5.10 Examine influence of C_{l_β} and C_{n_r} on the lateral roots. Use 747 data.

$$\begin{pmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{pmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & \frac{Y_p}{u_0} & -\left(1-\frac{Y_r}{u_0}\right) & \frac{g}{u_0}\cos\theta_0 \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix}$$

$$Y_\beta = \frac{QSC_{y_\beta}}{m} \qquad L_\beta = \frac{QSbC_{l_\beta}}{I_x} \qquad N_p = \frac{QSb^2C_{n_p}}{2I_z u_0}$$

$$Y_p = \frac{QSbC_{y_p}}{2mu_0} \qquad L_p = \frac{QSb^2C_{l_p}}{2I_x u_0} \qquad N_\beta = \frac{QSbC_{n_\beta}}{I_z}$$

$$Y_r = \frac{QSbC_{y_r}}{2mu_0} \qquad L_r = \frac{QSb^2C_{l_r}}{2I_x u_0} \qquad N_r = \frac{QSb^2C_{n_r}}{2I_z u_0}$$

Sea level:

$M = 0.14 \rightarrow a = 1117\ ft/s \Rightarrow u_0 = 156.4\ ft/s$

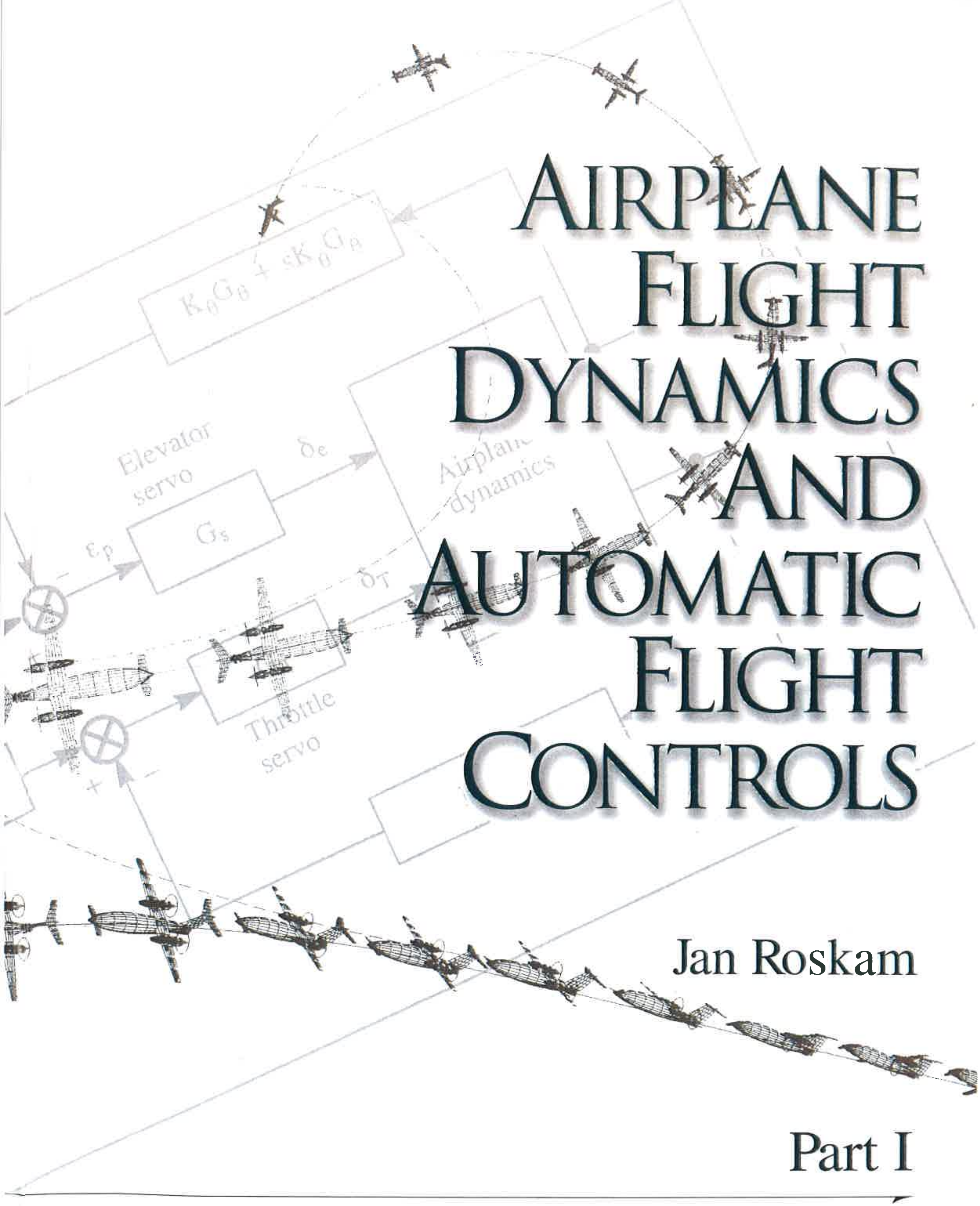
$Q = \frac{1}{2}\rho V^2 = \frac{1}{2}\, 0.00237 \frac{slug}{ft^3} (156.4)^2 \frac{ft^2}{s^2} \frac{lbf\ s^2}{slug\ ft\ ft^2} = 28.9\ psf$

