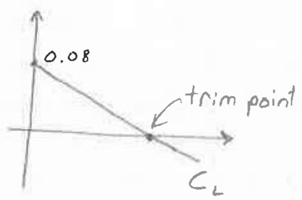
2.1 Determine trim C_L and X_{np}.

4

$$\frac{\partial C_m}{\partial C_L} = -0.15 \qquad C_{m_0} = 0.08 \qquad \frac{X_{cg}}{\bar{c}} = 0.3$$

32/32

trim C_L:



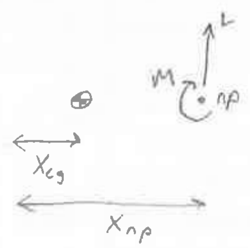
The trim pt is at $C_m = 0$.

$$C_m = C_{m_0} + \frac{\partial C_m}{\partial C_L} C_L = 0 \Rightarrow C_L = \frac{-C_{m_0}}{\frac{\partial C_m}{\partial C_L}} = \frac{-0.08}{-0.15}$$

$$\boxed{C_{L_{trim}} = 0.53}$$

X_{np}:

X_{np} is where the total aircraft $C_{L_\alpha} = 0$.



$$M_{cg} = M - L(X_{np} - X_{cg})$$

$$C_{M_{cg}} = C_{m_{np}} - C_L\left(\frac{X_{np}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}\right)$$

take derivative WRT C_L

$$\frac{\partial C_{m_{cg}}}{\partial C_L} = \cancel{\frac{\partial C_{m_{np}}}{\partial C_L}}^{\,0} - \cancel{\frac{\partial C_L}{\partial C_L}}^{\,1}\left(\frac{X_{np}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}\right)$$

so

$$\frac{\partial C_{m_{cg}}}{\partial C_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{np}}{\bar{c}}$$

$$\frac{X_{np}}{\bar{c}} = \frac{X_{cg}}{\bar{c}} - \frac{\partial C_{m_{cg}}}{\partial C_L}$$

$$= 0.3 - (-0.15) = 0.45$$

$$\boxed{X_{np} = 0.45\,\bar{c}}$$

2.2 Determine from Fig P2.2 and $W = 2750\,lb$, $S = 180\,ft^2$, $cg = 0.25\bar{c}$

4

a) Stick fixed np.

$$\frac{\partial C_{m_{cg}}}{\partial C_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{np}}{\bar{c}} \Rightarrow \frac{X_{np}}{\bar{c}} = \frac{X_{cg}}{\bar{c}} - \frac{\partial C_{m_{cg}}}{\partial C_L}$$

From the Figure, and a convienient line representing $\delta_e = -15°$

$$\frac{\partial C_{m_{cg}}}{\partial C_L} = \left.\frac{\Delta C_m}{\Delta C_L}\right|_{\delta_e = -15°} = \frac{0 - .15}{1.0 - 0} = -0.15$$

Also,

$$cg = 0.25\bar{c} \Rightarrow \frac{X_{cg}}{\bar{c}} = 0.25$$

so,

$$\frac{X_{np}}{\bar{c}} = 0.25 - (-0.15) = 0.40$$

$$\boxed{X_{np} = 0.4\,\bar{c}}$$

b) Elevator angle to trim at $125\frac{ft}{s}$ at SSL.

$$W = L = C_L \tfrac{1}{2}\rho V^2 S \Rightarrow C_L = \frac{L}{\tfrac{1}{2}\rho V^2 S}$$

AT SSL, $\rho = 2.3769 \times 10^{-3}\,\frac{slug}{ft^3}$

$$C_L = \frac{2750\,lb}{0.5} \cdot \frac{\cancel{ft^3}}{2.3769\times10^{-3}\,\cancel{slug}} \cdot \frac{\cancel{s^2}}{125^2\,\cancel{ft^2}} \cdot \frac{1}{180\,\cancel{ft^2}} \cdot \frac{\cancel{slug\,ft}}{lbf\,s^2}$$

$$\boxed{C_{L_{trim}} = 0.82}$$

From the figure,

C_m must be zero and $C_L = 0.82$.

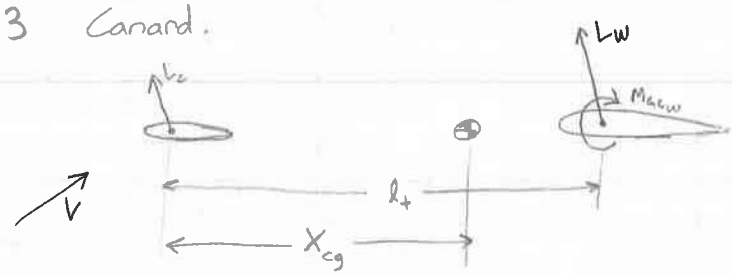
$C_L(\delta_e = -10) = .76$

$C_L(\delta_e = -15) = 1.0$

Interpolate for $C_L = 0.82$ $\Rightarrow$ $\boxed{\delta_e = -11.25°}$

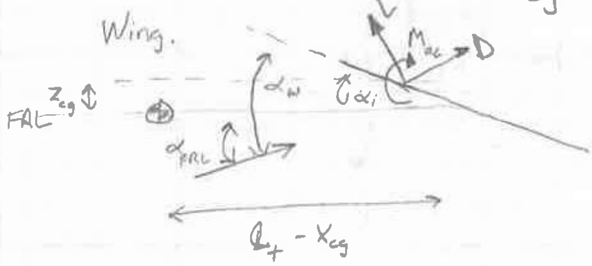
4

2.3 Canard.



a) Moment coefficient about the cg

Wing.

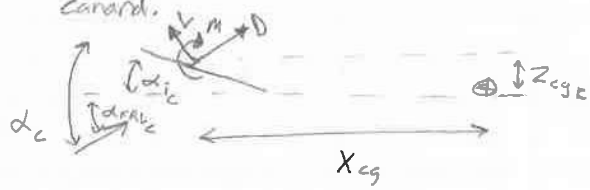


$$M_{cg_w} = M_{ac} - L\cos(\alpha_w - \alpha_i)(l_t - X_{cg}) - L\sin(\alpha_w - \alpha_i)(Z_{cg})$$
$$+ D\cos(\alpha_w - \alpha_i)(Z_{cg}) - D\sin(\alpha_w - \alpha_i)(l_t - X_{cg})$$

Neglect Drag and Z_{cg}. Small angle approx for cos, sin.

$$M_{cg_w} = M_{ac} - L_w(l_t - X_{cg})$$

canard.



$$M_{cg_c} = M_{ac} + L\cos(\alpha_c - \alpha_{i_c})(X_{cg}) + L\sin(\alpha_c - \alpha_{i_c})(Z_{cg_c})$$
$$+ D\sin(\alpha_c - \alpha_{i_c})(X_{cg}) + D\cos(\alpha_c - \alpha_{i_c})(Z_{cg_c})$$

