3.) Y Force SDT /m

The fully non-linear Y force equ is:

$$Y + mg\cos(\theta)\sin(\phi) = m\dot{v} + mru - mpw$$

Linearization:

$$Y = Y_0 + \Delta Y$$
$$\theta = \theta_0 + \Delta\theta$$
$$\phi = \phi_0 + \Delta\phi$$
$$V = V_0 + \Delta V$$
$$r = r_0 + \Delta r$$
$$U = U_0 + \Delta U$$
$$p = p_0 + \Delta p$$
$$W = W_0 + \Delta W$$

$\Rightarrow$ plug into Y equ. $\Rightarrow$

$\phi_0 = 0$
$\theta_0 \neq 0$
$r_0 = 0$
$U_0 \neq 0$
$V_0 = 0$
$p_0 = 0$
$W_0 = 0$

Substitute....

$$Y_0 + \Delta Y + mg\underbrace{\cos(\theta_0+\Delta\theta)}_{\cos\theta_0 - \Delta\theta\sin\theta_0}\overbrace{\sin(\phi_0+\Delta\phi)}^{\sin\phi_0 + \Delta\phi\cos\phi_0} = m\underbrace{\frac{d}{dt}}_{0}\underbrace{(V_0+\Delta V)}_{\frac{d}{dt}\Delta V} + m\underbrace{(r_0+\Delta r)(U_0+\Delta U)}_{r_0U_0 + r_0\Delta U + \Delta r U_0 + \Delta r\Delta U} - m\overbrace{(p_0+\Delta p)(W_0+\Delta W)}^{\overbrace{p_0W_0 + p_0\Delta W + \Delta p W_0 + \Delta p\Delta W}^{0}}$$

$\sin\phi_0 = 0$, $\cos\phi_0 = 1$; $\Delta r\Delta U$: HOT; $\Delta p\Delta W$: HOT; $\Delta r U_0 \neq 0$

0 if $\theta_0 = 0$ or $\cos\theta_0$ otherwise

Reduces to

$$Y_0 + \Delta Y + mg\,\Delta\phi\cos\theta_0 = m\Delta\dot{V} + m\Delta r U_0$$

Divide by m

$$\frac{Y_0 + \Delta Y}{m} + g\,\Delta\phi\cos\theta_0 = \Delta\dot{V} + \Delta r U_0$$

If the disturbances are zero, then $\frac{Y_0}{m} + \frac{0}{m} + g\cdot 0 = 0 + 0 \Rightarrow \frac{Y_0}{m} = 0$

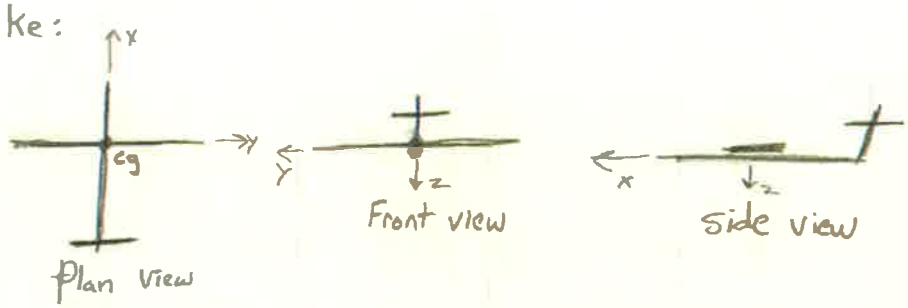
Thus

$$\boxed{\frac{\Delta Y}{m} + g\,\Delta\phi\cos\theta_0 = \Delta\dot{V} + \Delta r U_0}$$

Expanding ΔY as a linearized derivative gives table 3.2

3.4 Discuss why I_{yz} and I_{xy} are usually 0 and I_{xz} is not. 10

The reason why the products of inertia I_{yz} and I_{xy} are usually zero and I_{xz} is not is due to the symmetry of aircraft. A general airplane looks like:

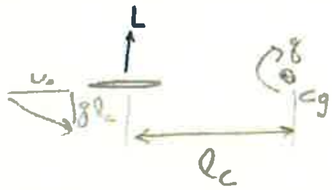


I_{yz} is defined as $\iiint yz\,dm$. From the front view, the airplane has a symmetry line along the z axis. Since the sign of yz will be opposite across the symmetry line, the sum of all of the yzdm's will add to zero. Similarly, I_{xy} is defined as $\iiint xy\,dm$. The airplane has a symmetry line about the x axis. Thus $I_{xy} = 0$.