$\theta_0 = ?$

$$C_L \approx C_{L_0} + C_{L_\alpha}\alpha \Rightarrow \theta_0 = \alpha = \frac{C_L - C_{L_0}}{C_{L_\alpha}}$$

$$= \frac{1.5 - \ldots}{5.24} = 0.286\ rad$$

$\theta_0 \approx 0.286\ rad$



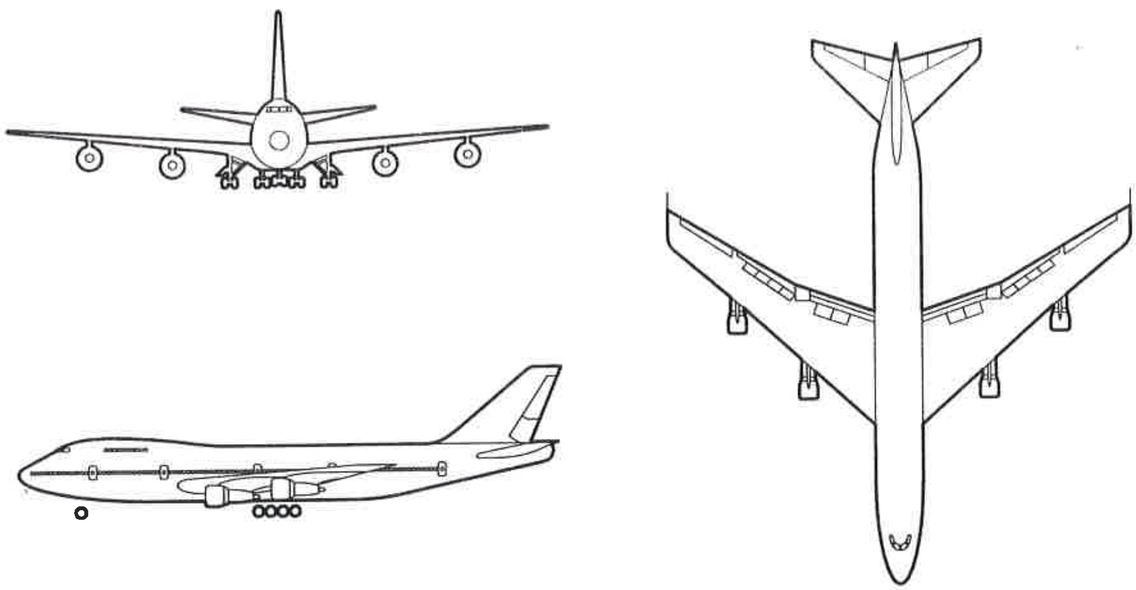
AIRPLANE FLIGHT DYNAMICS AND AUTOMATIC FLIGHT CONTROLS

Jan Roskam

Part I

Table B10 Stability and Control Derivatives for Airplane J (Pages 543–549)

Three–view



Reference Geometry

S (ft^2)	5,500
$\bar{c}$ (ft)	27.3
b (ft)	196

Flight Condition Data	Approach	Cruise (low)	Cruise (high)
Altitude, h (ft)	0	20,000	40,000
Mach Number, M	0.198	0.650	0.900
TAS, U_1(ft/sec)	221	673	871
Dynamic pressure, $\bar{q}$ (lbs/ft^2)	58.0	287.2	222.8
C.G. location, fraction $\bar{c}$	0.25	0.25	0.25
Angle of attack, α_1 (deg)	8.5	2.5	2.4

Mass Data

	Approach	Cruise (low)	Cruise (high)
W (lbs)	564,000	636,636	636,636
I_{xx_B} (slugft^2)	13,700,000	18,200,000	18,200,000
I_{yy_B} (slugft^2)	30,500,000	33,100,000	33,100,000
I_{zz_B} (slugft^2)	43,100,000	49,700,000	49,700,000
I_{xz_B} (slugft^2)	830,000	970,000	970,000

