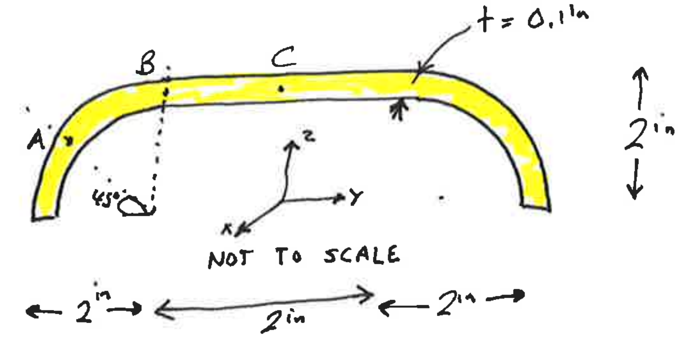
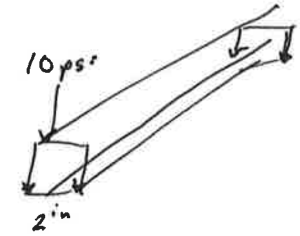
4.26

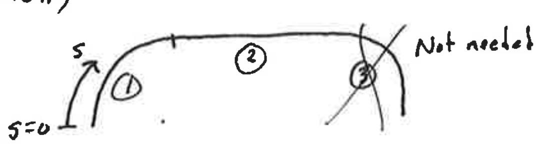




① Loading

$$V_z(x=0) = (\text{pressure} \cdot \text{Area})\hat{z} = -10\,\text{psi} \,|\, 2\,\text{in} \,|\, 100\,\text{in} = -2000\ \text{lbf}$$

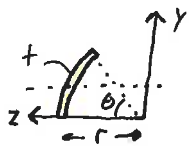
② Geometry



$$q(s_1) = q(0) - \frac{V_z Q_y}{I_{yy}}$$

Compute I_{yy}

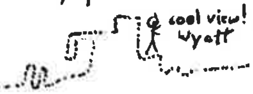
Luckily, in the back of my book, I wrote in the geometric properties of a thin arc.



$\text{Area} = r t \theta$

$\bar{y} = \frac{r}{\theta}(-\cos\theta + 1)$

yep, it's irrational.



Arc of $\theta = 90° = \pi/2$

r = 2 in

$A = 2 \cdot 0.1 \cdot \frac{\pi}{2} = 0.314159265358979...$

$I_{zz} = r^3 t\left(\frac{\theta - \sin\theta\cos\theta}{2}\right) - \frac{r^3 t}{\theta}(-\cos\theta + 1)^2$

$\bar{y} = \frac{2}{\pi/2}(-\cos 90° + 1) = \frac{4}{\pi} = 1.273$

$I_{zz} = \text{plug in} = 2^3 \cdot 0.1\left(\frac{\pi/2 - 1 \cdot 0}{2}\right) - \cancel{\ldots} = 0.2\pi = 0.6283\ \text{in}^4$

about the xyz axis NOT centroid

part	A	$\bar{z}$	$\bar{z}A$	I_{zz}
1	0.31415	1.273	0.4 !!!	0.6283
2	0.2	2.0	0.4	$0.000167 + 2^2 \cdot 2 \cdot 0.1 = 0.8$
3	0.31415	1.273	0.4 !!!	0.6283
	0.8283 ✓		1.2	2.057

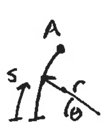
$$\bar{z}' = \frac{1.2}{0.8283} = 1.449\ \text{in} \checkmark$$

$$I_{z'z'} = I_{zz} - \bar{z}'^2 A = 2.057 - 1.449^2 \cdot 0.8283 = 0.318\ \text{in}^4$$

My CAD program says 0.31524 ✓

③ Compute Q_y

$$Q_y = \int_0^s z t\, ds$$



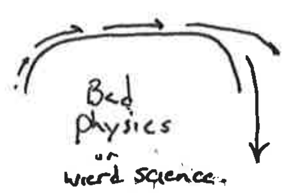
$z = r\sin\theta$
$s = \theta r$

$$\Rightarrow Q_{y_A} = \int_0^{\pi/4} r\sin\theta \cdot t \cdot r\, d\theta = r^2 t \int_0^{\pi/4} \sin\theta\, d\theta$$

$$Q_{y_A} = 2^2 \cdot 0.1 \cdot 0.2929 = 0.11716$$

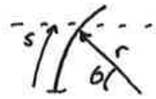
positive?!?!