However, there is no symmetry in the xz plane. The fuselage, vertical, horizontal and wing are not symmetric along any axis. So, $\iiint xz\,dm$ must be non zero.

3.8 Develop an expression for C_{m_q} due to a canard surface.

10



$$M_{cg} = L\,\ell_c$$

so non dimensionalise

$$C_{m_{cg_c}} = \frac{M_{cg}}{QS_w\bar{c}} = \frac{C_L\,QS\,\ell_c}{QS_w\,\bar{c}} = V_c C_L = V_c C_{L_{\alpha_c}} \alpha_c$$

$$\alpha_c \approx -\frac{q\ell_c}{U_o}$$ for small angles.

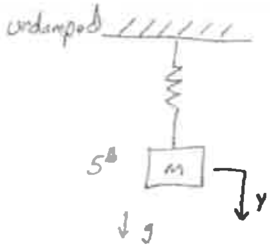
$$C_{m_{cg_c}} = V_c C_{L_{\alpha_c}} - q\frac{\ell_c}{U_o} \Rightarrow \frac{\partial C_{m_{cg_c}}}{\partial q} = -V_c C_{L_{\alpha_c}} \frac{\ell_c}{U_o}$$

$$C_{m_q} \equiv \frac{\partial C_m}{\partial\left(\frac{q\bar{c}}{2U_o}\right)} = \frac{2U_o}{\bar{c}}\frac{\partial C_m}{\partial q} = \frac{-2U_o}{\bar{c}} V_c C_{L_{\alpha_c}} \frac{\ell_c}{U_o}$$

$$\boxed{C_{m_q} = \frac{-2V_c\,\ell_c\,C_{L_{\alpha_c}}}{\bar{c}}}$$

4.2

a) undamped



$5^{lb} = \frac{5}{32}$ slug

8

$F = ma = -ky + mg = m\ddot{y} \Rightarrow \ddot{y} + \frac{k}{m}y = +g$

Initial defl.

$\ddot{y} = 0 \Rightarrow k = \frac{gm}{y} = \frac{32.2\,ft}{s^2}\Big|\frac{1}{1/12\,ft}\Big|\frac{5\,slug}{32.2}\Big|\frac{lbf\,s^2}{slug\,ft} = 60\,\frac{lbf}{ft}$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60\,slug}{s^2}\Big|\frac{32.2}{5\,slug}} = 19.6\,\frac{1}{s} \Rightarrow 3.1\,Hz$

A soln is expected of the form.

$y = C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t) + \cancel{C_3}\ 1in$

so,

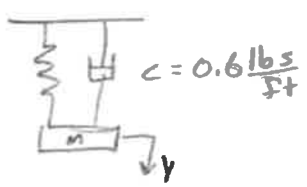
$\dot{y} = C_1 \omega_n \cos(\omega_n t) - C_2 \omega_n \sin(t)$

$\dot{y}(0) = 10\,\frac{ft}{s} \Rightarrow C_1 = \frac{10\,ft}{s}\Big|\frac{1}{\omega_n} = \frac{10\,ft}{s}\Big|\frac{s}{19.6} = 0.51\,ft$

$\boxed{y = 0.51\,ft \sin(19.6\,t) + \frac{1}{12}\,ft}$

the cosine term is useless here.

b) damped



$F = ma = -ky - c\dot{y} + mg = m\ddot{y} \Rightarrow \ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = mg$

From above, $k = 60\,\frac{lbf}{ft}$

$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = mg$

$\omega_n = \sqrt{\frac{k}{m}} = 19.6\,\frac{1}{s}$

$\zeta = \frac{c}{2\omega_n m} = \frac{0.6\,lbf\,s}{ft}\Big|\frac{1}{2}\Big|\frac{s}{19.6}\Big|\frac{32.2}{5\,slug}\Big|\frac{slug\,ft}{lbf\,s^2}$

$= 0.0986$

$\omega = \omega_n\sqrt{1-\zeta^2} = 19.6\sqrt{1-0.0986^2} = 19.5\,\frac{1}{s}$

A soln is expected of the form.