Table B10 (Continued) **Stability and Control Derivatives for Airplane J (Pages 543–549)**

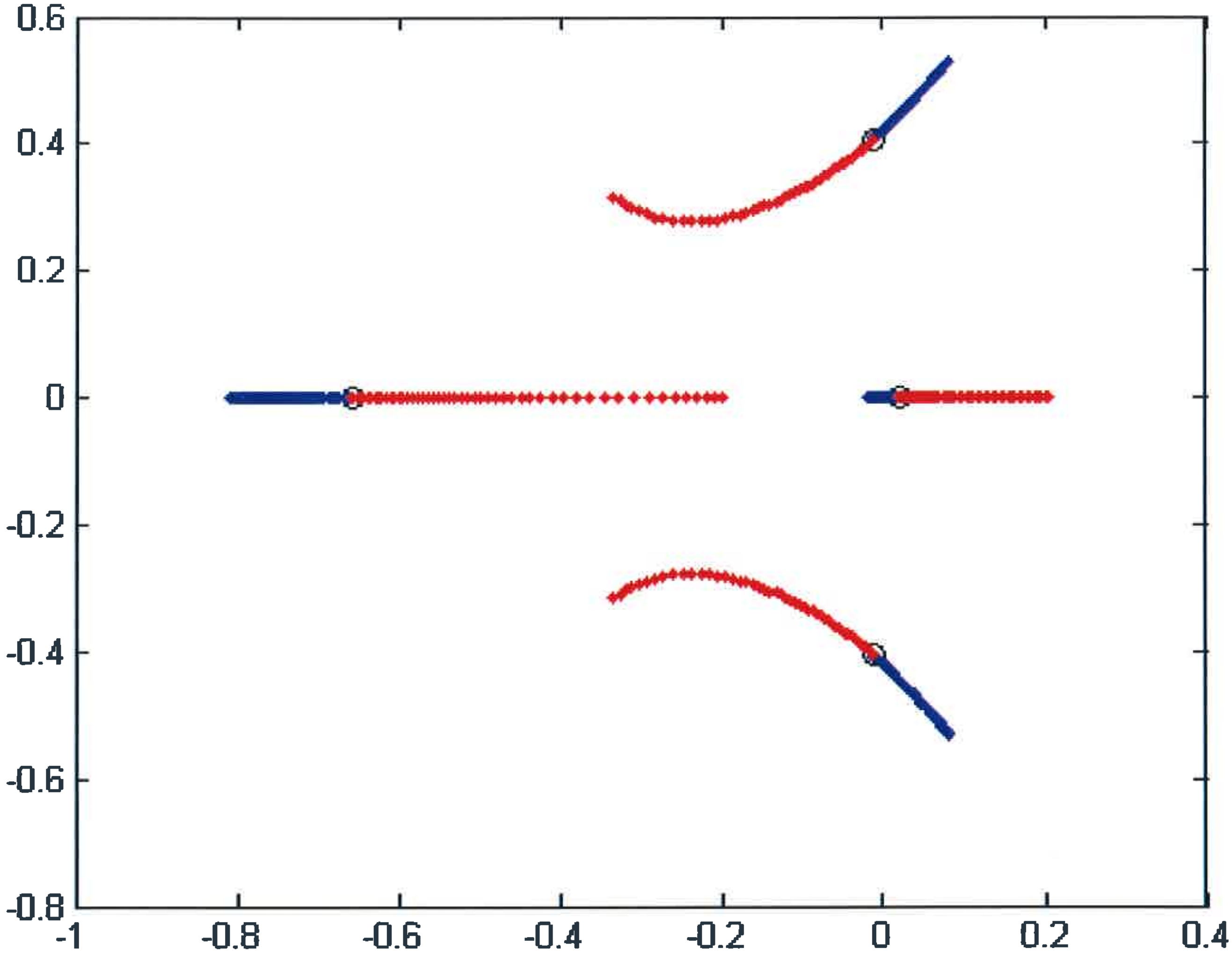
Flight Condition	Approach	Cruise (low)	Cruise (high)
Steady State Coefficients			
C_{L_1}	1.76	0.40	0.52
C_{D_1}	0.2630	0.0250	0.0450
$C_{T_{x_1}}$	0.2630	0.0250	0.0450
C_{m_1}	0	0	0
$C_{m_{T_1}}$	0	0	0
Longitudinal Coefficients and Stability Derivatives (Stability Axes, Dimensionless)			
C_{D_0}	0.0751	0.0164	0.0305
C_{D_u}	0	0	0.22
C_{D_α}	1.13	0.20	0.50
$C_{T_{x_u}}$	–0.5523	–0.055	–0.950
C_{L_0}	0.92	0.21	0.29
C_{L_u}	–0.22	0.13	–0.23
C_{L_α}	5.67	4.4	5.5
$C_{L_{\dot{\alpha}}}$	6.7	7.0	8.0
C_{L_q}	5.65	6.6	7.8
C_{m_0}	0	0	0
C_{m_u}	0.071	0.013	–0.09
C_{m_α}	–1.45	–1.00	–1.60
$C_{m_{\dot{\alpha}}}$	–3.3	–4.0	–9.0
C_{m_q}	–21.4	–20.5	–25.5
$C_{m_{T_u}}$	0	0	0
$C_{m_{T_\alpha}}$	0	0	0
Longitudinal Control and Hinge Moment Derivatives (Stability Axes, 1/rad)			
$C_{D_{\delta_e}}$	0	0	0
$C_{L_{\delta_e}}$	0.36	0.32	0.30
$C_{m_{\delta_e}}$	–1.40	–1.30	–1.20
$C_{D_{i_h}}$	0	0	0
$C_{L_{i_h}}$	0.75	0.70	0.65
$C_{m_{i_h}}$	–3.0	–2.7	–2.5

