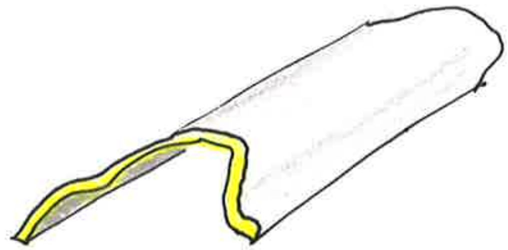
Torsion in Thin Walled Sections

What about m_x?

$$\frac{dM_x}{dx} = -m_x(x)$$

Types of sections

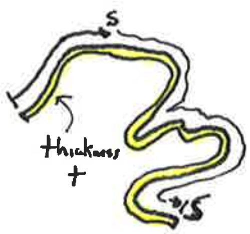
Closed



Open

Membrane Analysis for thin homogeneous Open Sections ONLY

(do not use for closed sections!)
do not use for multi material sections.
do not use for thick sections (i.e. σ_{xs} uniform across wall)

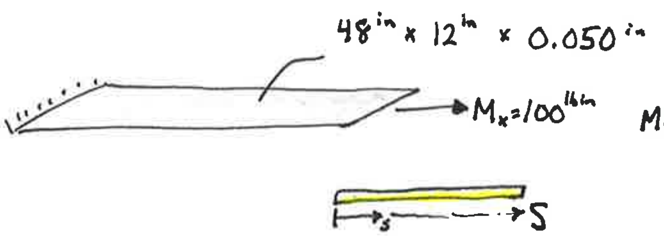


$$J = \frac{S t^3}{3}$$

Maximum shear stress

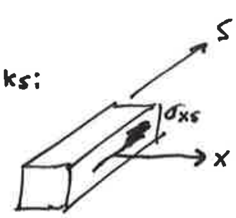
$$\sigma_{xs} = \frac{M_x t}{J} = \frac{M_x 3}{S t^2}$$

Ex:

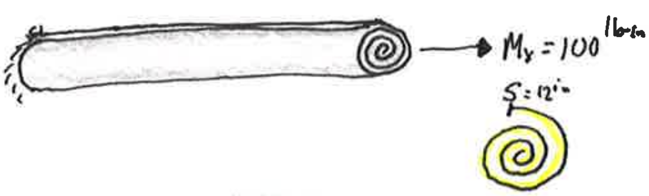


Maximum shear stress

$$\sigma_{xs} = \frac{100^{\text{lb-in}} \cdot 3}{12^{\text{in}} \cdot 0.050^{2}\,\text{in}^2} = 10^{\text{ksi}}$$



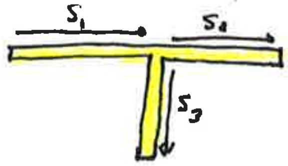
Ex: Roll up the above sheet



$$\sigma_{xs} = (\cdots) = 10^{\text{ksi}}$$

The configuration does not matter!

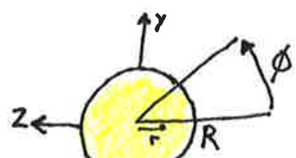
multisection Open sections



$$J = \Sigma \frac{S t^3}{3} = \frac{S_1 t^3}{3} + \frac{S_2 t^3}{3} + \frac{S_3 t^3}{3}$$

$$\sigma_{xs} = \frac{M_x t}{J}$$

Circular Bar Torsion



$$\varepsilon_{x\theta} = r \frac{\phi(L)}{L}$$

$$\varepsilon_{x\theta} = \frac{R\,\phi(L)}{L} \Rightarrow \varepsilon_{x\theta}(r) = \frac{r}{R}\varepsilon_{x\theta}(b) \Rightarrow \sigma_{x\theta} = \underbrace{G}_{\text{shear modulus}} \varepsilon_{x\theta}$$

