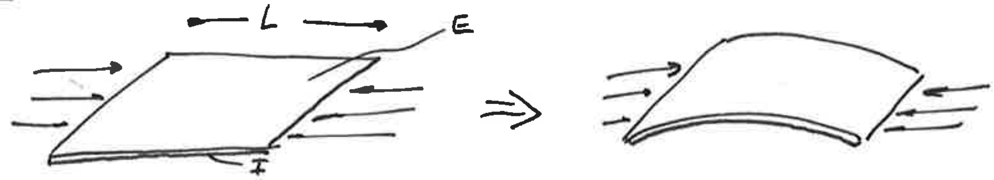
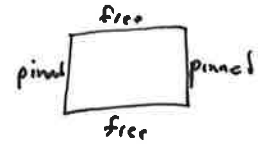
Buckling of flat plates

Similar to Euler beam buckling



$$P_{cr} = \frac{\pi^2 E I}{(1-\nu^2)L^2}$$



Ex:

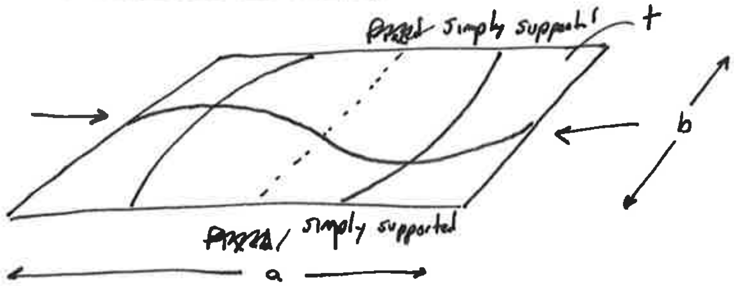
$E = 300\,000$ psi

$t = 0.004$ (I measured with calipers)

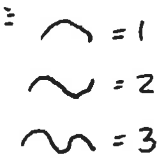
$$I = \frac{1}{12}\, 8.5\,(0.004)^3 = 4.5 \times 10^{-8}\ in^4$$

$$P_{cr} = \frac{\pi^2 \cdot 300\,000\ \frac{lbf}{in^2} \cdot 4.5\times 10^{-8}\ in^4}{(1-0.3^2) \cdot 11^2\ in^2} = 0.013\ ^{lb}\ !!$$

Flat plate fixed on all 4 edges



$m \equiv$ # waves in the sheet



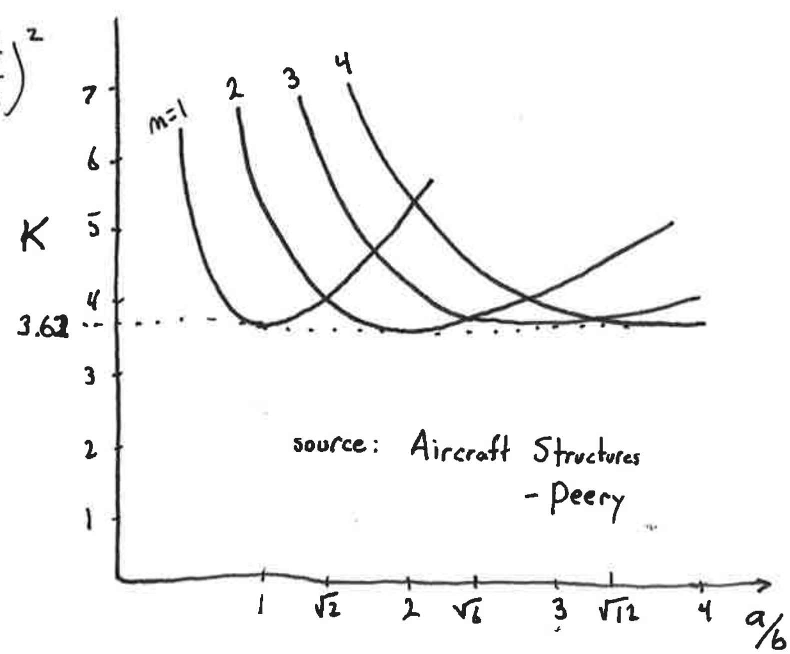
$$P_{cr} = tb \cdot \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{bm}{a} + \frac{a}{bm}\right)^2 \left(\frac{t}{b}\right)^2$$

$$= tb \cdot K \cdot E \cdot \left(\frac{t}{b}\right)^2$$

The minimum value of K is 3.62

The plate will buckle at the <u>lowest</u> of all K_m values.

