Shear in advanced beams - part 2

The shear flow derivation has A_s^*, Q_y^*, Q_z^* which are integrated terms as a function of the cross section geometry, materials, and location.

$$A_s^* = \int_0^{s_1} \frac{E}{E_1} t\, ds \qquad Q_y^* = \int_0^{s_1} \frac{E}{E_1} z t\, ds \qquad Q_z^* = \int_0^{s_1} \frac{E}{E_1} y t\, ds$$

These can be a challenge to calculate. But for many actual as-built structures, we can simplify the integrals by assuming a summation of parts.

$$A_s^* = \int_0^{s_1} \frac{E}{E_1} t\, ds = \underbrace{\sum \int_{s_{i-1}}^{s_i} \frac{E_i}{E_1} t\, ds}_{\text{composed of parts}}$$

$$= \underbrace{\sum \frac{E_i}{E_1} \int_{s_{i-1}}^{s_i} t\, ds}_{\text{pull out constants since each piece has constant } E_i} = \sum \frac{E_i}{E_1} A_i$$

$dA = t\,ds$

$$Q_y^* = \int_0^{s_1} \frac{E}{E_1} z t\, ds = \sum \int_{s_{i-1}}^{s_i} \frac{E_i}{E} z t\, ds = \sum \frac{E_i}{E_1} \int_{s_{i-1}}^{s_i} z t\, ds = \sum \frac{E_i}{E} \int_{s_{i-1}}^{s_i} z\, dA$$

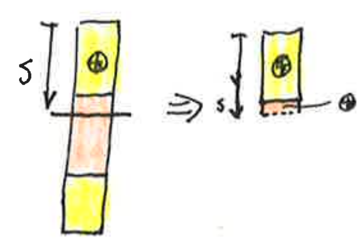
do you recognize $\int z\, dA$? $\bar{z} = \frac{1}{A} \int z\, dA$

$$\boxed{Q_y^* = \sum \frac{E_i}{E_1} A_i \bar{z}_i}$$

Likewise

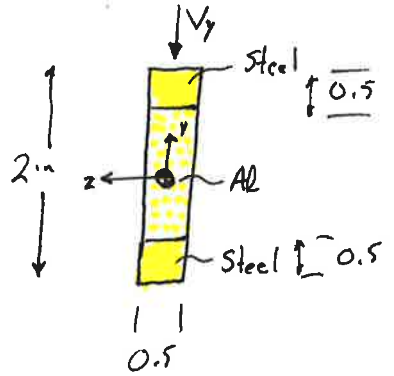
$$\boxed{Q_z^* = \sum \frac{E_i}{E_1} A_i \bar{y}_i}$$

But remember that there may be a partial part at distance s.



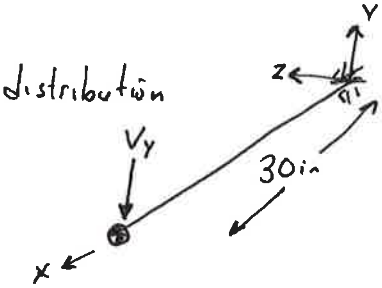
Still complicated, but doable

Ex



plot the shear stress distribution

$E_1 = 10\times10^6$ psi



Part	E/E_1	A_i	$\frac{E}{E_1}A_i$	$I_{z_0z_0}$	$\bar{z}$	$\frac{E}{E_1}\bar{z}A$	$\frac{E}{E_1}(I_{z_0z_0}+y^2A)$
1	3	0.25	0.75	0.005208	0.75	0.5625	0.4375
2	1	0.5	0.5	0.041667	0	0	0.041667
3	3	0.25	0.75	0.005208	-0.75	-0.5625	0.4375
			$A^* = 2.0$			$\bar{z}=0$	I_{zz}^* 0.91665

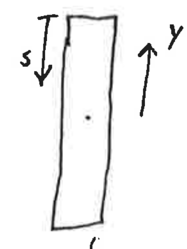
At root:
$M_y = -V_y \cdot 30$

$$q(s_1) = q(0) - p^T \cancel{\frac{A_s^*}{A^*}}^{0} + \left(\frac{(\cancel{-m_z} - V_y - \cancel{m_z^T}) I_{yy}^* + (\cancel{\cdots})}{I_{yy}^* I_{zz}^* - \cancel{I_{yz}^{*2}}^{0}} \right) Q_z^* - (\cancel{\cdots}) Q_y^* + E\cancel{\int \alpha \frac{dT}{dx} t\, ds}^{0}$$

$$= q(0) - \frac{V_y Q_z^*}{I_{zz}^*}$$

$$Q_z^* = \sum \frac{E_i}{E_1} A_i \bar{z}_i = \begin{cases} \underbrace{3}_{E/E}\cdot\underbrace{t\cdot s}_{A_i}\cdot\underbrace{\left(\frac{h}{2}-\frac{s}{2}\right)}_{\bar{z} \text{ since centroid is } 1/2 \text{ part height}} & \text{in upper } 1 \\ 3\cdot t\cdot\frac{h}{2}\left(\frac{h}{2}-\frac{h}{4}\right) + 1\cdot t\cdot\left(s-\frac{h}{2}\right)(\cdots) & \text{in } 2 \\ \text{What a mess!} & \text{in } 3 \end{cases}$$

