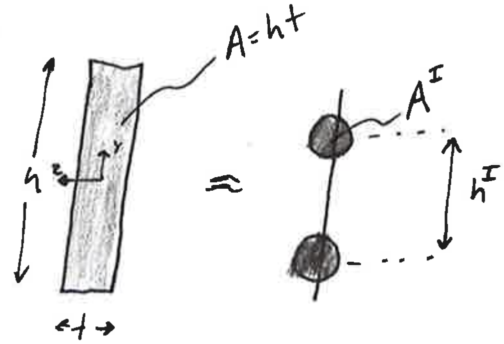
Idealized Cross Sections

Actual aircraft/rocket structures can be complicated with many spars, ribs, stringers, and cells. Can we solve the structural analysis exactly? Yes. but expensive. Or, we could directly go to a computer simulation. Expensive and not good for conceptual understanding.

Simplify the structural analysis.

Lumped properties



Can we find A^I and h^I such that the areas ~~are~~ is identical and the moments of inertia are identical.? Yes.

$$A = ht = 2A^I$$

$$A^I = \frac{ht}{2}$$

$$I_{zz} = \frac{1}{12}bh^3 = 2\left(\frac{h^I}{2}\right)^2 A^I = \frac{h^{I^2} A^I}{2}$$

$$= \frac{h^{I^2} ht}{4}$$

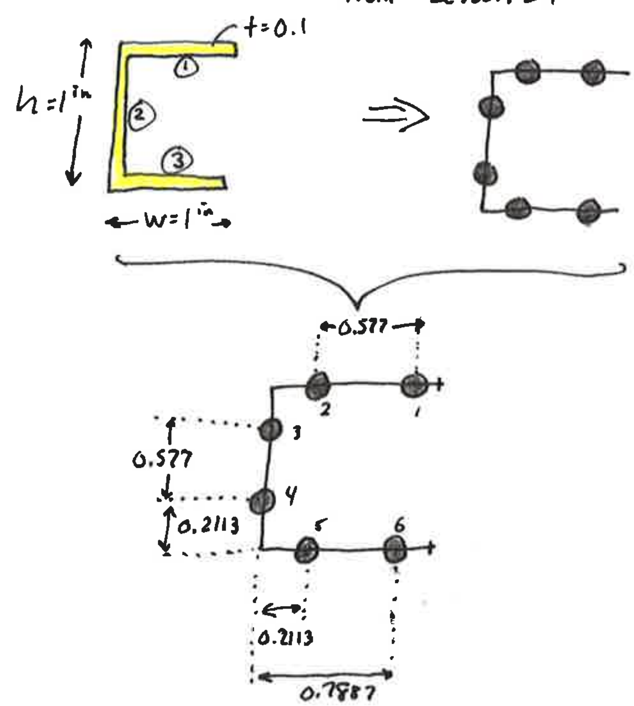
$$h^{I^2} = \frac{1}{12}\cancel{b}^{t} h^3 \cdot \frac{4}{ht} = \frac{1}{3}h^2$$

The lumped area and distance is

$$A^I = \frac{ht}{2} = \frac{A}{2} \quad \text{Kind of obvious!}$$

$$h^I = \sqrt{1/3}\, h = \frac{h}{\sqrt{3}} \approx 0.577\, h$$

Ex: Revisit C channel from Lesson 24



① Geometry

In part 1 and 3:

$$A^I_1 = \frac{(0.1)(1)}{2} = 0.05$$

$$h^I_1 = 0.577 \cdot 1^{in} = 0.577^{in}$$

In part 2:

$$A^I_2 = \frac{0.1}{2} = 0.05$$

$$h^I_1 = \text{same as above} = 0.577^{in}$$

Other important lengths

$$h^I_1/2 = 0.2887$$

$$1/2 - h^I_1/2 = 0.2113$$

$$h^I_1 + 1/2 - h^I_1/2 = \frac{1}{2} + \frac{1}{2}h^I_1/2 = 0.7887$$

② Compute I_{zz}

$$I_{zz} \equiv \int y^2 dA \underset{\text{lumped area}}{\Rightarrow} \sum y^2 A = \overbrace{\underbrace{0.2887^2}_{y} \cdot \underbrace{0.05}_{A}}^{\text{part 2 top}} + \overbrace{0.2887 \cdot 0.05}^{\text{part 2 bottom}}$$

$$+ \underbrace{\left(0.5^2 \cdot 0.05\right)}_{\text{parts 1, 3}} \underbrace{4}_{\text{four lumped areas}}$$

$$= 0.058335$$

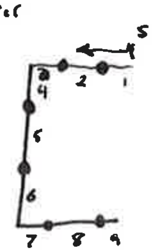
Not the same as what we found because in lumping the area, I used 0.5 up/down for parts 1, 3 rather than 0.45. And I double counted the web/~~...~~ cap interface.