So the buckling shape (m) depends on the a/b ratio.



Different Boundary Conditions

$$P = tb\,F_{c_{cr}} = tb\,K\,E\left(\frac{t}{b}\right)^2$$

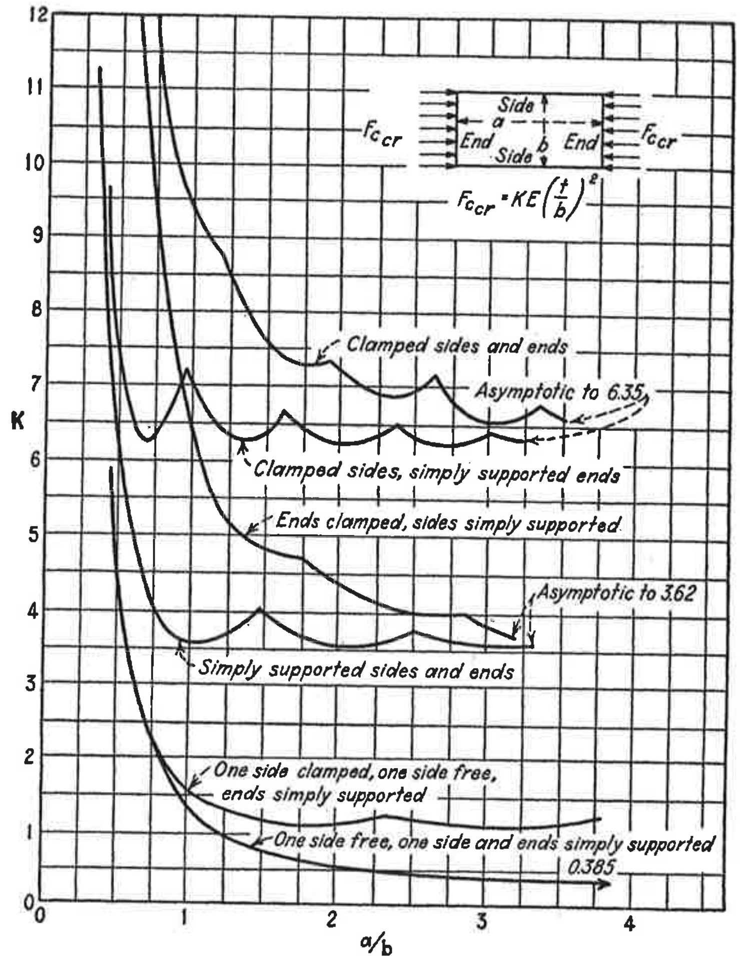
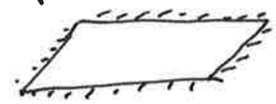
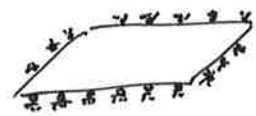


Fig. 14.25.

Clamped

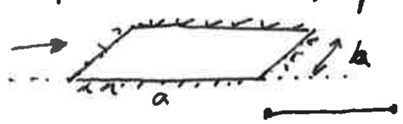


Simply Supported

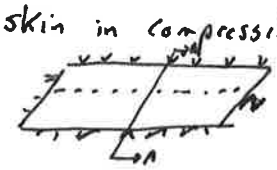


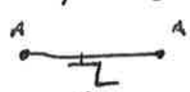
Ex: Compare the buckling of an upper skin in compression w/wo stringers

P



VS





acts as simple support $\Rightarrow$

4 x 8' Al 0.040 thick

Simply supported sides + ends

$a/b = 2 \Rightarrow K = 3.5$

$a/b = 4 \Rightarrow K = 3.5$

$$P = t \cdot b \cdot K \cdot E \left(\frac{t}{b}\right)^2$$

$$= 0.040\,\text{in} \,\big|\, 48\,\text{in} \,\big|\, 3.5 \,\big|\, 10\times10^6\,\text{psi} \,\big|\, \frac{(0.040)^2\,\text{in}}{48^2\,\text{in}^2}$$

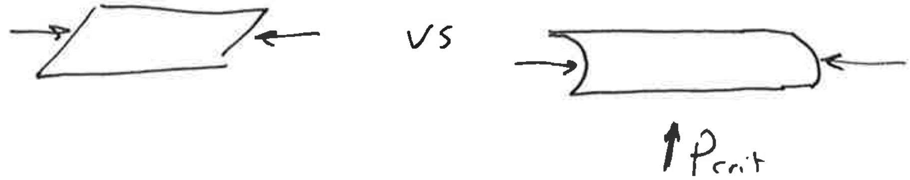
$$= 46.6\ \text{lbf}$$

$$P = 0.040 \,\big|\, 24 \,\big|\, 3.5 \,\big|\, 10\times10^6 \,\big|\, \frac{0.040^2}{24^2}$$