$$\sigma_{x\theta}(r) = \frac{r}{R}\sigma_{x\theta}(R)$$

Static equilibrium

$$dA = r\,dr\,d\theta$$

$$\Sigma M_x = 0 = -M_x + \int_A \sigma_{x\theta}\, r\, dA \Rightarrow M_x = \int_0^{2\pi}\int_{R_{inner}}^{R_{out}} \overbrace{\frac{r}{R_{out}}\sigma_{x\theta}(R)}^{\text{stress}}\overbrace{r}^{\text{distance}}\overbrace{r\,dr\,d\theta}^{dA}$$

define J : polar moment of inertia $\equiv \int_0^{2\pi}\int_{R_{in}}^{R} r^3\,dr\,d\theta$ $\qquad M_x = \frac{\sigma_{x\theta}(R_o)}{R}\int_0^{2\pi}\int_{R_{in}}^{R_{out}} r^3\,dr\,d\theta = \frac{\sigma_{x\theta}}{R} J$

solve for $\sigma_{x\theta}$

$$\sigma_{x\theta}(R) = \frac{M_x R}{J}$$ substitute above $$\sigma_{x\theta}(r) = \frac{r}{R}\sigma_{x\theta}(R) = \frac{M_x R}{J}\frac{r}{R}$$

$$\boxed{\sigma_{x\theta}(r) = \frac{M_x r}{J}}$$

with a rotation angle $\boxed{\phi(x) = \frac{M_x x}{JG}}$

Ex:

Thin tube in torsion

$s = 2\pi R$

$$J = \int_0^{2\pi}\int_{R-t}^{R} r^3\,dr\,d\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_{R-t}^{R} d\theta = \frac{r^4}{4}2\pi\Big|_{R-t}^{R} \approx \frac{\pi}{2}\left(4R^3 t - 6R^2t^2 + 4Rt^3 - t^4\right)$$

(higher order terms H.O.T)

$$= 2\pi R^3 t$$ compare with table in book.

$$\sigma_{x\theta_{max}} = \frac{M_x R}{J} = \frac{M_x}{2\pi R^2 t} = \frac{M_x}{S R t} = \frac{M_x 2\pi}{S^2 t}$$ Compare with open section of same S !!

4.8 Thermal

① Temperature $\Delta T = T_2 - T_1 = (-110^F) - (90^F) = -200\ F$ Brrr.....

② Heterogeneous beam with Thermal loads (4.55)

$$\sigma_{xx} = \frac{E}{E_1}\frac{\cancel{P}^{0} + P^T}{A^*} + \underbrace{\frac{M^T}{\cdots} y}_{0} + \underbrace{\frac{M^T}{\cdots}}_{0} z - E\alpha\Delta T$$

$$P^T = \int E\alpha\Delta T\, dA = \sum E_i \alpha_i \Delta T_i A_i \quad \text{and} \quad M_y^T = \sum E\alpha\Delta T \bar{z} A$$

but this is a symmetric beam about the centroid!

③ Calculate P^T

$$P^T = 30\times10^6 \cdot 6.78\times10^{-6} \cdot (-200) \cdot 12 \cdot 0.4375 \quad \text{steel}$$

$$+ 10\times10^6 \cdot 13\times10^{-6} \cdot (-200) \cdot 10 \cdot 7 \cdot 0.1 \quad \text{aluminum}$$

$$= -395570\ ^{lbf}$$

④ Find A^*

From earlier, $A^* = 22.75\ ^{in^2}$

⑤ Stress

$$\sigma_{xx_{steel}} = \frac{30}{10} \cdot \frac{-395570}{22.75} - 30\times10^6 \cdot 6.78\times10^{-6} \cdot (-200)$$

$$= -11.5\ ^{ksi}$$

$$\sigma_{xx_{al}} = \frac{1}{1} \cdot \frac{-395570}{22.75} - 10\times10^6 \cdot 13\times10^{-6} \cdot (-200)$$

$$= 8.6\ ^{ksi}$$

⑥ Visual

Nuts

Goes on the threaded portion of the bolt.....