There is an easier way

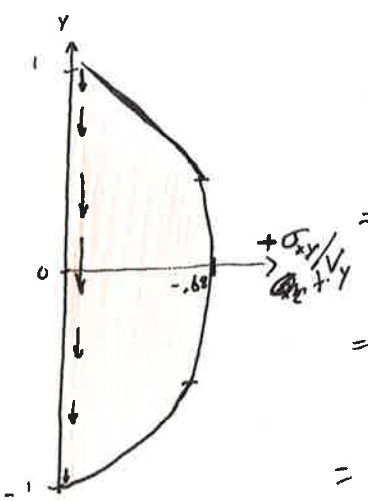


$y = \frac{h}{2} - s$

$s = \frac{h}{2} - y$

$$= \int \frac{E}{E_1} \cancel{z}\, t\, ds = \int_0^{s_1} \frac{E}{E_1}\left(\frac{h}{2}-s\right) t\, ds$$

$$= \underbrace{\int_0^{\langle s,\,0.5\rangle} 3\left(\cancel{\tfrac{h}{2}}^{1}-s\right)\cancel{t}^{0.5} ds}_{} + \int_{0.5}^{\langle s,\,1.5\rangle}\left(\cancel{\tfrac{h}{2}}^{1}-s\right)\cancel{t}^{0.5} ds + \int_{1.5}^{\langle s,\,2.0\rangle} 3\left(\cancel{\tfrac{h}{2}}^{1}-s\right)\cancel{t}^{0.5} ds$$



$$= 1.5\int_0^s (1-s)\,ds = 1.5\left(s - \frac{s^2}{2}\Big|_0^s\right) = \left(s-\frac{s^2}{2}\right)1.5 \quad \text{when } s < 0.5$$

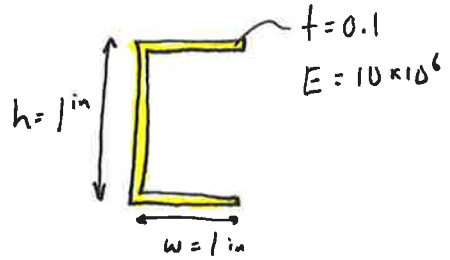
$$= 1.5\left(0.5 - \frac{0.25}{2}\right) + 0.5\int_{0.5}^{s}(1-s)\,ds = 0.5625 - 0.25 s^2 + 0.5 s - 0.1875 \quad \text{when } 0.5 < s < 1.5$$

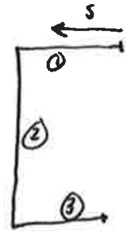
$$= 0.5625 + 0 + 1.5\int_{1.5}^{s}(1-s)\,ds \quad -0.75 s^2 + 1.5 s - 0.5625 \quad \text{when } s > 1.5$$

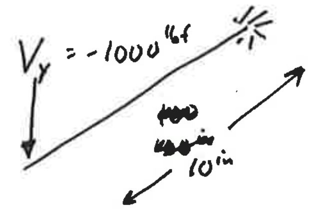
$$= 0.5625 - 0.75 s^2 + 1.5 s - 0.5625$$

The integral is easier if you want the distribution, Summation is better for specific points

Ex: C channel

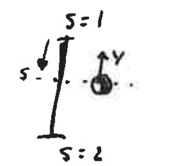






① $Q^*_{z_1} = \int \cancel{\frac{E}{E_1}}^{1}\, y\, t\, ds = \int = \int_0^{s_1} \underbrace{0.5}_{y}\cdot\underbrace{0.1}_{t}\, ds = 0.05\,s$

② $Q^*_{z_2}$ in | $= \int y\, t\, ds = \frac{E_i}{E_1} A_i \bar{y}_i + \int_1^{s} y\, t\, ds$



$= \underbrace{0.1}_{A}\cdot\underbrace{0.5}_{\bar{y}} + \int_1^{s} (1.5 - s)\, 0.1\, ds = 0.05 + \left(0.15s - \frac{0.1s^2}{2}\Big|_1^{s}\right)$ $y = 1.5 - s$

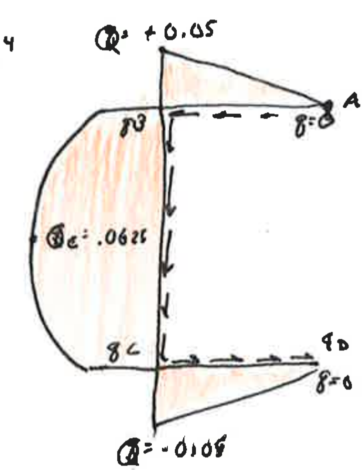
$= 0.05 + \left(0.15s - 0.05s^2 - 0.15 + 0.05\right)$