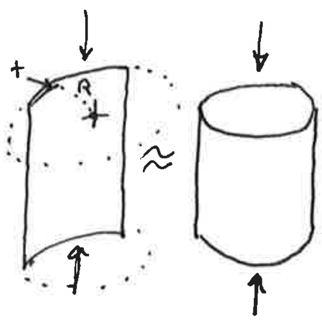
$$= 93\ \text{lbf}$$

Curved Sheet Buckling

We can immediately see a difference in the buckling load from a flat sheet to a curved sheet.

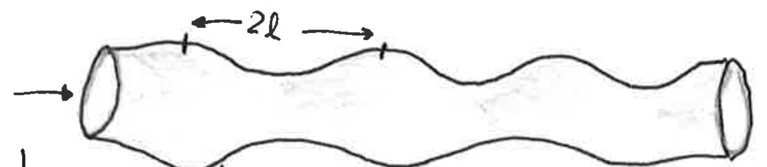


A curved sheet is a special case of the tube buckling.



Theory says: <u>Incorrectly</u>

$$\sigma_{xx_{crit_{theory}}} = 0.606\, E \frac{t}{R} \quad \text{and} \quad \ell = 1.73\sqrt{Rt}$$



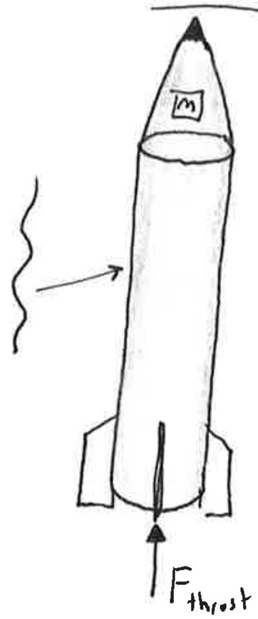
Actual experiments show $\sigma_{xx_{crit_{ex}}} \approx 40\%\ \sigma_{xx_{crit_{theory}}}$

A better approximation is.

$$-\sigma_{xx_{crit}} = 9E\left(\frac{t}{R}\right)^{1.6} + 0.16\left(\frac{t}{L}\right)^{1.3} \quad \text{when} \quad 500 < \frac{R}{t} < 3000$$

$$\text{and} \quad 0.1 < \frac{L}{R} < 2.5$$

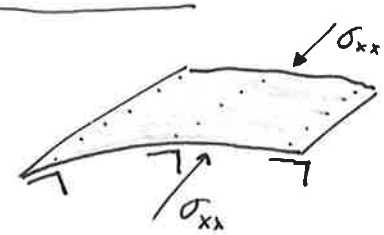
This is a <u>critical</u> design constraint for long, thin, tubular structures



<u>Rockets</u>

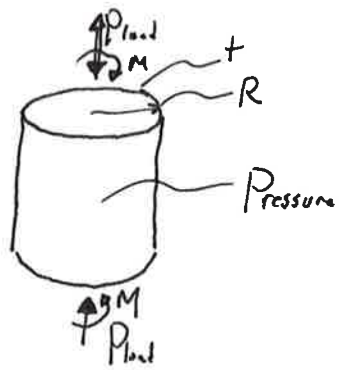
youtu.be/nUjpVBktTAI
Shell buckling test of SLS at NASA Marshall

<u>Aircraft Skins</u>



Atlas and Centaur Rockets

The axial stress in a rocket body depends on loading and internal pressure. and bending.



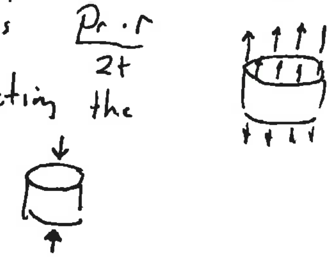
$$\sigma_{xx} = \frac{P}{A} + \frac{\text{Pressure} \cdot r}{2t} + \frac{My}{I}$$

Given a high enough pressure, the σ_{xx} stress term can be + or −.

Lifted!

The Centaur Rocket used internal pressurization as a concept to minimize structural weight. The skins were so thin that the rocket could not support itself without internal pressurization of the tanks.

