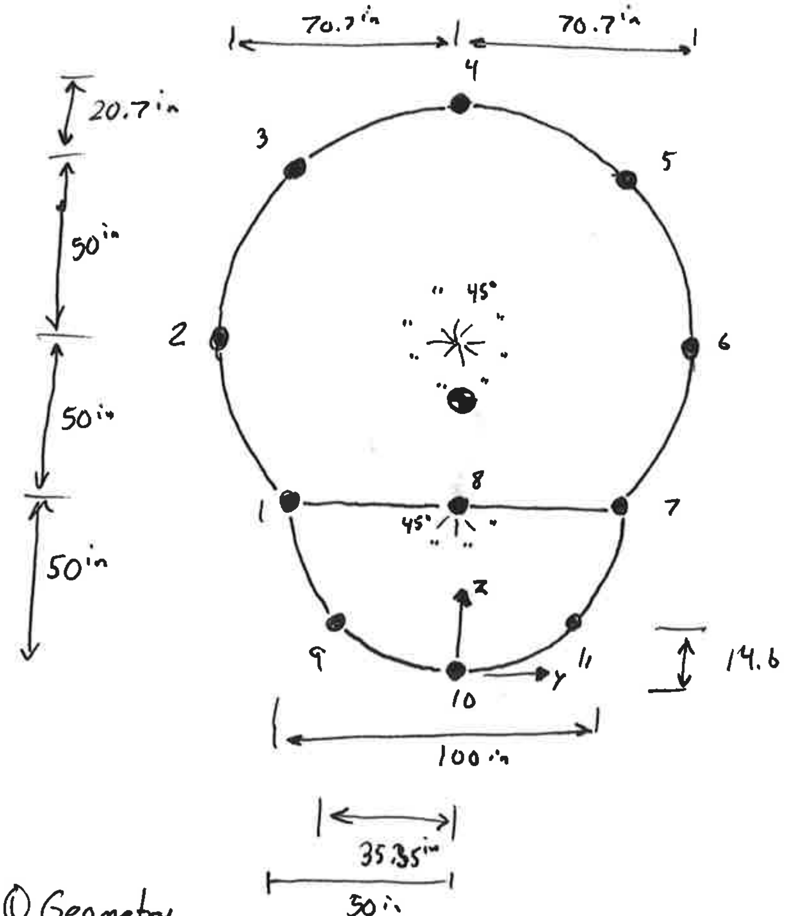
4.37 The ultimate 341 problem (from the textbook)



$M_y = 500\,000$ lbf in

$M_z = 1\,000\,000$ lbf in

$V_y = 10000$ lbf

$V_z = 50000$ lbf

$A_1 \rightarrow A_7 = 10\ in^2$

$A_8 \rightarrow A_{11} = 5\ in^2$

$t = 0.10$ in

① Geometry

part	A	z	y	z^2A	y^2A	zA	yA
1	10	50	-50	25000	25000	500	-500
2	10	100	-70.7	100000	50000	1000	-707
3	10	150	-50	225000	25000	1500	-500
4	10	171	0	292410	0	1710	0
5	10	150	50	225000	25000	1500	500
6	10	100	70.7	100000	50000	1000	707
7	10	50	50	25000	25000	500	500
8	5	50	0	12500	0	250	0
9	5	14.6	-35.4	1066	6266	73	-177
10	5	0	0	0	0	0	0
11	5	14.6	35.4	1066	6266	73	177
	90					8106	0

$\bar{z} = 90$ $\qquad \bar{y} = 0$ $\qquad I_{yy} = 1.0 \times 10^6\ in^4$ $\qquad I_{zz} = 212.5 \times 10^3\ in^4$

$$I_{y'y'} = I_{yy} - \bar{z}^2 A = 1\times10^6 - 90^2 \cdot 90 = 271000 \text{ in}^4$$

$$I_{z'z'} = I_{zz} - \bar{y}^2 A = 212500 \text{ in}^4$$

② Stress from bending (no thermal load and all the same material)

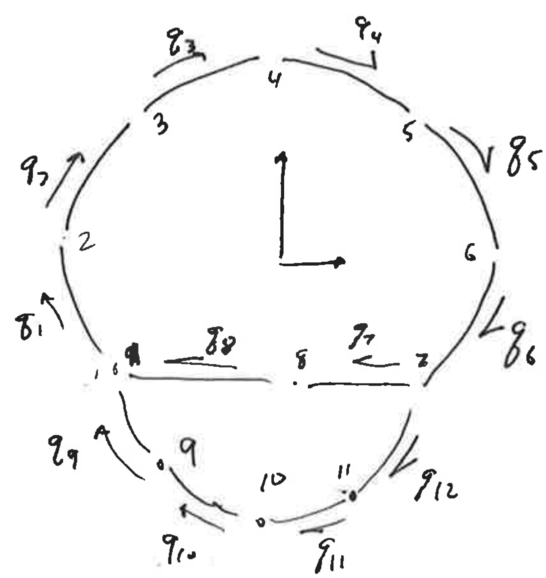
$$\sigma_{xx} = \frac{P}{A} - \frac{M_z y}{I_{zz}} + \frac{M_y z}{I_{yy}} = -\frac{1\times10^6 \cdot y}{212500} + \frac{500000}{271000} z = -4.71y + 1.85z$$