③ Where is the centroid?

Lumped ~~Part~~	A	z	zA	y
1	0.05	-0.7887		0.5
2	|	-0.2113	short cut	0.5
3	|	0		0.2887
4	|	0		0.2887
5	|	-0.2113		-0.5
6	0.05	-0.7887		-0.5
	3.0 = 0.05·6		0.05·(-1 + -1)	

$$\bar{z} = \frac{0.05(-2)}{0.05(6)} = \underline{-1/3} \qquad \bar{y} = \underline{0}$$

④ Shear



$$Q^*_{z_1} = \sum yA = 0 \qquad q_1 = 0$$

$$Q^*_{z_2} = \sum yA = (0.5)(0.05) = 0.025$$

$$Q^*_{z_3} = \sum yA = (0.5)(0.05) + (0.5)(0.05) = 0.05$$

$$Q^*_{z_4} = Q^*_{z_3}$$

$$Q^*_{z_5} = Q^*_{z_3} + (0.2887)(0.05) = 0.0644$$

$$Q^*_{z_6} = Q^*_{z_5} + (-0.2887)0.05) = 0.05$$

$$Q^*_{z_7} =$$

$$Q^*_{z_8} = \cancel{Q^*_{z_7}}^{0.05} + (-0.5)(0.05) = 0.025$$

$$Q^*_{z_9} = \cancel{Q^*_{z_8}}^{0.025} + (-0.5)(0.05) = 0$$

$$\sigma_{xs} = \frac{q}{t} = -\frac{(-V_y)\,Q_z}{I_{zz}\,t} = \frac{1000 \cdot 0}{0.058335} = 0$$

$\sigma_{xs_2} = 4.3$ ksi

$\sigma_{xs_3} = 8.6$ ksi

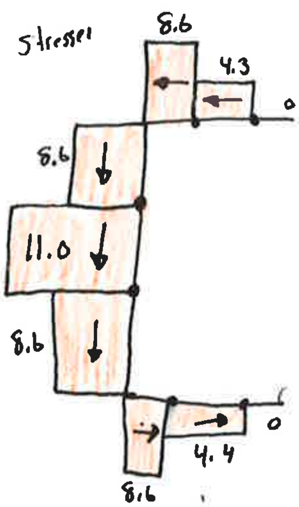
$\sigma_{xs_5} = 11.0$ ksi

$\sigma_{xs_6} = 8.6$ ksi

$\sigma_{xs_8} = 4.3$ ksi

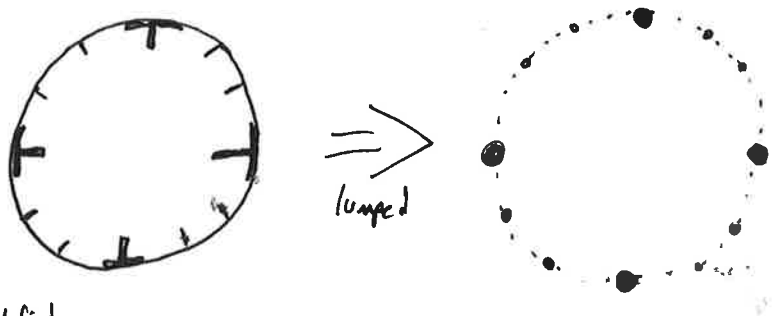
$\sigma_{xs_9} = 0$ ksi

stresses

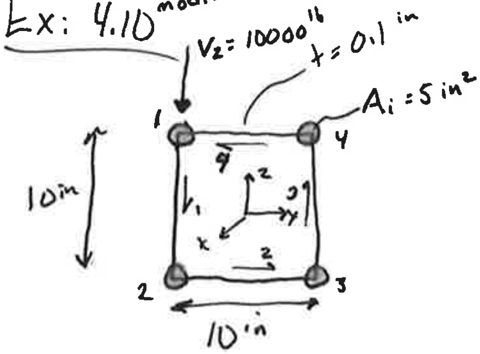


Previously we found $\sigma_{xs_{max}}$ was at centerline and 10.7 ksi

Shear in closed sections (did you notice that all of the shear loading is open?!)