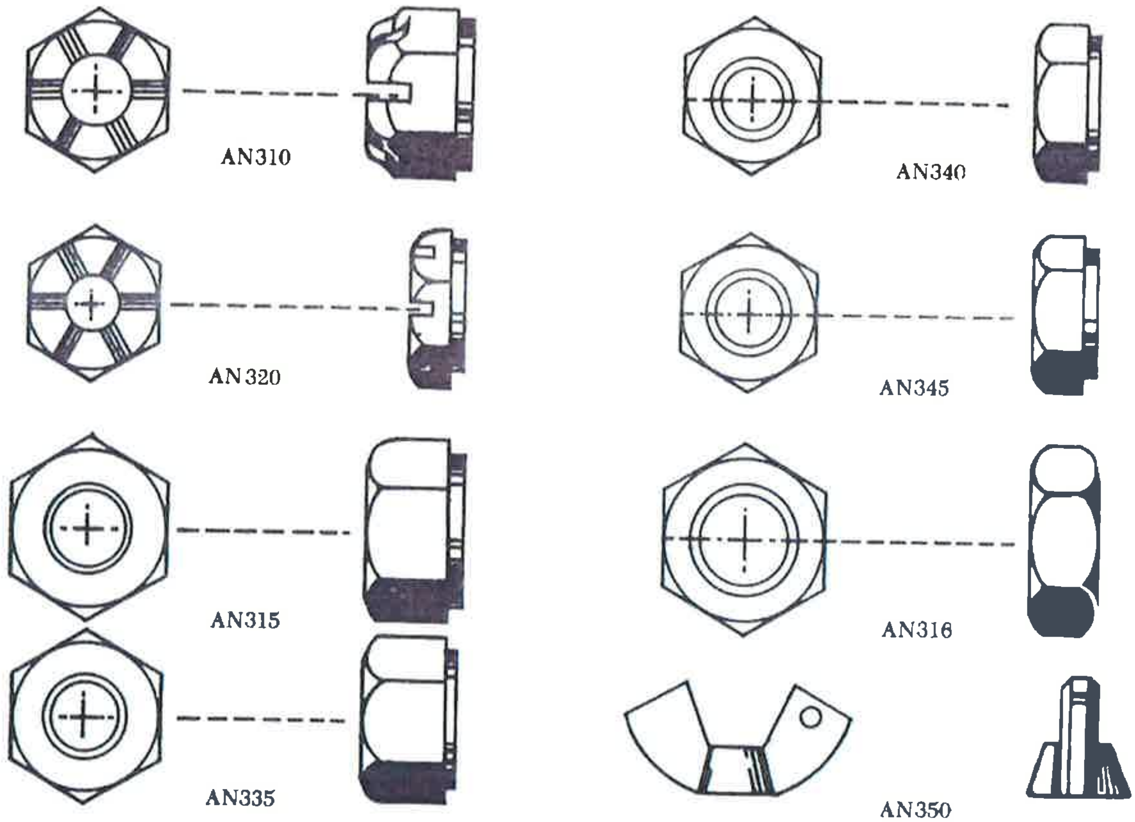
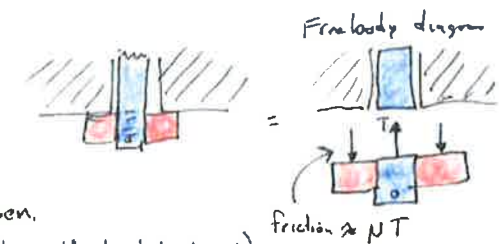


FIGURE 6–6. Non-self-locking nuts.

