AEM 341 Final Exam Review

Major Concepts in course:

- Nomenclature
- Loads (Aero, Inertial, Thermal, etc)
- 3D principal stresses
- Materials (Isotropic and anisotropic)
- Modulus weighted beam bending with thermal loads + Diff Eqns of beams
- Buckling
- Torsion and Shear
- Lumping and Idealized Sections

In other words

$$F = ma = a_{cg} + \dot{\omega} \times r_{pt} + \omega \times (\omega \times r_{pt}) + 2\omega \times V_{pt} + a_{pt}$$

$\sigma_{prin} = \text{eigenvalues}(\sigma)$ $\quad v_{prin\ direction} = \text{eigenvectors}(\sigma)$

$(\varepsilon) = [C](\sigma) + \alpha \Delta T \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ where C is linear Hookean but perhaps not isotropic

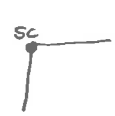
$$\sigma_{xx} = \frac{E}{E_1 A^*}(P + P^T) - \frac{E}{E_1}\left(\frac{(M_z - M_z^T) I_{yy}^* + (M_y - M_y^T) I_{yz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}}\right) y + \frac{E}{E_1}\left(\frac{(M_y - M_y^T) I_{zz}^* + (M_z - M_z^T) I_{yz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}}\right) z$$
$$- E \alpha \Delta T$$

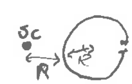
$P_{buckle} = \dfrac{\pi^2 E I}{(1-\nu^2) L^2}$ for  or $\sigma_{cr} = K E \left(\dfrac{t}{b}\right)^2$ for 

K ← lookup constant

$q = \sigma_{xs} \cdot t = \dfrac{M_x}{2A}$ for a single cell $\qquad M_x = 2 \sum q_i A_i$ for multicell