Neglect Drag and z_{cg}. Small Angle approx. Neglect M_{ac}

$$M_{cg_c} = L_c X_{cg}$$

Sum the moment and Nondimensionalize:

$$M_{cg} = M_{ac_w} - L_w(l_t - X_{cg}) + L_c X_{cg}$$

divide by $\frac{1}{2}\rho V^2 \bar{c} S_w$

$$C_{m_{cg}} = C_{m_{ac_w}} - C_{L_w}\left(\frac{l_t}{\bar{c}} - \frac{X_{cg}}{\bar{c}}\right) + C_{L_c}\frac{X_{cg}}{\bar{c}}\frac{S_t}{S_w}$$

Assume linear C_L vs α

$$C_{m_{cg}} = C_{m_{ac_w}} + \left(C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w\right)\left(\frac{X_{cg}}{\bar{c}} - \frac{l_t}{\bar{c}}\right) + \frac{S_c}{S_w} C_{L_{\alpha_c}} \alpha_c \frac{X_{cg}}{\bar{c}}$$

and $\alpha_c = \alpha_w - \alpha_{i_w} + \alpha_{i_c}$

$$C_{m_{cg}} = C_{m_{ac_w}} + \left(C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w\right)\left(\frac{X_{cg}}{\bar{c}} - \frac{l_t}{\bar{c}}\right) + \frac{S_c}{S_w}\frac{X_{cg}}{\bar{c}} C_{L_{\alpha_c}} \left(\alpha_w - \alpha_{i_w} + \alpha_{i_c}\right)$$

so,

$$\boxed{\begin{aligned} C_{m_{cg_0}} &= C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{l_t}{\bar{c}}\right) + \frac{S_c}{S_w}\frac{X_{cg}}{\bar{c}} C_{L_{\alpha_c}}\left(\alpha_{i_c} - \alpha_{i_w}\right) \\ C_{m_\alpha} &= C_{L_{\alpha_w}}\left(\frac{X_{cg}}{\bar{c}} - \frac{l_t}{\bar{c}}\right) + \frac{S_c}{S_w}\frac{X_{cg}}{\bar{c}} C_{L_{\alpha_c}} \\ \text{where } & C_{m_{cg}} = C_{m_{cg_0}} + C_{m_{\alpha_w}} \alpha_w \end{aligned}}$$

b) Neutral point.

$AR_c = AR_w \qquad S_c = 0.2 S_w \qquad \bar{c}_c = 0.45\bar{c}$

At the np, $C_{m_\alpha} = 0$ so,

$$0 = C_{L_{\alpha_w}}\left(\frac{X_{AB}}{\bar{c}} - \frac{l_t}{\bar{c}}\right) + \overset{0.2}{\frac{S_c}{S_w}} \frac{X_{AB}}{\bar{c}} C_{L_{\alpha_c}}$$

and since $AR_c = AR_w$, $C_{L_{\alpha_w}} \approx C_{L_{\alpha_c}}$

$$0 = \frac{X_{AB}}{\bar{c}} - \frac{l_t}{\bar{c}} + 0.2\frac{X_{AB}}{\bar{c}} = 1.2\frac{X_{AB}}{\bar{c}} - \frac{l_t}{\bar{c}} = 0$$

$$X_{np} = \frac{l_t}{1.2} = \boxed{0.83 l_t = X_{np}}$$

good

2.5 Determine the following for the Jetstar $2 \times 4 = 8$

a) Wing Contribution to pitch moment.

The airfoil looks symmetrical (hard to tell from 3view drawings)

So, $C_{m_{0_w}} = 0$ $C_{L_{0_w}} = 0$ From App B $C_{L_{\alpha_w}} = 5.0$ $AR = \frac{b^2}{S} = 5.3$

$\frac{X_{cg}}{\bar{c}} = 0.25$

$$C_{m_{\alpha_w}} = C_{L_{\alpha_w}} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) = 5.0(0.25 - 0.25) = 0$$

The wing has no influence on $C_{m_{cg}}$!

b) tail moments ✓

$\lambda_t = \frac{3.3}{9.8} = 0.33$ $C_0 = 9.8$ $C_t = 3.3$ $b_t =$

$$\bar{c} = \frac{2}{3} C_0 \frac{1 + \lambda + \lambda^2}{1 + \lambda} = \frac{2}{3} 9.8 \frac{1 + 0.33 + 0.33^2}{1 + 0.33} = 7.0^{ft}$$

$$S_t = \left(\frac{9.8 + 3.3}{2}\right)(23.75^{ft}) = 155.6^{ft^2}$$

$$\ell_t = 21.7^{ft}$$

$$V_H = \frac{S_t \ell_t}{S \bar{c}} = \frac{(155.6)(21.7)}{(542.5)(10.93)} = 0.57$$

Estimate $\eta = 1$ (due to horz stab up out of the fuselage and wing wake.)

$$C_{L_{\alpha_t}} \approx \frac{2\pi}{1 + \frac{2\pi}{3.6\pi}} = 4.0 \tfrac{1}{rad}$$

$$\frac{d\epsilon}{d\alpha} \approx \frac{2 C_{L_{\alpha_w}}}{\pi AR_w} = \frac{(2)(5.0)}{\pi (5.3)} = 0.6 \tfrac{1}{rad}$$

$$C_{L_{0_w}} = 0 \Rightarrow \epsilon_0 = 0$$

From the side view, $i_w - i_t = 2°$

$$C_{m_{0_t}} = \eta V_H C_{L_{\alpha_t}} (\epsilon_0 + i_w - i_t) = (1)(0.57)(4.0)(0 + 2°)\left(\frac{1}{57.3}\right) = 0.080$$

$$C_{m_{\alpha_t}} = -\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right) = -(1)(0.57)(4.0)(1 - 0.6) = -0.912 \tfrac{1}{rad}$$

$$\boxed{\begin{aligned} C_{m_{0_t}} &= 0.080 \\ C_{m_{\alpha_t}} &= -0.912 \tfrac{1}{rad} \end{aligned}}$$

c) Fuselage contribution ✓

$$\frac{l_f}{d_{max}} = \frac{60.5^{ft}}{7.2^{ft}} = 8.4 \Rightarrow K_2 - K_1 = 0.9$$

From the spreadsheet,

$$\sum w_f^2 (\alpha_{0_w} + i_f)\Delta X = 3365$$

$$\sum w_f^2 \frac{\partial \epsilon_u}{\partial \alpha}\Delta X = 1866$$

So,

$$C_{m_{0_f}} = \frac{0.9 \quad 3365}{(36.5)(542.5)(10.93)} = 0.013$$

$$C_{m_{\alpha_f}} = \frac{1866}{(36.5)(542.5)(10.93)} = 0.0086 \frac{1}{deg} = 0.\cancel{49} \frac{1}{rad} = 0.054$$