Ex: 4.10 modified



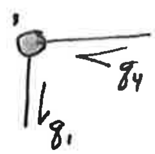
① Geometry

$$I_{yy} = 4\underset{\bar{z}}{(5)^2}\underset{A}{(5)} = 500^{in^4}$$

② Shear flow

$$q(s) = q(0) - \frac{V_z Q_y}{I_{yy}}$$

- from shear flow 4 to 1 we pass through lump 1.

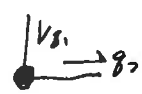


$$Q = A\bar{y} = 5^{in^2} \cdot 5^{in}$$

$$q_1 = q_4 - \frac{V_z Q_y}{I_{yy}}$$

$$= q_4 - \frac{-10000^{lb} \cdot 5^{in^2} \cdot 5^{in}}{500^{in^4}} = q_4 + 500$$

- from q_1 to q_2, we pass through lump 2



$$Q = A\bar{y} = 5 \cdot (-5)$$

$$q_2 = q_1 - \frac{-10000 \cdot -25}{500} = q_4 - 500 + 500 = q_4$$

- from q_2 to q_3, we pass through lump 3

$$Q = Ay = -25$$

$$q_3 = q_2 - \frac{-10000 \cdot -25}{500} = q_4 \mp 500$$

- from q_3 to q_4, we pass thru lump 4

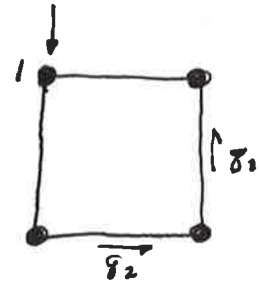
$$Q = Ay = 25$$

$$q_4 = q_4 + 500 - 500$$

$q_4 = q_4$ (not so usefull)

Need 1 more equation to solve.....

③ Static Equilibrium



$$\Sigma M_x = 0 = \Sigma \underbrace{q L}_{\text{force}} \underbrace{d}_{\text{distance}} + \text{loading}$$

$$\Sigma M_{x_1} = 0 = \underbrace{q_2 \cdot 10^{in} \cdot}_{\text{force}} \underbrace{10^{in}}_{\text{distance}} + q_3 \cdot 10^{in} \cdot 10^{in}$$

$$0 = q_4 \cdot 100^{in^2} + (q_4 \mp 500) \cdot 100^{in^2} \Rightarrow q_4 + q_4 \mp 500 = 0$$

$$2q_4 = +500 \qquad \boxed{q_4 = +250 \tfrac{lb}{in}}$$

④ Substitute into shear flows

$$q_1 = q_4 + 500 = +250 + 500 = +750$$

$$q_2 = q_4 = +250$$

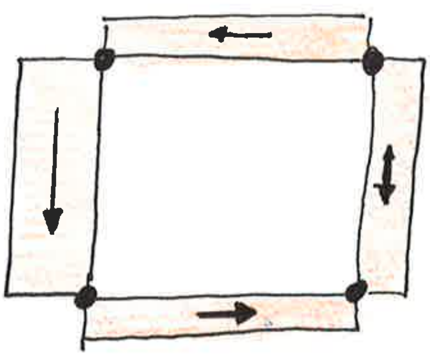
$$q_3 = q_4 \mp 500 = +250 \mp 500 = -250$$

$$q_4 = +250$$

⑤ Stresses

$$\sigma_{xs_1} = \frac{q}{t} = +7500^{psi} = +7.5^{ksi} \downarrow$$

$$\sigma_{xs_2} = +2.5^{ksi} \longrightarrow \qquad \sigma_{xs_3} = \mp 2.5^{ksi} \downarrow \qquad \sigma_{xs_4} = +2.5^{ksi} \longleftarrow$$



⑥ Twist

$$\theta = \frac{1}{2A} \oint \frac{q}{Gt} ds$$