$\theta = \dfrac{1}{2A} \oint \dfrac{q}{Gt} ds$ for any closed cell

Shear center is location where shear loads don't result in rotation 

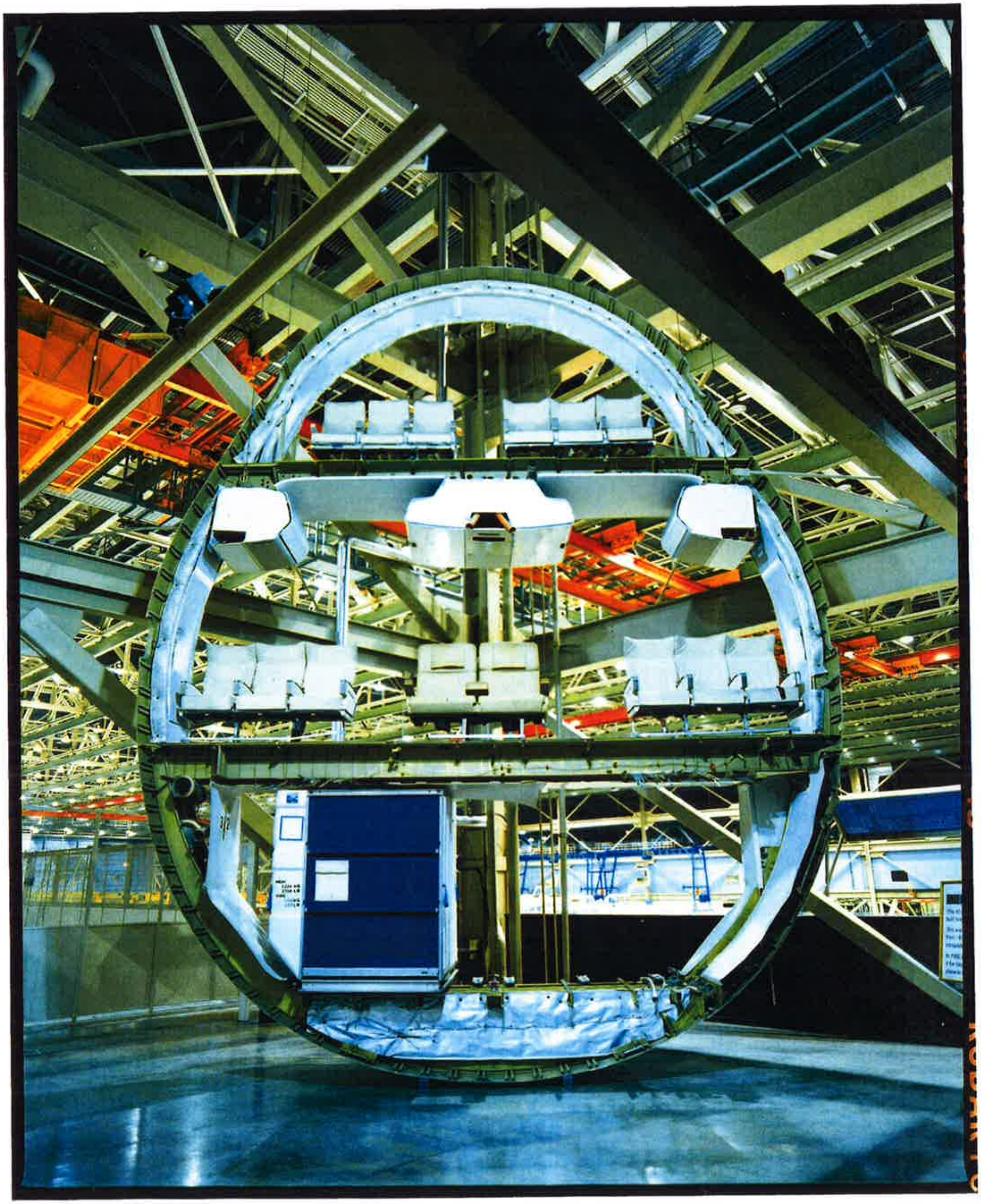


$$\sum q_i = -\frac{V_y Q_z}{I_{zz}} - \frac{V_z Q_y}{I_{yy}}$$ simplified shear flow

Big Ideas and what seperates Mechanical Engineers from Aero Engineers

	Why?
Thin structures	Light weight, more mission payload = more \$ Operate closer to yield $FOS_{ultimate} \approx 1.5$
Web carries shear and Stringers carry moments	
Design for stress, deflection and buckling	
Lumping + Idealized Cross Sections	Simplify analysis with little to no impact on solution accuracy for thin, light structures
Non Symmetric Parts mix loads and deflections/stress directions The shear center is not always at the centroid.	$I_{yz} \neq 0$ complicates the analysis
Twist rate depends on $1/A^2$ and $\frac{1}{t}$ and s alternatively $\left(\frac{1}{A}\right)\left(\frac{A_{material}}{A_{cross\ sect}}\right)\left(\frac{1}{t^2}\right)$	$\theta = \frac{1}{2A}\frac{q}{Gt}\Delta s = \frac{1}{2A}\frac{M_x}{2A}\frac{1}{Gt}S_{total}$ $A_{material} = st$
Shear flow in thin structures resembles water flow (conservation) with a source (shear)	$\sum q_i = -\frac{VQ}{I}$
Composites provide you significant structural advantages at the expense of analysis	

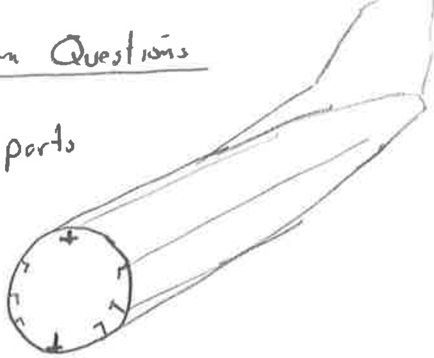
Boeing 747 Fuselage Cross-Section



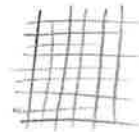
Composite Aircraft

Example Final Exam Questions

1) Name these parts



composite fabric



2) Determine the root bending moment of a 10000^{lb} 50^{ft} span aircraft in a $45°$ banked turn. The wing is generating lift exactly as an elliptical distribution.

42% semispan

$$n = \frac{1}{\cos\phi} = \frac{1}{\cos 45} = 1.41 \qquad L = nW = 14100^{lbf}$$

$$M = \frac{L}{2} \cdot 42.44\% \left(\frac{50^{ft}}{2}\right) = \frac{150\,000}{2}^{\;ft\ lbf}$$

3) A material fails when the principal stress exceeds 15^{ksi}. Your stresses are

$\sigma_{xx} = \sigma_{yy} = 5$, $\sigma_{xy} = 1$, $\sigma_{zz} = 10$ all others $= 0$

Is the stress state safe?

$$\text{eigs}\begin{pmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 10 \end{pmatrix} = 6, 4, 10 < 15 \qquad \underline{\text{safe}}$$

4) Find the strains of an Al (6061) bar if $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 5^{ksi}$ and $\Delta T = 500^{F}$

$\sigma_{others} = 0$

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \vdots \end{pmatrix} = \frac{1}{E}\begin{pmatrix} 1 & -\nu & -\nu & & \\ -\nu & 1 & -\nu & & \\ -\nu & -\nu & 1 & & \\ & & & 2(1+\nu) & \\ & & & & \ddots \end{pmatrix}\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \vdots \end{pmatrix} + \alpha \Delta T \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Symmetric!