Error!! What is wrong? $Q_c = 0.8$



③ Compute Q_y correctly. $\boxed{\text{the } z \text{ coordinate is to/from the Centroid!}}$

$$Q_{y_A} = \int_0^s z' t \, ds$$

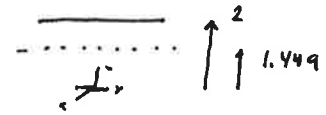


$z = r\sin\theta$ and $z' = z - \bar{z}' = r\sin\theta - 1.449$

$s = r\theta$

$$= \int_0^{\pi/4} (2\sin\theta - 1.449)(0.1)(2)\,d\theta = -0.11045$$

$$Q_{y_B} = \int_0^{\pi/2} \text{''} = -0.055217$$



$\updownarrow = 2 - 1.449 = 0.551$

$$Q_{y_C} = Q_{y_C} + \int_{\not{x}_B}^{\not{x}_C} z' t \, ds = Q_{y_C} + z' t \Delta s = -0.055217 + 0.551 \cdot 0.1 \cdot 1$$

$$\approx 0.0$$

$$= 0 \text{ by theory}$$

④ Shear flow

$$q_A = \frac{-V_z Q_y}{I_{yy}} = -\frac{\left(-2000 \text{ lbf}\right)\left(-0.11045 \text{ in}^3\right)}{0.318 \text{ in}^4} = -695 \frac{\text{lbf}}{\text{in}}$$

$$q_B = \cdots = -\frac{(-2000)(-0.055217)}{0.318} = -347 \frac{\text{lbf}}{\text{in}}$$

$$q_c = 0$$

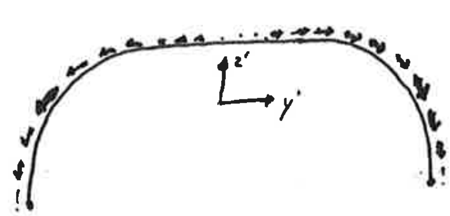
⑤ Shear Stress

$$\sigma_{xs_A} = \frac{q}{t} = \frac{-695 \frac{\text{lbf}}{\text{in}}}{0.1 \text{ in}} = -6.95 \text{ ksi}$$

in direction of s

$$\sigma_{xs_B} = \frac{q}{t} = -3.47 \text{ ksi}$$

$$\sigma_{xs_C} = \frac{q}{t} = 0$$



Where is σ_{xs} highest?

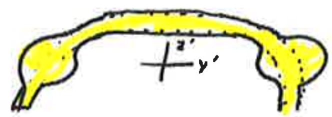
At $z' = 0$ ∩ where $z = 1.449 = r\sin\theta \Rightarrow \theta = 0.8103^{\text{rad}}$

$\approx 46° \approx 45°!!$

$Q_{y_{z'}} = -0.11054$ $\quad \sigma_{xs_{z'}} = -6.95$ ksi

So, mister smarty pants engineer.... if shear stresses are a problem should material be added at the $\sigma_{xs_{max}}$ location near $\bar{z}'$?

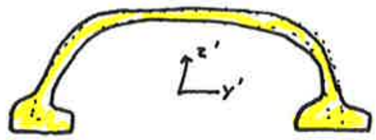
- thicker material means $\sigma_{xs} = \frac{q}{t + \Delta t} < \frac{q}{t}$



But if we put the material in the further away locations (say at the bottom)

- $I_{zz} = I_{zz_0} + z^2 A$ and $q = \frac{V_z Q_y}{I_{zz}}$ and $\bar{z}$ will move down

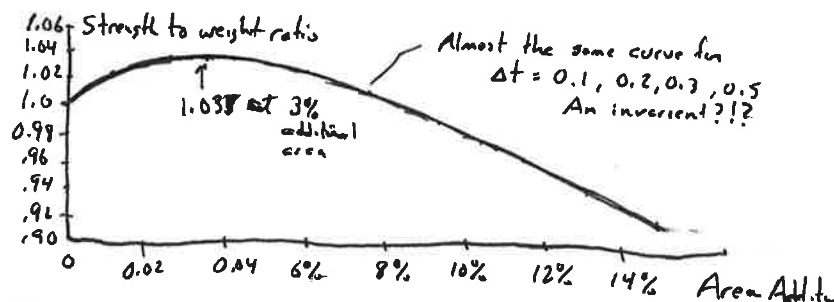
$$z'^{new} = z - \bar{z}'^{new} < z - \bar{z}^{orig}$$



The 2nd option is usually what we see.

If we look at the strength to weight ratio, add a small flange helps up to about 3.8%, but not much more. This is about 3% area on the lower flange.

This only considers shear stress.



Compute the axial stress in 4.26

$$\sigma_{xx} = \frac{-Mz}{I} = -\frac{(2000^{lbf})(50^{in})(-1.449\,in)}{0.318} = 455^{ksi} \;!$$

whatever it was, it is probably broken.