Math Error

$$\boxed{\begin{array}{l} C_{m_{0_f}} = 0.013 \\ C_{m_{\alpha_f}} = 0.\cancel{49} \frac{1}{rad} \end{array}}$$ ← this seems too high yes!

	section	del x	wf	if	x	x/c	dEu/da	wf^2*(if)*del x	wf^2 dEu/da delx
Fore	1	15	7.2	1	7.5	0.517241	1.4	777.6	1088.64
	2	2.73	6.42	-10	16.36	1.128276	1.2	-1125.20772	135.0249264
	3	2.73	5.46	1	18.97	1.308276	1.2	81.385668	97.6628016
	4	2.73	1.6	1	21.7	1.496552	1.2	6.9888	8.38656
	5	1.37	3	1	23.75	1.637931	1.2	12.33	14.796
	6	0.92	1.64	1	24.8	1.710345	1.2	2.474432	2.9693184
	7	5.47	2.2	1	3.3	0.3	1.6	26.4748	42.35968
	8	8.2	3	1	2	0.181818	1.8	73.8	132.84
Aft	9	5.46	2.3	1	1.36	0.093793	0.025069	28.8834	0.724081548
	10	5.46	1.36	1	4	0.275862	0.073733	10.098816	0.744613161
	11	2.73	4.91	1	12.3	0.848276	0.226728	65.815113	14.92213622
	12	2.73	3.82	1	15	1.034483	0.276498	39.837252	11.01490839
	13	2.73	2.73	1	17.7	1.22069	0.326267	20.346417	6.638370155
	14	1.9	1.64	1	19.9	1.372414	0.36682	5.11024	1.87453965
Engines	15	9.6	18.3	1	4.8	0.331034	0.088479	3214.944	284.4558747
	16	1.63	8.7	1	10.2	0.703448	0.188018	123.3747	23.19671779
								3364.255918	1866.250528

d) Total

$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_t}} + C_{m_{0_f}}$$

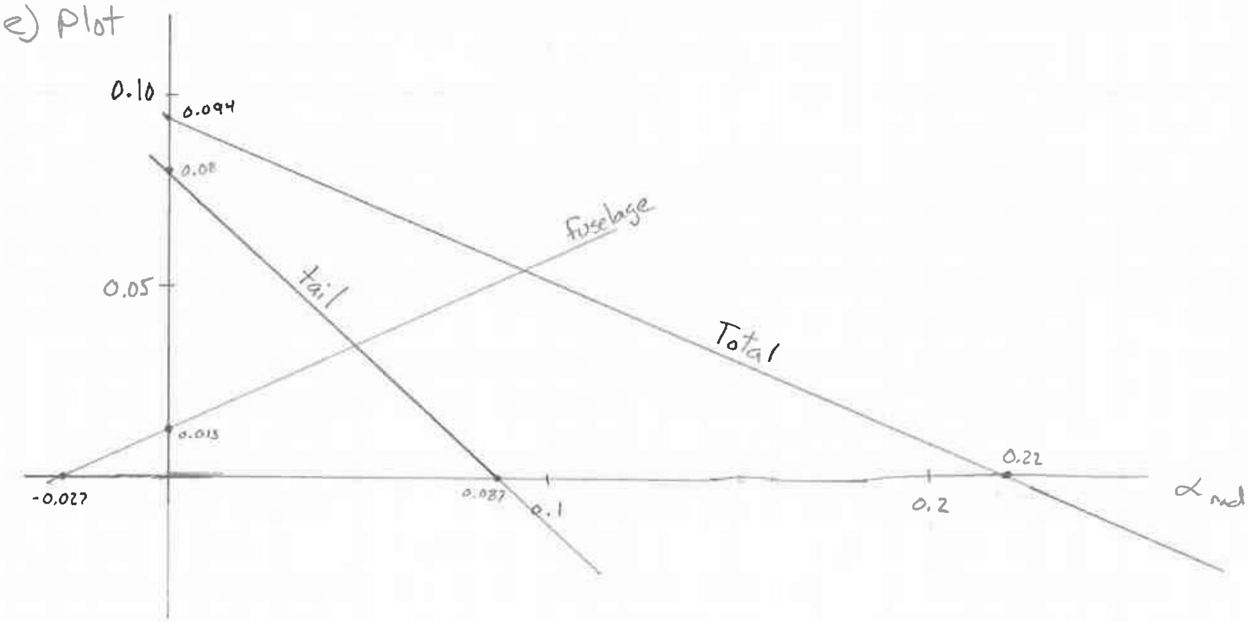
$$= 0 + 0.080 + 0.013 = 0.093 = C_{m_0}$$

$$C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_t}} + C_{m_{\alpha_f}}$$

$$= 0 - .912 + 0.49 = -0.422 \ \tfrac{1}{rad}$$

$$\boxed{\begin{aligned} C_{m_0} &= 0.094 \\ C_{m_\alpha} &= -0.422 \end{aligned}}$$

e) Plot



$$\alpha_{cm=0_t} = \frac{C_{m_{0t}}}{-C_{m_{\alpha t}}} = \frac{-0.080}{-0.912} = 0.087$$

$$\alpha_{cm=0_f} = \frac{C_{m_{0f}}}{-C_{m_{\alpha f}}} = \frac{0.013}{-0.49} = -0.027$$

$$\alpha_{cm=0_{total}} = \frac{-0.094}{-0.422} = 0.22$$

f) Stick fixed NP.

$$C_{m_\alpha} = 0$$

$$\Rightarrow \quad C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_t}} + C_{m_{\alpha_f}} = 0$$