part	$\bar{z}$	$\bar{y}$	σ_{xx} [psi]	$\sigma_{xx} A$
1	-40	-50	162	⋮
2	10	-70.7	352	
3	60	-50	347	
4	81	0	150	
5	60	50	-125	
6	10	70.7	-315	
7	-40	50	-309	
8	-40	0	-74	
9	-75.4	-35.4	27	
10	-90	0	-167	
11	-75.4	35.4	-306	= ?

③ Shear

Around the upper cell + lower cell

part	$\bar{y}$	$\bar{z}$	A	$\bar{y}A$ (Q_z)	$\bar{z}A$ (Q_y)
1	-50	-40	10	-500	-400
2	-70.7	10	10	-707	100
3	-50	60	10	-500	600
4	0	81	10	0	810
5	50	60	10	500	600
6	70.7	10	10	707	100
7	50	-40	10	500	-400
8	0	-40	5	0	-200
9	-35.4	-75.4	5	-177	-377
10	0	-90	5	0	-450
11	35.4	-75.4	5	177	-377



$$q_2 = q_1 - \frac{V_y Q_z}{I_{zz}} - \frac{V_z Q_y}{I_{yy}} = q_1 - \frac{10000}{271000}(-707) - \frac{50000}{212500}(100)$$

$$= q_1 - 0.0369(-707) - 0.2353(100) = q_1 + 2.56$$

$$q_3 = q_2 - 0.0369(-500) - 0.2353(600) = q_1 - 120.2$$

$$q_4 = q_3 - 0.0369(0) - 0.2353(810) = q_1 - 310.8$$

$$q_5 = q_4 \quad '' \quad (500) - \quad '' \quad (600) = q_1 - 470.4$$

$$q_6 = q_5 \quad '' \quad (707) - \quad '' \quad (100) = q_1 - 473.0$$

At. 7

$$q_7 - q_6 + q_{12} = -\frac{V_y Q_z}{I_{zz}} - \frac{V_z Q_y}{I_{yy}} = -0.0369(500) - 0.2353(-400)$$

$$= 75.7$$

At 1

$$q_1 - q_8 - q_9 = -0.0369(-500) - 0.2353(-400) = 112.6$$

$$q_8 = q_7 - 0.0369(0) - 0.2353(-200) = q_7 + 47.1$$

$$q_{11} - q_{12} = -0.0369(177) - 0.2353(-377) = 82.2$$

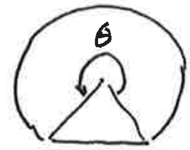
$$q_{10} - q_{11} = \quad '' \quad (0) - \quad '' \quad (-450) = 106$$

$$q_9 - q_{10} = \quad '' \quad (-177) - \quad '' \quad (-377) = 95.2$$

12 equations and 12 unknowns

④ Angle of Rotation

$$\theta = \frac{1}{2A}\oint \frac{q}{Gt}\,ds = \frac{1}{2A}\sum \frac{q\,\Delta s}{Gt}$$



$$A = \frac{D^2}{8}(\theta - \sin\theta)$$

$$\theta_{upper} = \frac{1}{2\cdot 14276\cdot G\cdot 0.1}\Big[(55.5)(q_1+q_2+q_3+q_4+q_5+q_6) + (50)(q_7+q_8)\Big]$$

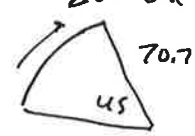
$$A_{upper} = \frac{(141.4)^2}{8}\left(\frac{3}{2}\pi - \sin\frac{3}{2}\pi\right) = 14276\ in^2$$

$$\theta_{lower} = \frac{1}{2\cdot 3927\cdot G\cdot 0.1}\Big[(39.3)(q_{12}+q_{11}+q_{10}+q_9) + 50(-q_8-q_7)\Big]$$

$$A_{lower} = \frac{100^2}{8}\left(\pi - \sin\pi\right) = 3927\ in^2$$

Thus, $\theta_{upper} = \theta_{lower}$

Arc lengths $\Delta s = \theta R = 55.5$ in



$\Delta s = 39.3$

~~We have a larger 12x12 matrix to solve~~

Static Equilibrium

$\sum M_{*} = 0 =$ Moment from V_z and V_y applied at centroid.
$+\sum q\,\Delta s\, d$

Solve a 12 x 12 matrix to give $q_1 \cdots q_{12}$

$$\sigma_{xs} = \frac{q}{t}$$

Web carries shear Stringers carry moments