Non-Self locking

Loosening rotation prevented only by tightness + friction

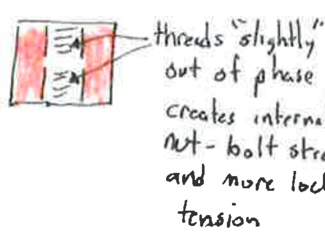


In a high vibration environment (like an airplane), these nuts will loosen.

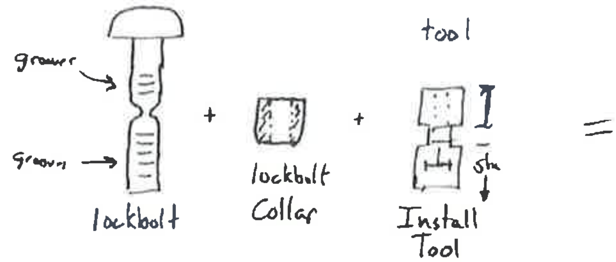
Must be safety wired or pinned. (Hence AN3-5 bolt with drilled shank!)

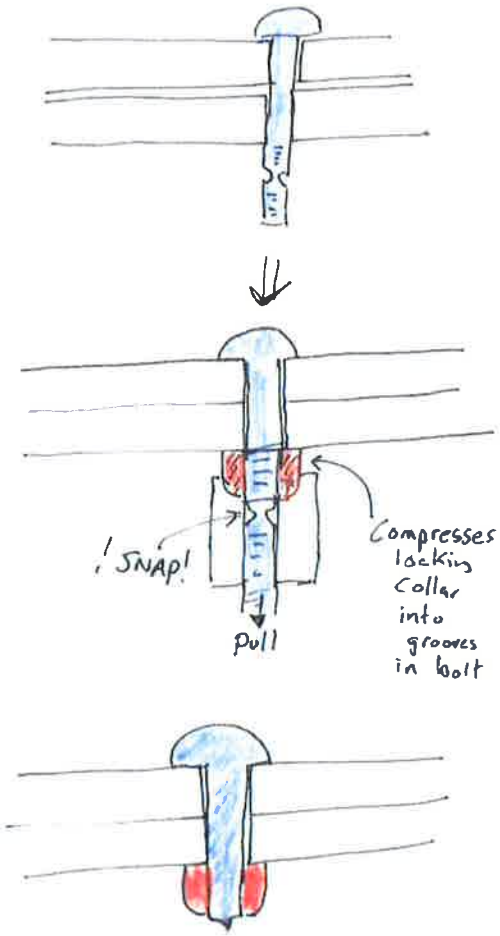
Self locking

Many ways to increase rotational static friction without bolt tension



Lockbolt (bolt + rivet)





ALPP H T 8 8

- ALPP — head type "pan head"
- H — fit "interference"
- T — pin type ↑ Steel
- 8 — diameter 1/32 in
- 8 — grip length 1/16 in

BL 8 4

- BL — blind lockbolt — x "pop" rivet cousin.