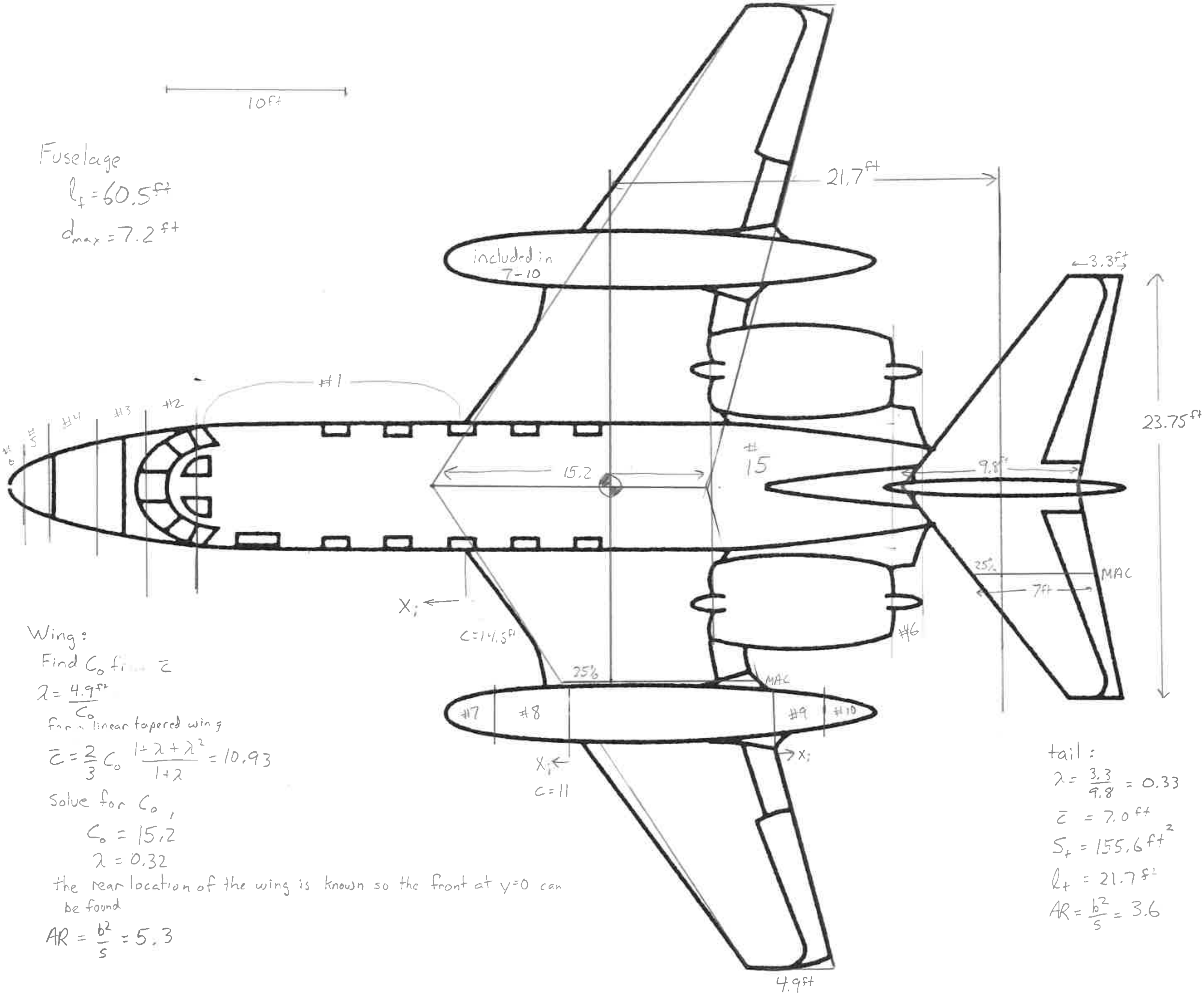
$$= \underbrace{C_{L_{\alpha_w}}}_{5.0}\left(\frac{X_{cg}}{\bar{c}} - \underbrace{\frac{X_{ac}}{\bar{c}}}_{0.25}\right) - 0.912 + 0.49 = 0$$

Solve for $\frac{X_{cg}}{\bar{c}}$

$$\frac{X_{np}}{\bar{c}} = 0.33 \quad \Rightarrow \quad X_{np} = (0.33)(10.93) = 3.65\,ft$$

$$\boxed{X_{np} = 3.65\,ft}$$

Not bad



Fuselage

$l_f = 60.5^{ft}$

$d_{max} = 7.2^{ft}$

Wing:

Find C_o fr... $\bar{c}$

$\lambda = \frac{4.9^{ft}}{C_o}$

for a linear tapered wing

$\bar{c} = \frac{2}{3} C_o \frac{1+\lambda+\lambda^2}{1+\lambda} = 10.93$

Solve for C_o,

$C_o = 15.2$

$\lambda = 0.32$

the rear location of the wing is known so the front at y=0 can be found

$AR = \frac{b^2}{S} = 5.3$

tail:

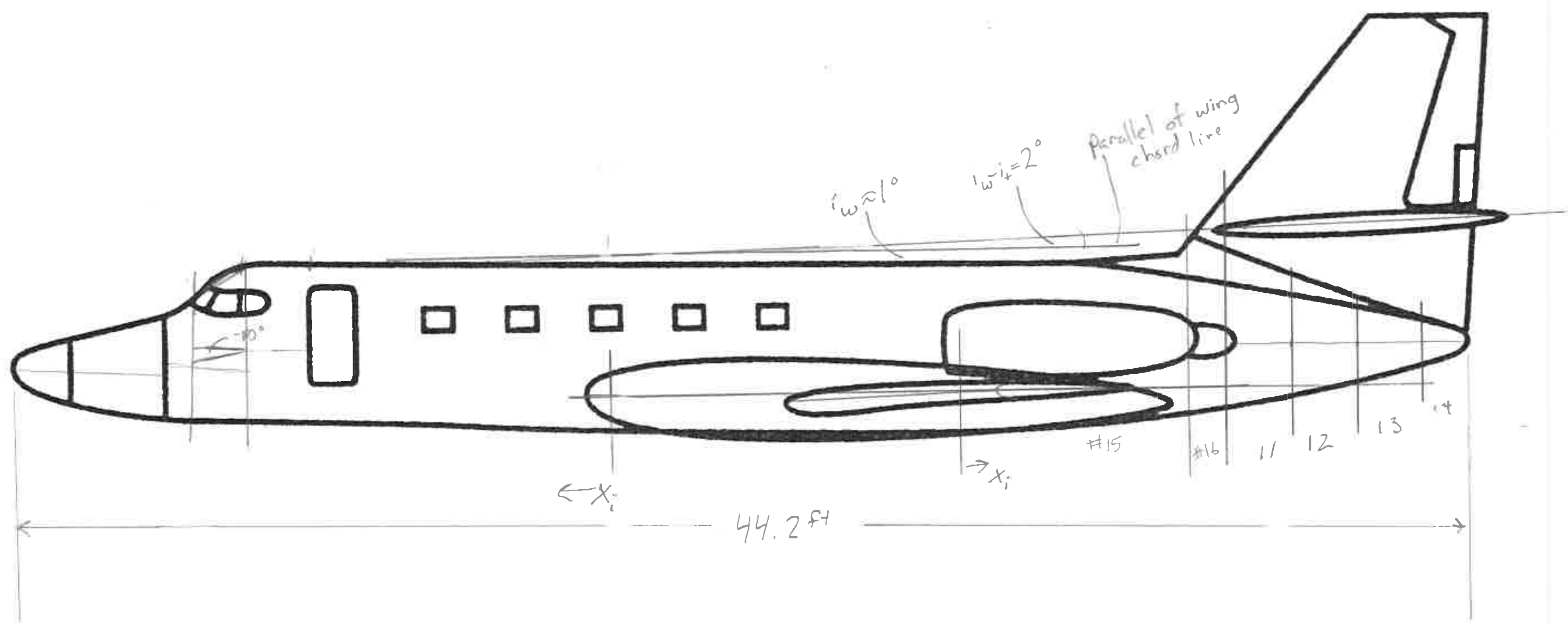
$\lambda = \frac{3.3}{9.8} = 0.33$

$\bar{c} = 7.0^{ft}$

$S_t = 155.6 ft^2$

$l_t = 21.7^{ft}$

$AR = \frac{b^2}{S} = 3.6$



or a Jetstar executive business jet.