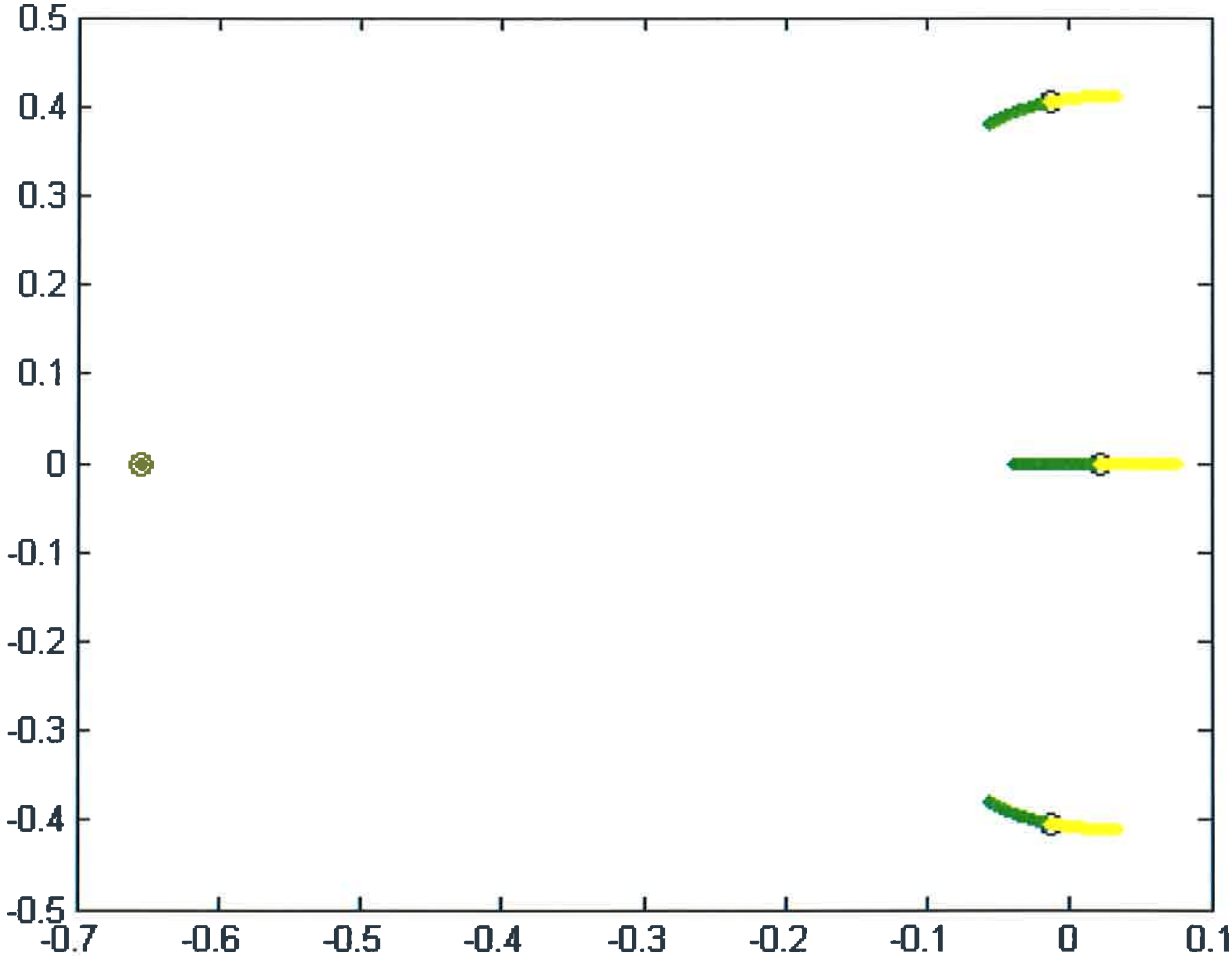
Table B10 (Continued) Stability and Control Derivatives for Airplane J (Pages 543–549)