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \frac{\begin{pmatrix} 1 & -\nu & -\nu \end{pmatrix}}{E}\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{pmatrix} + \alpha \Delta T$$

$$= \sigma_{xx} - 0.33\,\sigma_{yy} - 0.33\,\sigma_{zz}$$

$$= \sigma_{xx}\left(1 - \cancel{0.33 - 0.33}\right) \rightarrow 0.33$$

$$= \frac{5^{ksi}(0.33)}{9.9 \times 10^{6}} + 13 \times 10^{6}\,{}^{1}/_{F} \cdot 500^{F} = 0.0065$$

$\underline{6.5\ \mu\text{strains}}$

5) Is $\phi = Ax^2y^2$ a permissible Airy stress function?

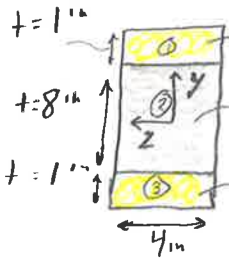
$$\nabla^4\phi \overset{?}{=} 0 = \frac{d^4\phi}{dy^4} + \frac{d^4\phi}{dx^4} + 2\frac{d^4\phi}{dx^2dy^2}$$

$$= 0 + 0 + 2(2A) = 4A \neq 0$$

No

Not physically consistent

6) Determine A^* for the following, with $E_1 = 10\times10^6$



Al $E=10\times10^6$

Steel $E=30\times10^6$

Al above

Part	E/E_1	A	$\frac{E}{E_1}A$
1	1	4	4
2	3	32	96
3	1	4	4

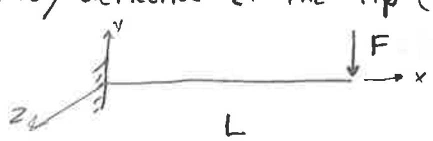
$A^* = 104$

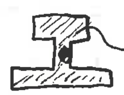
7) Determine M_z^T if $\Delta T = 100^F$

part	ΔT	E	α	y	A	$E\alpha\Delta T y A$
1	100	10×10^6	13×10^{-6}	4.5	4	234000
2	100	30×10^6	7×10^{-6}	0	32	0
3	100	10×10^6	13×10^{-6}	-4.5	4	-234000
						0

By inspection, heating will not result in bending so $M_z^T = 0$!

8) Find the y deflection at the tip (symbolic)





I_{zz}

$I_{yz} = 0$

$$\frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}}$$

$M_z = FL - Fx$

Integrate

$$\frac{dv}{dx} = \int \frac{FL - Fx}{EI}\,dx = \frac{1}{EI}\int FL - Fx\,dx$$

$$= \frac{1}{EI}\left(FLx - \frac{Fx^2}{2}\right)\Big|_0^x = \frac{F}{EI}\left(Lx - \frac{x^2}{2}\right)$$

constraints

$\frac{dv}{dx}$ at $x=0 = 0$

$v(0) = 0$

Integrate

$$v = \int \frac{dv}{dx}\,dx = \frac{F}{EI}\int\left(Lx - \frac{x^2}{2}\right)dx$$

$$= \frac{F}{EI}\ \frac{Lx^2}{2} - \frac{x^3}{6}\Big|_0^x =$$

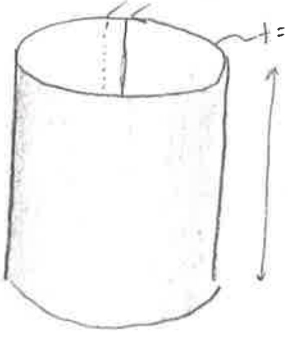
$$v = \frac{F}{EI}\ \frac{Lx^2}{2} - \frac{x^3}{6}$$