2.7 From Fig P2.7 and $X_{cg} = 0.25c$
$S = 150 ft^2$
$\bar{c} = 5 ft$

4

a) Where is the stick fixed NP?
From Fig P2.7, at $\delta_e = -8°$

$$\frac{\partial C_{m_{cg}}}{\partial C_L} = \left.\frac{\Delta C_m}{\Delta C_L}\right|_{\delta_e=-8°} = \frac{0-0.4}{1.6-0} = -0.25$$

Also

$$\frac{\delta C_{m_{cg}}}{\delta C_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{np}}{\bar{c}} \Rightarrow \frac{X_{np}}{\bar{c}} = \frac{X_{cg}}{\bar{c}} - \frac{\delta C_{m_{cg}}}{\delta C_L} = 0.25 - (-0.25) = 0.50$$

so

$$X_{np} = 0.5\bar{c} = 0.5(5^{ft}) = \boxed{2.5^{ft} = X_{np}}$$

b) What is δ_e for trim?
$W = 2500^{lb}$
$V = 150 \frac{ft}{s}$
$\rho = 0.002378 \frac{slug}{ft^3}$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{2500^{lb}}{0.5} \cdot \frac{ft^3}{0.002378\ slug} \cdot \frac{s^2}{150^2\ ft^2} \cdot \frac{1}{150 ft^2} \cdot \frac{slug\ ft}{lbf\ s^2}$$

$$C_L = 0.62$$

From Fig p2.7
$C_{m_{cg}} = 0$ and $C_L = 0.62$

$C_L(\delta_e = 0) = 0.4$
$C_L(\delta_e = -4) = 1.0$
Interpolate for $C_L = 0.62 \Rightarrow \boxed{\delta_e = -1.5°}$

c)
Extending a landing gear will lower the pitch moment curve. The Landing gear's low location and drag cause a nose down moment.

Lowering flaps has two major influences on the pitch moment. First, the increased camber causes a nose down pitching moment. The second effect due to a drag increase depends on where the wing is located relative to the cg. A low wing aircraft tends to nose down more than a high wing due to the nose down moment caused by the drag below the cg.

Downwash too!

6

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2.9 Estimate ϵ_0 and $\frac{d\epsilon}{d\alpha}$

4

Assume linear ϵ vs α.

$$\epsilon = \epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha = \alpha_w - i_w + i_t$$

intersection #1:

$\alpha_w = 7.4°$ $i_w = 1°$ $i_t = -3°$

$$\Rightarrow \epsilon_0 + \frac{d\epsilon}{d\alpha} 7.4° = 7.4° - 1° + (-3°) = 3.4°$$

intersection #2:

$\alpha_w = 2°$ $i_w = 1°$ $i_t = 1°$

$$\Rightarrow \epsilon_0 + \frac{d\epsilon}{d\alpha} 2° = 2° - 1° + 1° = 2°$$

solve simutaneously,

$\boxed{\frac{d\epsilon}{d\alpha} = 0.260}$ $\boxed{\epsilon_0 = 1.48°}$

so, ϵ vs α is

$$\boxed{\epsilon = 1.48° + 0.260\,\alpha}$$

2.12 Determine the forward cg limit for the airplane in ex pr 2.2

$$C_{L_{max}} = 1.4 \qquad C_{m_{\delta_e}} = -1.03/\text{rad} \qquad \delta_e = \begin{cases} +10^\circ = +0.175\ \text{rad} \\ -20^\circ = -0.350\ \text{rad} \end{cases}$$

4

From ex pr 2.2

$$C_{L_{\alpha_w}} = 4.3\ \tfrac{1}{\text{rad}} \quad \text{and} \quad \alpha_{0_L} = -5^\circ$$

thus for the landing C_L,

$$C_{L_w} = C_{L_{\alpha_w}}(\alpha - \alpha_{0_L}) = 4.3\ \tfrac{1}{\text{rad}}\left(\tfrac{1}{57.3}\right)(\alpha + 5^\circ) = 1.4$$

$$\Rightarrow \alpha_w = 13.7^\circ = 0.24\ \text{rad}$$

The forward cg is determined by the up elevator, thus $\delta_e = -20^\circ$. From Ex 2.2

$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + C_{m_{0_f}} + C_{m_{\delta_e}}\delta_e$$

$$= -0.116 + 0.375\left(\frac{X_{cg}}{\bar{c}} - 0.25\right) + 0.194 - 0.037 - 1.03(\delta_e)$$

$$C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_f}} + C_{m_{\alpha_f}}$$

$$= 4.3\left(\frac{X_{cg}}{\bar{c}} - 0.25\right) - 1.42 + 0.12 = 4.3\left(\frac{X_{cg}}{\bar{c}}\right) - 2.375$$

$$C_m = 0 = C_{m_0} + C_{m_\alpha}\alpha_w$$

where $\alpha_w = 0.24$, $\delta_e = -0.350$

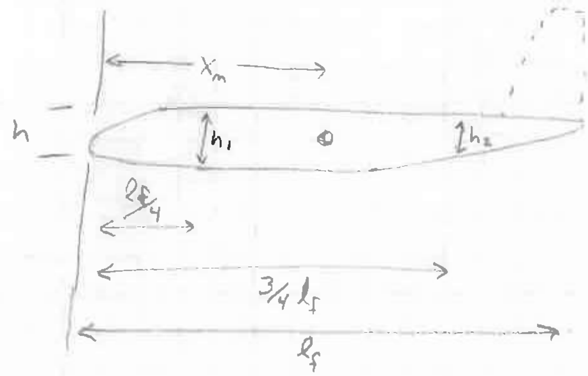
solve

$$\boxed{\frac{X_{cg}}{\bar{c}} = 0.186}$$

ok

Forward CG

2.13) Size the vertical tail so that $C_{n\beta} = 0.1 \frac{1}{rad}$ 4



$V = 150 \frac{m}{s}$ S.L.
$S = 21.3\ m^2$
$b = 10.4\ m$
$z_w = 0.4\ m$
$d = 1.6\ m$
$l_f = 13.7\ m$
$x_m = 8.0\ m$