Flight Condition	Approach	Cruise (low)	Cruise (high)
Longitudinal Control and Hinge Moment Derivatives: Cont'd (Stability Axes, 1/rad)			
C_{h_α}	???	???	???
$C_{h_{\delta_e}}$	???	???	???
Lateral–Directional Stability Derivatives (Stability Axes, Dimensionless)			
C_{l_β}	–0.281	–0.160	–0.095
C_{l_p}	–0.502	–0.340	–0.320
C_{l_r}	0.195	0.130	0.200
C_{y_β}	–1.08	–0.90	–0.90
C_{y_p}	0	0	0
C_{y_r}	0	0	0
C_{n_β}	0.184	0.160	0.210
$C_{n_{T_\beta}}$	0	0	0
C_{n_p}	–0.222	–0.026	0.020
C_{n_r}	–0.360	–0.280	–0.330
Lateral–Directional Control and Hinge Moment Derivatives (Stability Axes, Dimensionless)			
$C_{l_{\delta_a}}$	0.053	0.013	0.014
$C_{l_{\delta_r}}$	0	0.008	0.005
$C_{y_{\delta_a}}$	0	0	0
$C_{y_{\delta_r}}$	0.179	0.120	0.060
$C_{n_{\delta_a}}$	0.0083	0.0018	–0.0028
$C_{n_{\delta_r}}$	–0.113	–0.100	–0.095
$C_{h_{\alpha_a}}$	???	???	???
$C_{h_{\delta_a}}$	???	???	???
$C_{h_{\beta_r}}$	???	???	???
$C_{h_{\delta_r}}$	???	???	???

```
%
%
%

plot(eig(Boeing747Lateral(1,1)), 'ok');
hold on

%Vary Clb (dihedral)
N=50;
vary = linspace(1,3,N);
for ratio  = 1:N
    plot(eig(Boeing747Lateral(vary(ratio),1)), '.b');
end
vary = linspace(1,-1,N);
for ratio  = 1:N
    plot(eig(Boeing747Lateral(vary(ratio),1)), '.r');
end

figure
plot(eig(Boeing747Lateral(1,1)), 'ok');
hold on
%Vary Cnr (directional stab)
N=50;
vary = linspace(1,3,N);
for ratio  = 1:N
    plot(eig(Boeing747Lateral(1,vary(ratio))), '.g');
end
vary = linspace(1,-1,N);
for ratio  = 1:N
    plot(eig(Boeing747Lateral(1,vary(ratio))), '.y');
end
```

```
function A = Boeing747Lateral( ratioClb, ratioCnr )
%UNTITLED Summary of this function goes here
%   Detailed explanation goes here

W = 636600 ; %lbf
g = 32.2; % ft/s^2
m = W/32.2; %slug
S = 5500;
b = 195.68; % ft
q = 28.9 ; % psf
u0 = 156.4 ; % ft/s
theta0 = 0.286; % rad

% Inertia Matrix
Ix = 18.2E6 ; % slug ft^2
Iz = 49.7E6 ; % slug ft^2

Cyb = -0.362;
Cyp = 0;
Cyr = 0;

Clb = -0.125 * ratioClb;
Clp = -0.53;
Clr = 0.410;

Cnb = 0.101;
Cnp = -0.283;
Cnr = -0.188 * ratioCnr;

Yb = q*S*Cyb/m;
Yp = q*S*b*Cyp/2/m/u0;
Yr = q*S*b*Cyr/2/m/u0;

Lb = q*S*b*Clb/Ix;
Lp = q*S*b^2*Clp/2/Ix/u0;
Lr = q*S*b^2*Clr/2/Ix/u0;

Nb = q*S*b*Cnb/Iz;
Np = q*S*b^2*Cnp/2/Iz/u0;
Nr = q*S*b^2*Cnr/2/Iz/u0;

A =[ Yb/u0,  Yp/u0, -(1-Yr/u0), g/u0*cos(theta0);
    Lb, Lp, Lr, 0;
    Nb, Np, Nr, 0;
    0, 1, 0, 0];

end
```


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1