9) Find and precisely mark the shear center of the following cross sections

10) Estimate the buckling stress of this paper rolled into a tube

0.5 inch overlap

$t = 0.004^{in}$



11^{in}

$E \approx 290^{ksi}$

$C = \pi D = 8.0^{in} \Rightarrow D = 2.54^{in} \Rightarrow R = 1.27^{in}$

$\frac{t}{R} = \frac{0.004}{1.27} = 0.00314 = \frac{\pi}{1000}!$

$\frac{t}{L} = \frac{0.004}{11} = 0.000364$

correct your notes for missing E

$\sigma_{xx_{crit}} = -9E\left(\frac{t}{R}\right)^{1.6} - 0.16E\left(\frac{t}{L}\right)^{1.3}$ Approximately since $\frac{L}{R} > 2.5$

$= -260^{psi}$

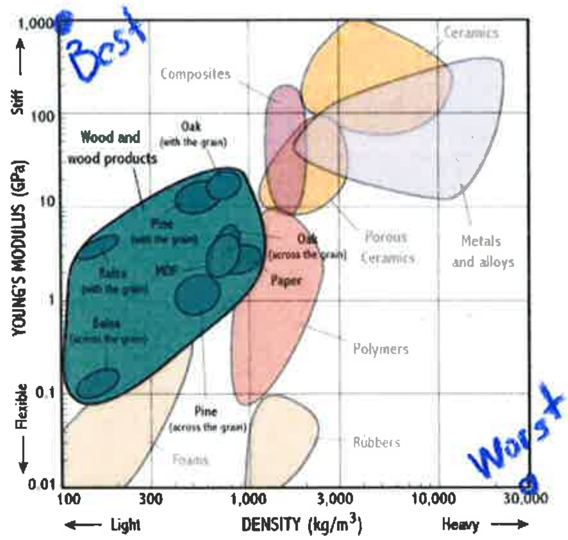
$F = \sigma \cdot A = \sigma \cdot C \cdot t = -260^{psi} \cdot 8^{in} \cdot 0.004^{in}$

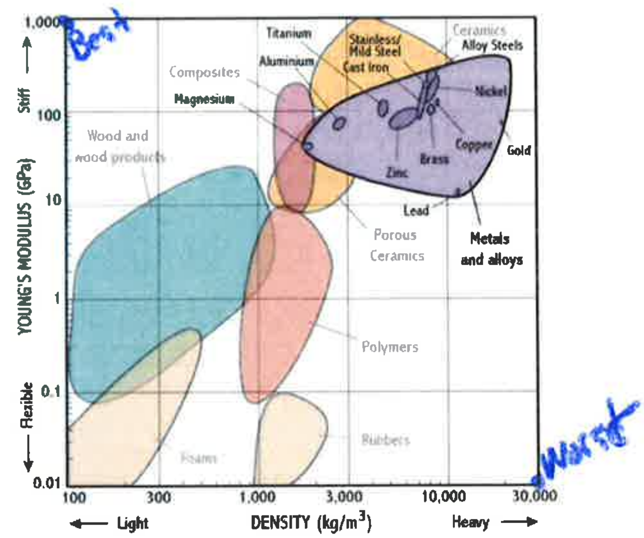
$= -8.3^{lbf}$

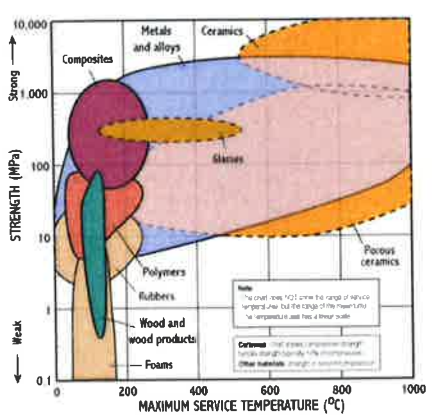
11) Why are certain materials favored in aerospace applications?

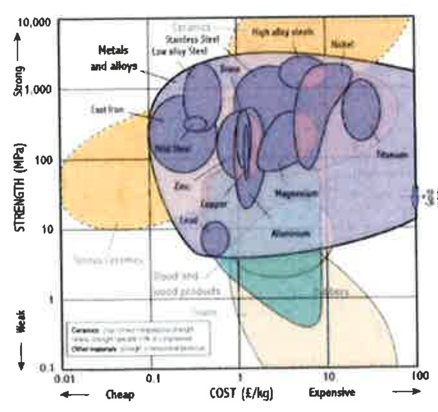
http://www-materials.eng.cam.ac.uk/mpsite/interactive_charts/stiffness-density/basic.html