LC C 8

- LC — lockbolt Collar
- C — material
- 8 — diameter 1/32 in

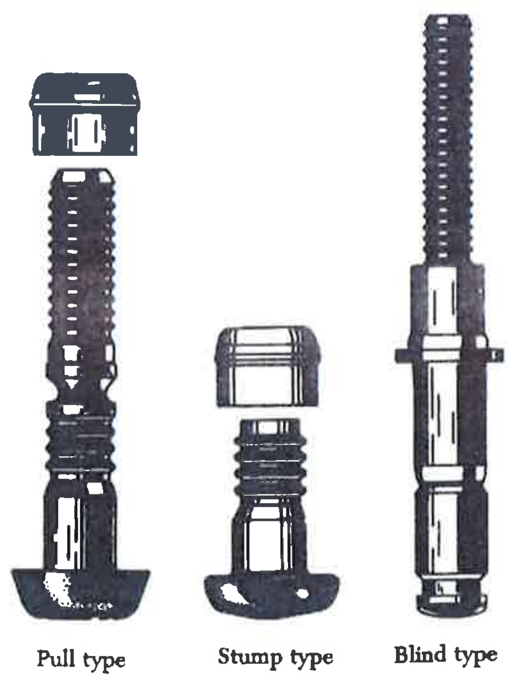
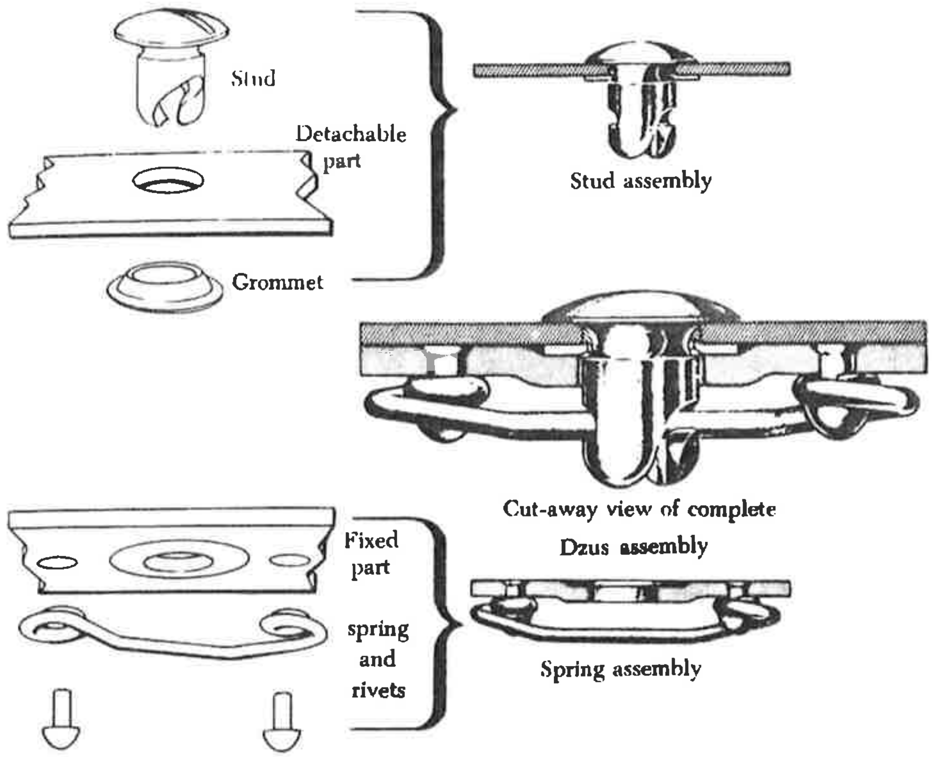


FIGURE 6–2. Lockbolt types.

Dzus "Zeus" / Camloc / Airloc



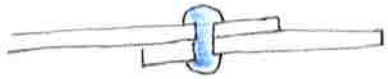
F – Flush head

6½–Body diameter in 16th's of an inch

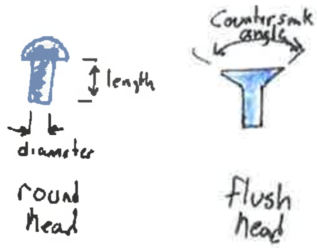
.50 – Length (50/100th's of an inch)

FIGURE 6–18. Dzus identification.