Increasing pressure creates a σ_{xx} stress $\frac{Pr \cdot r}{2t}$ counteracting the load P



However, the rocket system had many public failures. A decision was made (with the urging of W. vB) to a self supporting system .... the Saturn V

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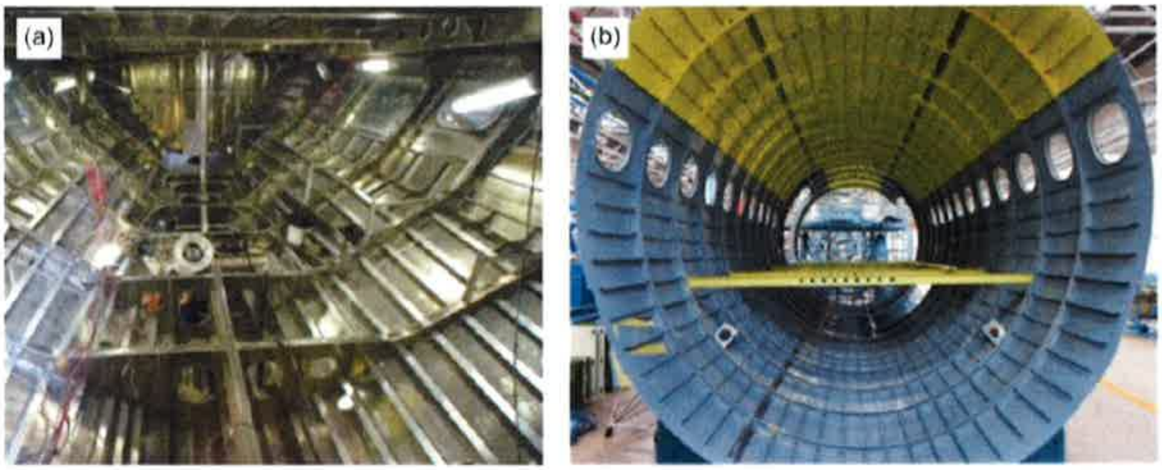
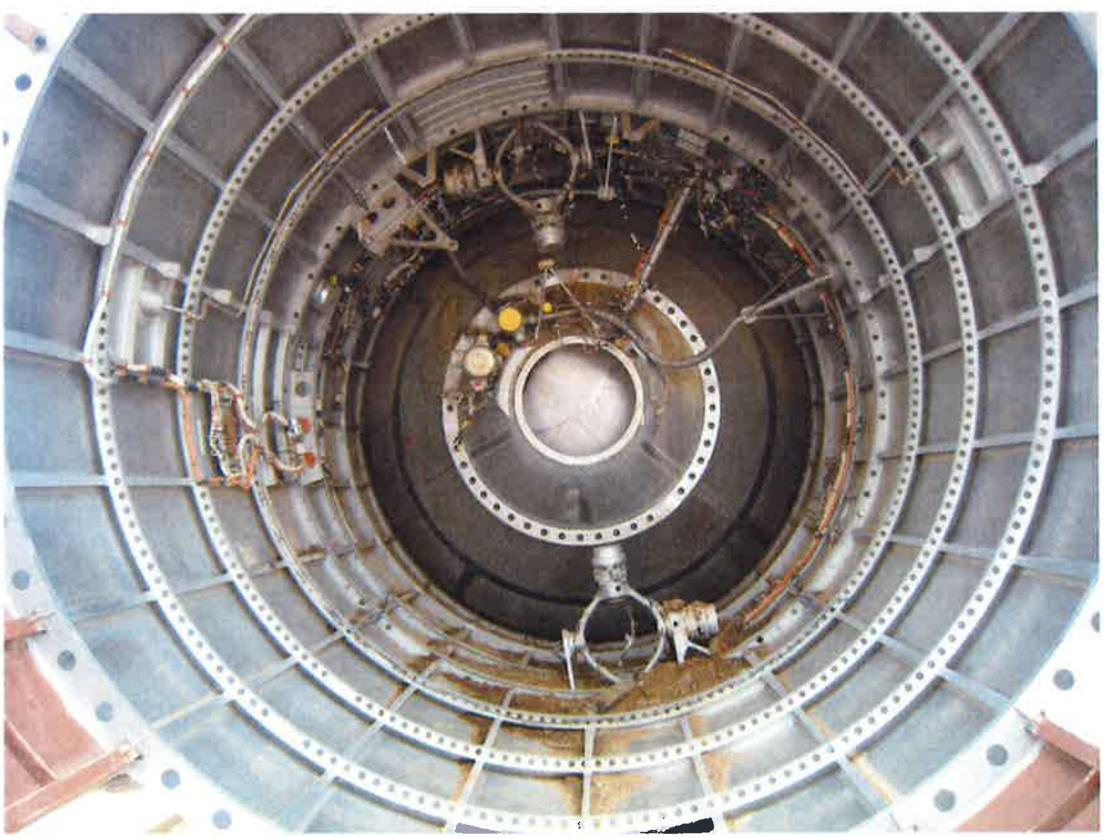
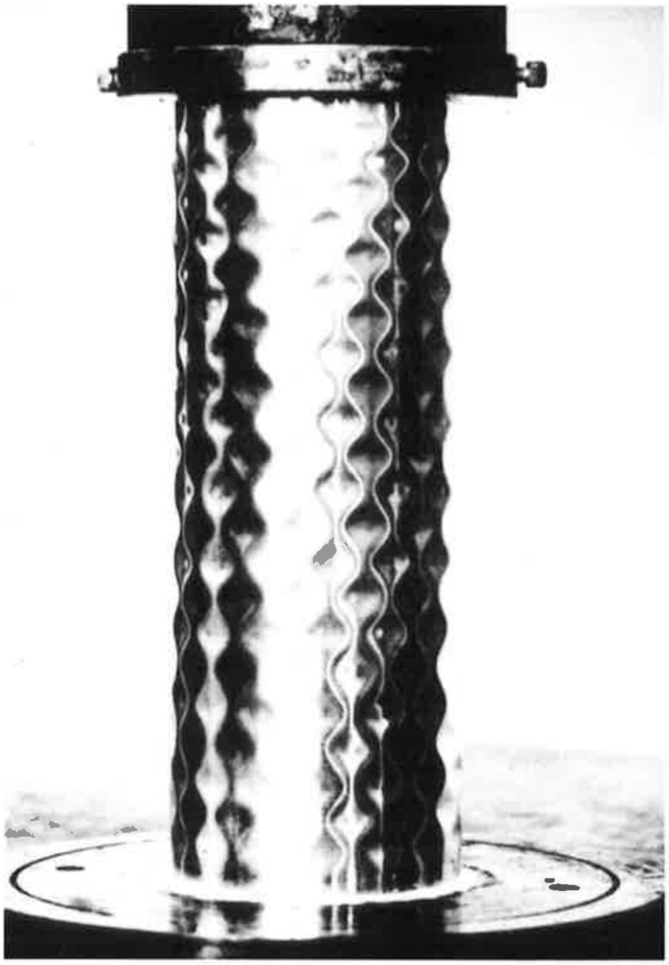


