\frac{M_z I_{yy} + M_y \cancel{I_{yz}}^{0}}{I_{yy} I_{zz} - \cancel{I_{yz}^2}^{0}}\right) y + \left(\frac{M_y I_{zz} + M_z \cancel{I_{yz}}^{0}}{I_{yy} I_{zz} - \cancel{I_{yz}^2}^{0}}\right) z$$

Cross off terms that are zero

$$\sigma_{xx} = \frac{P}{A} - \frac{M_z}{I_{zz}} y + \frac{M_y}{I_{yy}} z$$

$$\boxed{\sigma_{xx} = \frac{+1000 \text{ lbf}}{26.28 \text{ in}^2} - \frac{-\overset{1}{\cancel{1000}}\left(100x^2 - \frac{x^3}{6}\right)}{43.77 \text{ in}^4} y + \frac{1140 + \overset{1}{\cancel{50}}\left(200x - \frac{x^2}{2}\right)}{724.69 \text{ in}^4} z}$$

$$\boxed{\varepsilon_{xx} = \frac{\sigma_{xx}}{E}}$$

⑤ Evaluate at specific points

$(x, y, z) = (200, 2, 0)$ ~~Not on the part!!~~ $\boxed{0}$ — $\frac{+1000}{26.28} + \frac{1000}{43.77}\left(100 \cdot 200^2 - \frac{200^3}{6}\right) z$

$= 1270$ psi

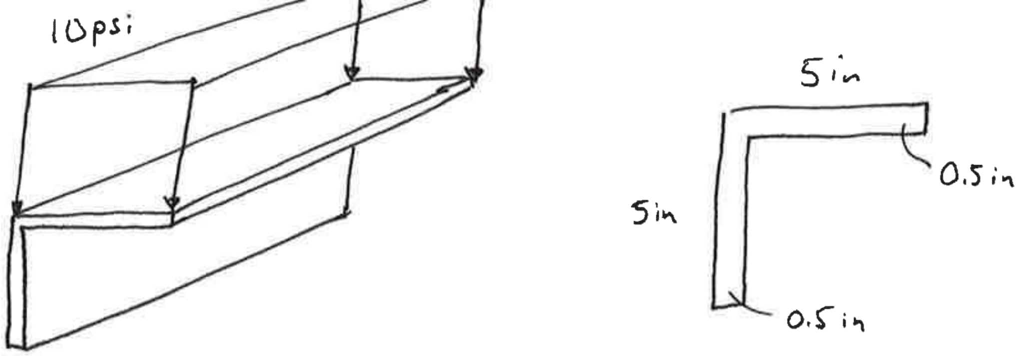
$(x, y, z) = (200, -2, 0)$ ~~" " " "~~ -1195 psi $\boxed{0}$

~~$(x,y,z) = (200, 0, 8.86) = 38$ psi~~

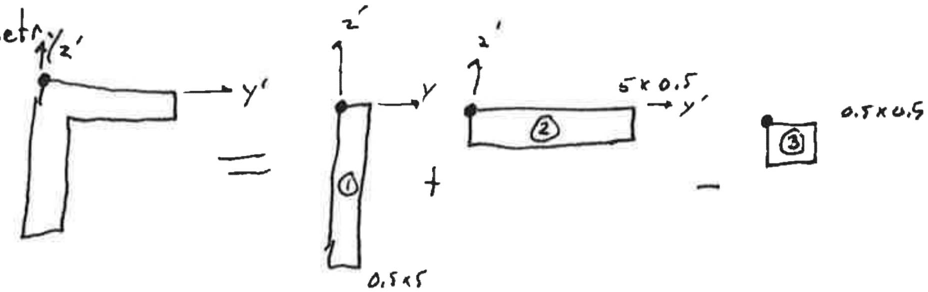
$(x, y, z) = (200, 0, 8.86) = \text{TE} =$ ~~38 psi~~ 56 psi

$(x, y, z) = (200, 0, -8.14) = \text{LE} = 20.7$ psi

Ex 4.17



① Geometry



part	$\bar{y}'$	$\bar{z}'$	A	$\bar{y}'A$	$\bar{z}'A$	$I_{y_0y_0}$	$I_{z_0z_0}$	$\bar{y}'^2A$	$\bar{z}'^2A$
1	0.25	-2.5	2.5	0.625	-6.25	5.208	0.05208	0.15625	by inspection $= I_{y_0y_0}$
2	2.5	-0.25	2.5	6.25	-0.625	0.05208	5.208	15.625	
-3	0.25	-0.25	-0.25	-0.0625	0.0625	-0.005208	-0.005208	-0.0156	
			$\boxed{4.75}$	6.8125	6.8125	5.255	5.255	15.77	15.77

$$\bar{y}^* = \frac{6.8125}{4.75} = \boxed{1.43 \text{ in}} \qquad \bar{z}^* = \frac{6.8125}{4.75} = \boxed{1.43 \text{ in}}$$

$$I_{yy}^* = I_{y_0y_0} + \bar{z}'^2A - \bar{z}^{*2}A^* = 5.255 + 15.77 \cdot 4.75 - 1.43^2\, 4.75$$

$$= \boxed{70.45 \text{ in}^4}$$

$$I_{zz}^* = I_{z_0z_0} + \bar{y}'^2A - \bar{z}^{*2}A^* = 5.255 + 15.77 \cdot 4.75 - 1.43^2\, 4.75$$

$$= \boxed{70.45 \text{ in}^4}$$

what about I_{yz}?
Not zero!