$w_f = 1.6\ m$
$S_{fs} = 15.4\ m^2$
$h_1 = 1.6\ m$
$h_2 = 1.07\ m$
$h = 1.6\ m$

20/20

$$C_{n\beta} = C_{n\beta_f} + C_{n\beta_w} + C_{n\beta_v} = 0.1 \frac{1}{rad}$$

$$C_{n\beta_{wf}} + C_{n\beta_v}$$

From Eq 2.73 an estimate of the wing and fuselage contribution to $C_{n\beta}$ is

$$C_{n\beta_{wf}} = -K_n K_{Rl} \frac{S_{fs}\, l_f}{S_w\, b} \frac{1}{deg}$$

K_n: Found From Fig 2.29

$$\frac{x_m}{l_f} = \frac{8m}{13.7m} = 0.58 \qquad \frac{l_f^2}{S_{fs}} = \frac{(13.7m)^2}{15.4m^2} = 12.2$$

$$\sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{1.6m}{1.07m}} = 1.22 \qquad \frac{h}{w_f} = \frac{1.6m}{1.6m} = 1$$

$$\Rightarrow K_n \approx 0.0015$$

K_{Rl}:

$$R_{lf} = \frac{V l_f}{\nu} \qquad @\ SSL,\ \nu = 1.46 \times 10^{-5} \frac{m^2}{s}$$

$$= \frac{150\,m}{s} \cdot 13.7m \cdot \frac{s}{1.46\times10^{-5}\,m^2} = 1.4\times10^{8} = 140\times10^{6}$$

$$\Rightarrow K_{Rl} = 1.98$$

so,

$$C_{n\beta_{wf}} = -(0.0015)(1.98) \frac{15.4m^2 \cdot 13.7m}{21.3m^2 \cdot 10.4m} \, 57.3 \frac{deg}{rad}$$

$$= -0.162 \frac{1}{rad}$$

$C_{n_{\beta_v}}$:

$$C_{n_{\beta_v}} = V_v \eta_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$$

For the vertical stab, Assume a linearly tapered w̄ elliptical "lift" distr.

$AR_v = 2 \quad \lambda = 0.5 \qquad V_v = \frac{l_v S_v}{S b} = \frac{(13.7m - 8m) S_v}{(21.3 m^2)(10.4 m)} = 0.0257 S_v$

$$C_{L_{\alpha_v}} = \frac{2\pi}{1 + \frac{2\pi}{\pi 2}} = \frac{2\pi}{2} = \pi = 3.14 \ \frac{1}{rad}$$

From Eq 2.80,

$$\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) = 0.724 + 3.06 \frac{S_v/S}{1 + \cos \Lambda_{c/4_w}} + 0.4 \frac{z_w}{d} + 0.009 AR_w$$

$$AR_w = \frac{b^2}{S} = \frac{(10.4m)^2}{21.3 m^2} = 5.08$$

The plane looks like it could cruise w̄ $M > 0.7$ so sweep the wings 20°

$\Lambda = 20°$

$$\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) = 0.724 + 3.06 \frac{S_v}{(1 + \cos 20°) 21.3 m^2} + 0.4 \frac{0.4m}{1.6m} + 0.009(5.08)$$

$$= 0.8697 + 0.074 S_v$$

So adding all of the components

$$0.1 = -0.162 + (0.8697 + 0.074 S_v)(3.14)(0.0257 S_v)$$

Solve for S_v

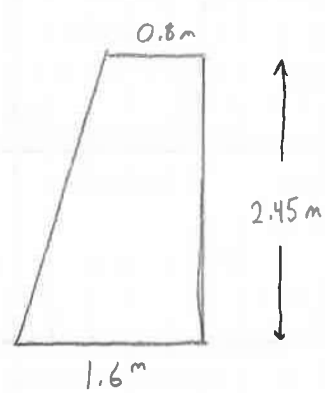
$$S_v = 2.98 \approx \boxed{3 m^2 = S_v}$$

$$\frac{b^2}{S} = AR \Rightarrow b = \sqrt{S\, AR} = \sqrt{(3m^2)(2)} = 2.45 m$$

for lin tapered

$$S = C_r \frac{b}{2}(\lambda + 1) = C_r \left(\frac{2.45}{2}\right)(1.5) = 3$$

$$\Rightarrow C_r = 1.6 m$$



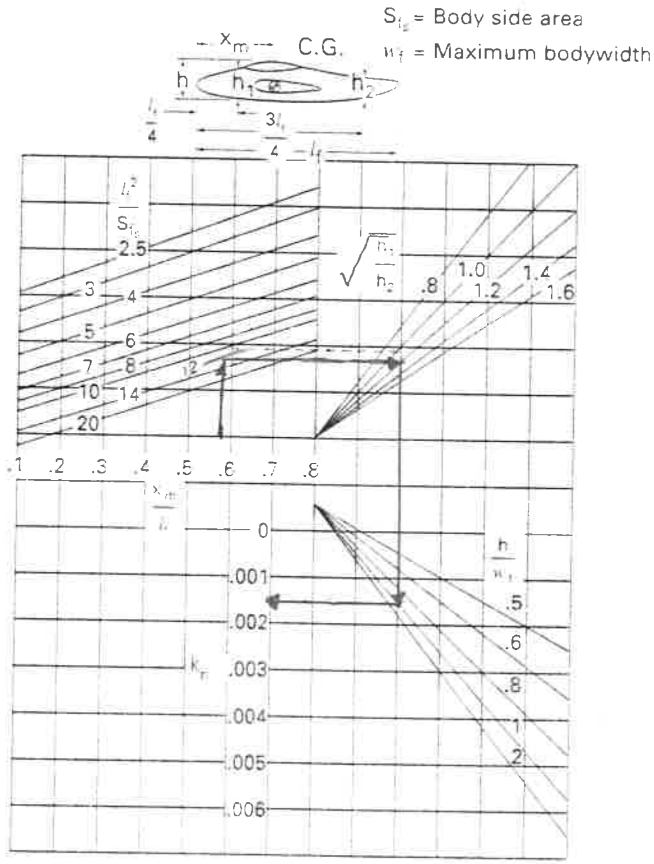


FIGURE 2.29
Wing body interference factor.

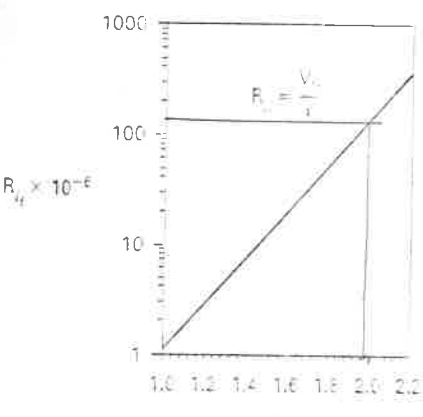


FIGURE 2.30
Reynolds number correction factor.

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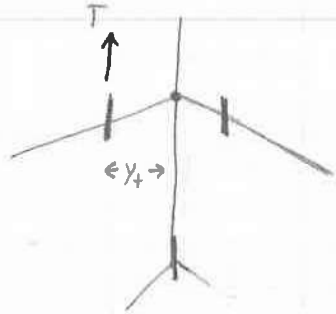
4

2.16) Determine the rudder size for single engine flight.