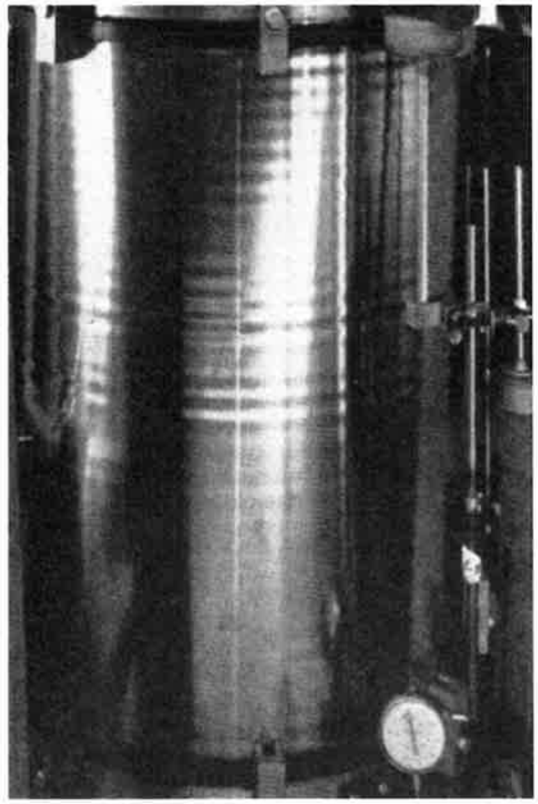
Figure 1.2: Structures including stiffened panels: (a) boat hull structure [1] and (b) an airplane fuselage [2].

Rui Miguel Ferreira Paulo, "Modelling of friction stir welding processes and their influence on the structural behaviour of aluminium stiffened panels", Ph.D dissertation, Dept. of Mechanical Engineering, University of Aveiro, 2015



--Robert Ball





Buckling with small hole