$$y = C_1 e^{-\xi \omega_n t} \sin(\omega t + \phi)$$

So,

$$\dot{y} = -\xi \omega_n C_1 e^{-\xi \omega_n t} \sin(\omega t + \phi) + C_1 \omega e^{-\xi \omega_n t} \cos(\omega t + \phi)$$

$$y(0) = 0 \Rightarrow \phi = 0$$

$$\dot{y}(0) = 10 \tfrac{ft}{s} \Rightarrow C_1 = \frac{\dot{y}(0)}{\omega} = \frac{10\,ft}{s} \cdot \frac{s}{19.5} = 0.51\,ft$$

$$y = 0.51\,ft\; e^{-(0.0986)(19.6)t} \sin(19.5t) + \frac{1}{12}\,ft$$

$$\boxed{y = 0.51\,ft\; e^{-1.93t} \sin(19.5t) + \cancel{\tfrac{1}{12}}\,ft}$$

4.4 Given $\ddot{\theta} + 2\dot{\theta} + 5\theta = -\delta$

a) Rewrite in state space form

This is a 2nd order equation, so there should be 2 states. We could rearrange the equation to $\ddot{\theta} = -2\dot{\theta} - 5\theta - \delta$ so that $\ddot{\theta}$ is a function of θ and $\dot{\theta}$ (and δ). Pick the states θ and $\dot{\theta}$

$$S = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$$

Thus the time derivative of S is $\frac{dS}{dt} = \dot{S} = \frac{d}{dt}\begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix}$

Well, the $\frac{d}{dt}(\theta)$ term (top row) is just the $\dot{\theta}$ state. And, $\ddot{\theta}$ can be written from $\ddot{\theta} = -2\dot{\theta} - 5\theta - \delta$. Putting both together gives

$$\dot{S} = \begin{pmatrix} \dot{\theta} \\ -2\dot{\theta} - 5\theta - \delta \end{pmatrix}$$

This is a linear function of $\underbrace{\text{states}}_{S}$ and $\underbrace{\text{inputs}}_{\eta}$

$$\dot{S} = \underbrace{\begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix}}_{\dot{S}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}}_{S} + \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{B} \underbrace{\delta}_{\eta}$$

$$\boxed{\dot{x} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} x + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \delta}$$

b) Determine the eigenvalues of the A matrix.

$$A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}$$

To find the eigenvalues, find $\underbrace{\left| \lambda I - A \right|}_{\text{determinant of } [\lambda] - [A]} = 0$

$$\det \begin{bmatrix} \lambda - 0 & -1 \\ +5 & \lambda + 2 \end{bmatrix} = \underbrace{\lambda^2 + 2\lambda + 5}_{} = 0$$

Not a coincidence that this is similar to $\frac{d^2\theta}{dt^2} + \frac{2d\theta}{dt} + 5\theta$

Both are equivalent characteristic equations

Quadratic Equ solution

$$\lambda^2 + 2\lambda + 5 = 0$$

$a = 1$

$b = 2$

$c = 5$

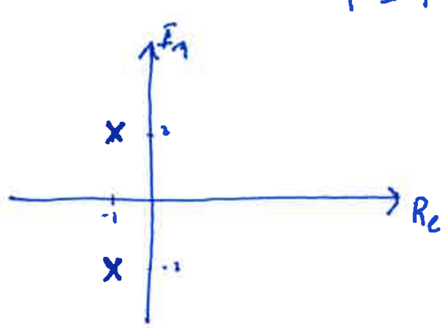
$$\Rightarrow \lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

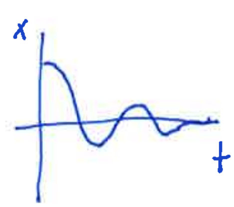
$$\lambda = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 5}}{2} = -1 \pm \frac{\sqrt{4 - 20}}{2}$$

$$= -1 \pm \frac{i\sqrt{16}}{2}$$

$$= -1 \pm i2$$

$$\boxed{\begin{aligned} \lambda_1 &= -1 + 2i \\ \lambda_2 &= -1 - 2i \end{aligned}}$$





verify with Matlab: eig([0,1;-5,-2]) ✓ (eigenvals)

Verify on TI calculator: czeros($x^2 + 2x + 5$, x) ✓