Rivets

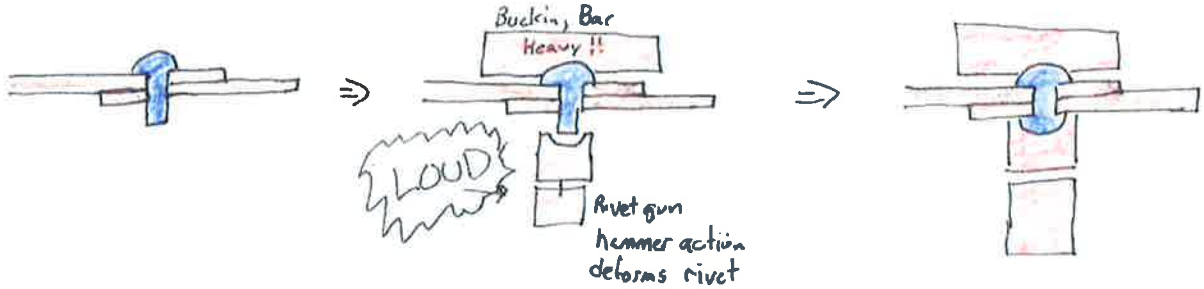


Light and strong joiner of sheet metal
(fast and cheap too!)



tiny.cc/AEM617Rivets
7:40

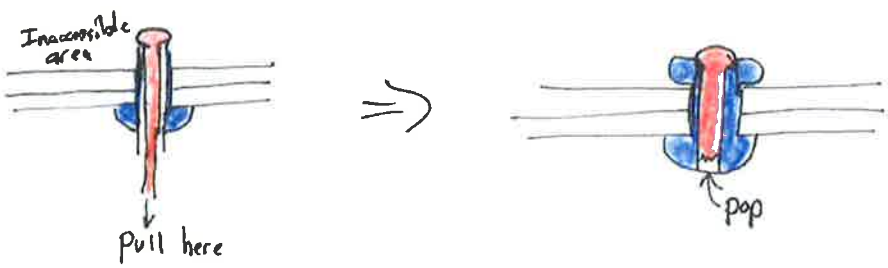
Install:



Heat Treating:

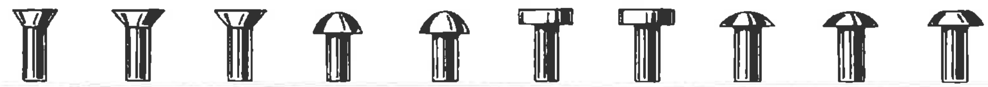
Some rivets require a special heat treating process before and during install.

"Pop" Rivets (call them blind rivets...)



Heavy, expensive, less strength, very convenient!

Rivet ID



Material	Head Marking	AN Material Code	AN425 78° Counter-Sunk Head	AN426 100° Counter-Sunk Head MS20426*	AN427 100° Counter-Sunk Head MS20427*	AN430 Round Head MS20470*	AN435 Round Head MS20613* MS20615*	AN441 Flat Head	AN442 Flat Head MS20470*	AN455 Brazier Head MS20470*	AN456 Brazier Head MS20470*	AN470 Universal Head MS20470*	Heat Treat Before Using	Shear Strength P.S.I.	Bearing Strength P.S.I.
1100	Plain	A	X	X		X			X	X	X	X	No	10000	25000
2117T	Recessed Dot	AD	X	X		X			X	X	X	X	No	30000	100000
2017T	Raised Dot	D	X	X		X			X	X	X	X	Yes	34000	113000
2017T-HD	Raised Dot	D	X	X		X			X	X	X	X	No	38000	126000
2024T	Raised Double Dash	DD	X	X		X			X	X	X	X	Yes	41000	136000
5056T	Raised Cross	B		X		X			X	X	X	X	No	27000	90000
7075-T73	Three Raised Dashes		X	X		X			X	X	X	X	No		
Carbon Steel	Recessed Triangle				X		X MS20613*	X					No	35000	90000
Corrosion Resistant Steel	Recessed Dash	F			X		X MS20613*						No	65000	90000
Copper	Plain	C			X		X	X					No	23000	
Monel	Plain	M			X			X					No	49000	
Monel (Nickel-Copper Alloy)	Recessed Double Dots	C					X MS20615*						No	49000	
Brass	Plain						X MS20615*						No		
Titanium	Recessed Large and Small Dot			MS 20426				X					No	95000	

* New specifications are for Design purposes

FIGURE 6–33. Rivet identification chart.