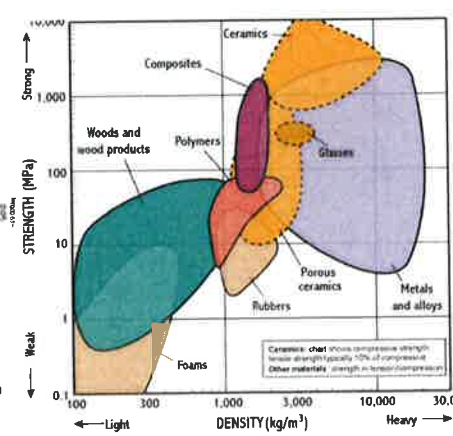
log scale!



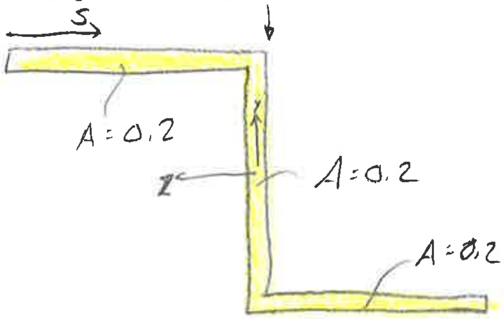






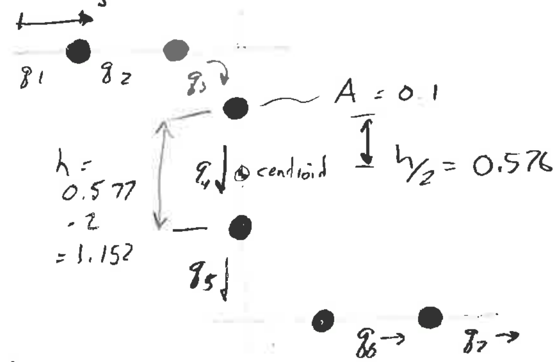


12) Estimate the maximum shear



≈

(Idealized Section)



$Q_z = Ay$

$I_{zz} = A\bar{y}^2$
$= 2(0.1)(1)^2$
$+ 2(0.1)(-1)^2$
$+ 2(0.1)\left(\frac{1.152}{2}\right)^2$
$= 0.466$

Section 1

$q = \cancel{q(0)}^{0} - \frac{V_y Q_z}{I_{zz}} = \frac{100 \cdot 0}{0.466}$

2:

$q = q_1 - \frac{\overset{-100}{\cancel{V_y}}(0.1)(0.1)(1)}{0.466} = 0 + \frac{1.00}{0.466} = 2.14$

3.

$q = 2.14 + \frac{100(0.1)(0.1)1}{0.466} = 4.28$

4:

$q = 4.28 + 100(0.1)(0.1)(0.576) = 4.28 + 1.23 = 5.52$

5: $q = 5.52 + 100(0.1)(0.1)(-0.576) = 4.28$

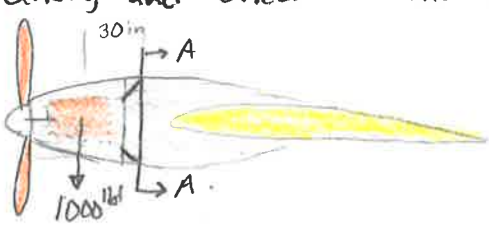
6: $q = 4.28 + 100(0.1)(0.1)(-1) = 2.14$

7: $q = 2.14 - 100(0.1)(0.1)(-1) = 0$

Max $q = 5.52$

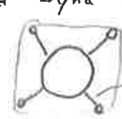
$\sigma_{xs} = \frac{q}{t} = 55$ psi

13) Find bending and shear in the following nacelle section



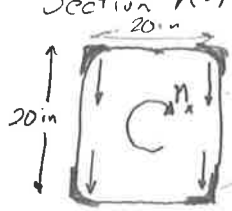
Torque = 5000 ft lbf

Quad "Dyna" focus mount



broken!

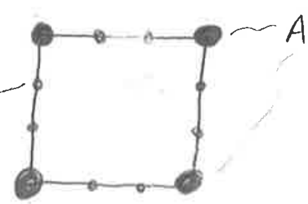
Section A-A



Stringers 1" x 1" x 0.1 in Al

web 0.050 Al 6061

Lump to



A = 0.5

A = 0.2

You can design the structure of an aerospace vehicle.