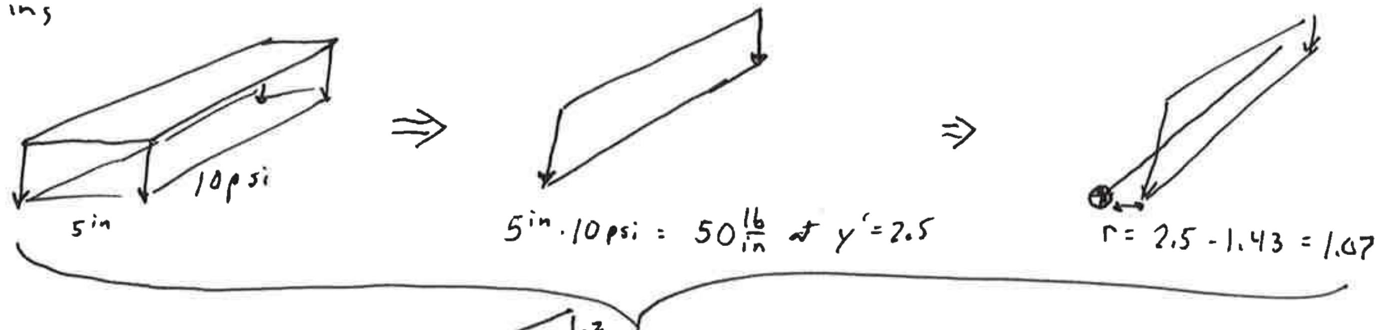
part	$\bar{y}'\bar{z}'A$
1	-1.56
2	-1.56
3	0.0156
	-3.11

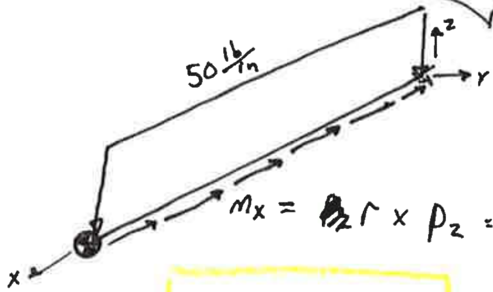
$$I_{yz}^* = \cancel{I_{y_0z_0}}^{\,0} + \bar{y}'\bar{z}'A - y^*z^*A^*$$

$$= -3.11 - 1.43^2\, 4.75$$

$$= \boxed{-12.8 \text{ in}^4}$$

② Loading





$$m_x = r \times p_z = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1.07 & 0 \\ 0 & 0 & -50 \end{vmatrix} = -53.5 \ \text{lbf}\ \hat{x}$$

$$\text{lbf} = \frac{\text{lbf}\cdot\text{in}}{\text{in}}$$

$$\boxed{\begin{aligned} m_x &= -53.5 \ \tfrac{\text{lbf}\cdot\text{in}}{\text{in}} \\ p_z &= -50 \ \text{lb/in} \end{aligned}}$$

$$\frac{dP}{dx} = -\cancel{p_x(x)}^{\,0} \Rightarrow P(x) - \cancel{P(L)}^{\,0} = 0 \Rightarrow \boxed{P(x) = 0}$$

$$\frac{dV_y}{dx} = -\cancel{p_y(x)}^{\,0} \Rightarrow V_y(x) = 0$$ Book solution is Vy=1000 lbf (There is a point load at the tip)

$$\frac{dV_z}{dx} = -p_z(x) = -50 \ \tfrac{\text{lb}}{\text{in}} \Rightarrow \int_{\cancel{V_z(L)}^{\,0}}^{V_z(x)} dV_z = \int_L^x -50 \ \tfrac{\text{lb}}{\text{in}} \, dx \Rightarrow V_z(x) = -50x + 50 \cdot \cancel{L}^{\,100}$$

$$\boxed{V_z(x) = -50x + 5000}$$

$$\frac{dM_x}{dx} = -m_x = 53.5 \Rightarrow \int_{M_x(L)}^{M(x)} dM_x = \int_L^x 53.5 \, dx$$

$$M_x(x) - \cancel{M_x(L)}^{\,0} = 53.5x - 53.5\cancel{L}^{\,100}$$

$$\boxed{M_x(x) = 53.5x - 5350.0}$$

$$\frac{dM_y}{dx} = -\cancel{m_y}^{\,0} + V_z$$

$$= 0 - 50x + 5000 \Rightarrow \int_{\cancel{M_y(L)}^{\,0}}^{M_y(x)} dM_y = \int_L^x (-50x + 5000)\, dx$$

$$M_y(x) = -\frac{50x^2}{2} + 5000x + \frac{50\cancel{L^2}^{\,100}}{2} - 5000\cancel{L}^{\,100}$$

$$= \boxed{-25x^2 + 5000x - 25000}$$

$$\frac{dM_z}{dx} = -\cancel{m_z(x)}^{\,0} - \cancel{V_y(x)}^{\,0}$$

$$\boxed{M_z(x) = 0}$$

③ stress

$$\sigma_{xx} = \frac{\cancel{P}^{\,0}}{A} - \left(\frac{\cancel{M_z}^{\,0} I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right) y + \left(\frac{M_y I_{zz} + \cancel{M_z}^{\,0} I_{yz}}{(I_{yy} I_{zz} - I_{yz}^2)}\right) z$$

$$= \frac{-M_y I_{yz}\, y}{I_{yy} I_{zz} - I_{yz}^2} + \frac{M_y I_{zz}\, z}{I_{yy} I_{zz} - I_{yz}^2} = \frac{M_y}{I_{yy} I_{zz} - I_{yz}^2} \cdot \left(-I_{yz}\, y + I_{zz}\, z\right)$$

$$\sigma_{xx} = \boxed{\frac{-25x^2 + 5000x - 25000}{70.45^2 - 12.8^2}\left(+12.8\, y + 70.45\, z\right)}$$