$S = 980\ ft^2$ $\qquad \delta_r = \pm 15°$

$b = 93\ ft$ $\qquad T = 14000\ lb$

$S_v = 330\ ft^2$ $\qquad y_t = 16\ ft$

$AR_v = 4.3$ $\qquad V = 250\ ft/s$

$l_v = 37\ ft$ $\qquad \rho = 0.002378\ \frac{slug}{ft^3}$

$\eta_v = 1.0$

$$N = T y_t = (14000\ lb)(16\ ft) = 224000\ ft\,lb$$

$$C_{n_{eng}} = \frac{N}{Qsb} = \frac{224000\ ft\,lb}{\frac{1}{2}(0.002378\ slug/ft^3)(250^2\ ft^2/s^2)(980\ ft^2)(93\ ft)} \left| \frac{slug\ ft}{lbf\ s^2} \right.$$

$$= 0.0332$$

$$C_n = C_{n_{eng}} + C_{n_{\delta_r}} \delta_r = 0 \quad \Rightarrow \quad C_{n_{\delta_r}} = \frac{-C_{n_{eng}}}{\delta_r} = \frac{-0.0332}{\delta_r}$$

$$C_{n_{\delta_r}} = -V_v \cancel{\eta}^{1} C_{L_{v_{\delta_r}}}$$

$$C_{L_{\delta_r}} = \frac{dC_L}{d\delta_r} = \frac{dC_L}{d\alpha_v}\frac{d\alpha_v}{d\delta_r} = C_{L_{\alpha_v}} \tau$$

$$C_{L_{\alpha_v}} \approx \frac{2\pi}{1 + \frac{2\pi}{\pi(4.3)}} = 4.3$$

$$V_v = \frac{l_v S_v}{Sb} = \frac{(37\ ft)(330\ ft^2)}{(980\ ft^2)(93\ ft)} = 0.134$$

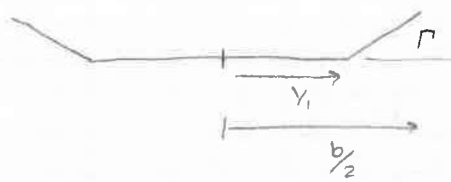
δ_r: The plane has lost the right eng, so must have left rudder $\Rightarrow \delta_r = +15°$
$= 0.262\ rad$

$$C_{n_{\delta_r}} = -(0.134)(1)(4.3)\tau = \frac{-0.0332}{0.262}$$

$$\Rightarrow \tau = 0.220 \quad \Rightarrow \text{Fig 2.21} \quad \frac{S_r}{S_v} \approx 0.09$$

$$\Rightarrow S_r = 0.09(330\ ft^2) = \boxed{29.7\ ft^2 = \text{rudder area}}$$

2.18) Develop an expression for $C_{\ell\beta}$ due to the dihedral.



4

From geometry and small angles,

$$\Delta\alpha = \Gamma\beta$$

From strip theory.

$$\mathcal{L} = -Ly \quad \Rightarrow \quad d\mathcal{L} = -dLy$$

$$\Rightarrow dC_\ell = \frac{d\mathcal{L}}{QSb} = \frac{-dLy}{QSb} = \frac{-C_\ell c\,dy\,y}{QSb}$$

$$\Rightarrow \int dC_\ell = -\int_{y_1}^{b/2} \frac{C_\ell c\,dy\,y}{QSb}$$

C_ℓ for the 3-D wing is equivalent to $C_L = C_{L\alpha}\alpha$

$$C_\ell = -\frac{C_{L\alpha}\alpha}{QSb}\int_{y_1}^{b/2} c\,y\,dy$$

For two outer wing panels and $\alpha = \Gamma\beta$,

$$\boxed{C_{\ell\beta} = -\frac{2C_{L\alpha}\Gamma\cancel{\beta}}{QSb}\int_{y_1}^{b/2} c\,y\,dy}$$

There is no β in $C_{\ell\beta}$

This not right

CO 11-5-00

The inner panels don't contribute to $C_{\ell\beta}$ because of $\Gamma = 0$

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2.20) Estimate the roll moment induced by 747 vortices.

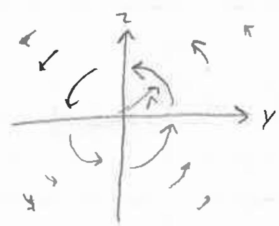
From Pr 2.19

$$V_\theta = \frac{\Gamma}{2\pi r} \qquad \Gamma = \frac{W}{\rho V b'} \qquad b' = \frac{\pi}{4} b \text{ (Elliptic dist)}$$

Vortexfrom the generating aircraft:

$$V_\theta = \frac{W \quad 4}{2\pi r \, \rho V \, \pi b} = \frac{2W}{\pi^2 \rho V b r}$$

The left and right vorticies rotate in opposite directions but the direction of rotation makes no difference to the calculations

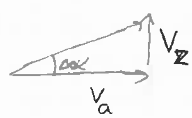


The worst case would be to fly directly up the vortex core.

So, convert to Cartesian coordinates along $\theta = 0$

$$V_z = \frac{2W_g}{\pi^2 \rho V_g b_g y}$$

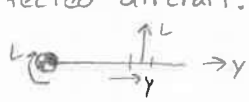
The angle of attack change depends on the velocity of the affected a/c.



$$\Delta\alpha = \tan^{-1}\left(\frac{V_z}{V_a}\right)$$ for small angles, $$\Delta\alpha = \frac{V_z}{V_a}$$

$$\Delta\alpha = \frac{2W_g}{\pi^2 \rho V_g b_g V_a y}$$

Roll moment on the affected aircraft:



$$dL = -dL_{ift}\, y$$

$$\frac{dL}{Qsb} = \frac{-C_{lift}\, c\, dy\, y\, Q}{Qsb}$$

$$\int dC_l = 2\int_0^{b/2} \frac{-C_{lift}\, c\, y\, dy}{S_a b_a}$$

C_{lift} for the whole wing is $C_{Lift} = C_{L_\alpha} \alpha$

$$C_l = \frac{-2C_{L_\alpha}}{S_a b_a} \int_0^{b/2} \alpha\, c\, y\, dy$$

For a 747 generating a/c,

$W = 636600^{lb}$ $b = 196^{ft}$

Method OK 4

The worst case for generating vortices is slow and clean (departing)
Assume Sea Level and 130^{kt} $(220 \frac{ft}{s})$ for th 747
↖ perhaps a little slow

$$\rho = 2.377 \times 10^{-3} \frac{slug}{ft^3}$$

$$\Delta\alpha = \frac{2(636600^{lb})}{\pi^2 (2.377\times10^{-3}\, slug/ft^3)(220 \frac{ft}{s})(196^{ft})\, V_a\, y} \left| \frac{slug\, ft}{lbf\, s^2} \right.$$

$$\Delta\alpha = \frac{1258.6}{V_a\, y} \left[\frac{ft^2}{s}\right]$$

Convair 880:

$b = 120^{ft}$ From B.26 at SL and slow, $C_{L_\alpha} = 4.52$ $V_a = 130^{kt}$ $(220 \frac{ft}{s})$