$= -0.05s^2 + 0.15s - 0.05$

③ $Q^*_{z_3}$ in — $= \int y\, t\, ds = \cancel{\frac{E}{E_1}}^{1} A_1 Y_1 + \cancel{\frac{E}{E}}^{1} A_2 Y_2 + \int_2^{s} y\, t\, ds = 0.05 + 0 + 0.1\int_2^{s} -0.5\; s$

$= 0.05 - 0.05(s-2) = 0.15 - 0.05s$

$I_{zz} = \underbrace{\frac{1}{12}\, 0.1 \cdot \cancel{1}^{3}\ 0.8^3}_{I_{z_1 z_0}} + \underbrace{\frac{2}{12}\, 1 \cdot 0.1^3}_{2\cdot I_{z_2 z_0}} + \underbrace{(0.1)(1)(2)}_{A}\underbrace{(0.5)^2}_{y^2} = 0.054433$ ~~0.0585~~ in^4



$q(s) = q(0) - \frac{\overset{-1000\text{ lbf}}{V_y}\, Q_z^*}{I_{zz}^*} \Rightarrow \sigma_{xy} = \frac{V_y Q_z}{I_{zz}\cdot t}$

$q_A = 0$, $q_B = \frac{1000 \cdot 0.05}{0.0585} = 854\ \frac{\text{lbf}}{\text{in}}$

$q_C = q_B$ $q_D = 0$ $q_E = 1068\ \frac{\text{lbf}}{\text{in}}$

$\sigma_{xy_E} = -\frac{V_y Q_z}{I_{zz}\, t} = \frac{1068}{0.1} = 10$ ksi   Compare to $\sigma_{xx} = -\frac{M y}{I} = \pm 85$ ksi

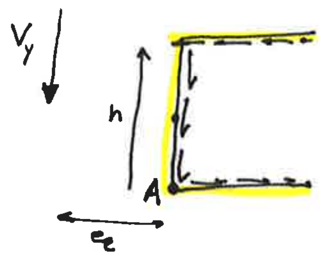
Shear Center

The point on the cross section's plane where a shear load creates no twisting

NOT ALWAYS THE CENTROID unless 2 symmetry planes

Ex:

The shear flow creates an applied torque/moment.



What is the moment about A due to shear flow?

$$M_A = \int q h \, ds = \int_0^1 \frac{V_y Q_z h \, ds}{I_{zz}} = \frac{V_y \cdot 1}{I_{zz}} \int_0^1 0.05 s \, ds$$

$$= \frac{V_y}{I_{zz}} 0.05 \int_0^1 s \, ds = \frac{V_y}{I_{zz}} 0.05 \frac{1}{2} = \frac{V_y \, 0.05}{2} \Big/ 0.054433$$

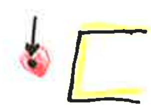
$$= 0.4593 V_y$$

What is the moment due to V_y in negative y direction at e_z from A?

$$M_A = V_y e_z$$

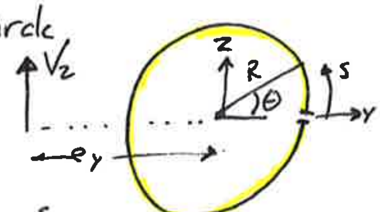
Where are these the same? $V_y e_z = 0.4593 V_y \Rightarrow \boxed{e_z = 0.4593}$

No twist



NOT ON CROSS SECTION!!

Ex: Cut circle

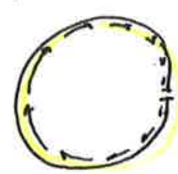


$s = R\theta$
$ds = R\,d\theta$
$Z = R\sin\theta$

$$I_{yy} = \pi R^3 t$$

$$Q_y = \int_0^s z t \, ds = \int_0^\theta \underbrace{R\sin\theta}_{z} \, \underbrace{t}_{t} \, \underbrace{R\,d\theta}_{ds} = R^2 t \int_0^\theta \sin\theta \, d\theta = R^2 t (1 - \cos\theta)$$

$$q = \frac{-V_z R^2 t (1 - \cos\theta)}{\pi R^3 t} = \frac{-V_z Q_y}{I_{yy}} = \frac{-V_z}{\pi R}(1 - \cos\theta)$$



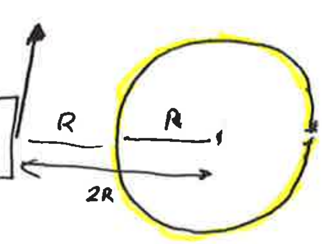
What is the moment about center from shear flow?

$$M_0 = \int_0^\theta q R \, ds = \int_0^\theta \frac{-V_z}{\pi R}(1 - \cos\theta) R^2 \, d\theta = \frac{-V_z R}{\pi} (\theta - \sin\theta) \Big|_0^{2\pi} = -\frac{V_z R}{\pi} 2\pi = -V_z 2R$$

What is the moment of V_z at e_y from origin?

$$M_0 = -V_z e_y$$

Where are these the same? $-V_z e_y = -V_z 2R \Rightarrow \boxed{e_y = 2R}$



Not dependent on V_z!