Assume a linear tapered wing

$c_r = 27^{ft}$ $c_t = 5.6^{ft}$ $\lambda = \frac{5.6}{27} = 0.20$

$$\Rightarrow c = \frac{c_r(\lambda - 1)}{b/2} y + c_r$$

so,

$$C_\ell = \frac{-2\,(4.52)}{(2000 ft^2)(120 ft)} \int_0^{b/2} \left[\frac{1258.6}{(220 \frac{ft}{s})(\cancel{y})}\right]\left[\frac{27^{ft}(0.2-1)}{(60^{ft})} y + 27^{ft}\right] \cancel{y}\, dy$$

$$= \frac{-2\,(4.52)\, 1258.6 \frac{ft^2}{s}}{(2000\, ft^2)(120\, ft)(220 \frac{ft}{s})} \int_0^{60} \left(\frac{27(-0.8)}{60} y + 27\right) dy$$

$$= \left(-0.000215 \frac{1}{ft^2}\right)\left(972\, ft^2\right) = \boxed{-0.209 = C_{\ell_{wake}}}$$

From B.26
$\delta_a = 25°$ $C_{\ell_{\delta_a}} = 0.038$ (The book says $C_{\ell_{\delta_a}} = -0.038$. I think that is the wrong sign)

$$\Rightarrow C_{\ell_{aileron}} = C_{\ell_{\delta_a}}\, \delta_a = (0.038)(25°)\left(\frac{1}{57.3}\right) = \boxed{0.0165 = C_{\ell_a}}$$

The ailerons are at least 12 times weaker than the induced roll moment.

<u>There will be an uncorrectable roll</u>

STOL Transport:

$S = 945\ ft^2$

$b = 96\ ft \qquad C_{L_\alpha} = 5.24 \qquad V = \text{Mach } 0.14 \text{ @ SL} \ (154\ \frac{ft}{s})$

The wing is almost a linearly tapered wing.

$C_t = 6.3\ ft \quad C_r = 14.8\ ft \qquad \lambda = \frac{6.3}{14.8} = 0.43$

$$\Rightarrow c = \frac{14.8\ ft}{48\ ft}(0.43 - 1)\, y + 14.8 = -0.1758\, y + 14.8$$

$$C_\ell = \frac{(-2)(5.24)}{(945\ ft^2)(96\ ft)} \int_0^{48\ ft} \left[\frac{1258.6\ \frac{ft^2}{s}}{(154\ \frac{ft}{s})(\cancel{y})}\right]\left[-0.1758\, y + 14.8\ ft\right] \cancel{y}\, dy$$

$$= \frac{(-2)(5.24)(1258.6\ \frac{ft^2}{s})}{(945\ ft^2)(96\ ft)(154\ \frac{ft}{s})} \ 507.9\ ft^2$$

$$\boxed{C_{\ell_{wake}} = -0.480}$$

From B.28, $C_{\ell_{\delta_a}} = 0.20 \qquad \delta_a = 25°$

$$C_\ell = C_{\ell_{\delta_a}} \delta_a = (0.20)(25°)\left(\frac{1}{57.3}\right) = \boxed{0.087 = C_{\ell_a}}$$

The STOL's ailerons are 5 times less effective than the vortices at roll moment creation.

Jetstar:

$b = 53.75^{ft}$ $\quad S = 542.5 ft^2$ $\quad C_{L_\alpha} = 5.0$ $\quad V = \text{Mach } 0.2 \left(220 \frac{ft}{s}\right)$

Assuming a linearly tapered wing,

$C_r = 16^{ft}$ $\quad C_t = 4.9^{ft}$ $\quad \lambda = \frac{4.9}{16} = 0.31$

$$c = \frac{C_r(\lambda - 1)}{b/2} y + C_r = \frac{16^{ft}(0.31-1)}{26.88^{ft}} y + 16^{ft} = -0.411 y + 16^{ft}$$

$$C_\ell = \frac{(-2)(5.0)}{(542.5 ft^2)(53.75 ft)} \int_0^{26.88 ft} \frac{1258.6 \frac{ft^2}{s}}{(220 \frac{ft}{s})(y)} \left[-0.411y + 16^{ft}\right] y \, dy$$

$$\boxed{C_{\ell_{wake}} = -0.552}$$

From B.25

$C_{\ell_{\delta_a}} = 0.054$ $\quad \delta_a = 25°$

$$C_\ell = C_{\ell_{\delta_a}} \delta_a = (0.054)(25°)\left(\frac{1}{57.3}\right)$$

$$\boxed{C_{\ell_a} = 0.0236}$$

The Jetstar's ailerons are 23 times less effective than the vortex at roll moment creation.

Navion:

From example pr 2.4

$C_{L_\alpha} = 4.44$ $\lambda = 0.54$ $b/2 = 16.7^{ft}$ $S = 184\,ft^2$ $C_t = 3.9^{ft}$
$C_r = 7.2^{ft}$

$$c = \frac{7.2^{ft}}{16.7^{ft}}(0.54-1)\,y + 7.2^{ft} = -0.198y + 7.2^{ft}$$

$V = 90^{kt} \approx 150\frac{ft}{s}$

$$C_\ell = \frac{(-2)(4.44)}{(184^{ft^2})(33.4^{ft})}\int_0^{16.7^{ft}}\left[\frac{1258.6\frac{ft^2}{s}}{(150\frac{ft}{s})\cancel{(y)}}\right]\left[-0.198y + 7.2^{ft}\right]\left[\cancel{y}\,dy\right]$$

$$\boxed{C_\ell = -1.12}$$!!!!

Ailerons:

$C_\ell = C_{\ell_{\delta_a}}\delta_a$ $C_{\ell_{\delta_a}} \equiv 0.134$ $\delta_a = 25°$

$$C_{\ell_a} = (0.134)(25°)\left(\frac{1}{57.3}\right) = \boxed{0.058 = C_{\ell_a}}$$

The wake is 19 times stronger than the ailerons.

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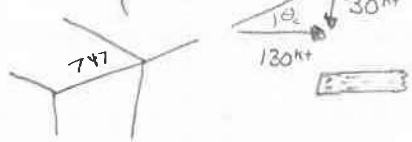
pr 4) Determine the sideslip angle and rudder angle just before touchdown.

4



$\theta_c = \tan^{-1}\left(\frac{30}{130}\right) = 13°$

$\Rightarrow$ $\boxed{\text{Sideslip at touchdown} = 13°}$

The pilot needs right rudder which is a negative δ_r

$C_n = 0 = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r$

From App B, at SL

$C_{n_\beta} \approx 0.150 \qquad C_{n_{\delta_r}} = -0.109$

$\beta = -13° \left| \frac{1}{57.3} = -0.227 \text{ rad} \right.$

$$\Rightarrow \quad \delta_r = \frac{-C_{n_\beta}\beta}{C_{n_{\delta_r}}} = \frac{-(0.150)(-0.227)}{-0.109} = 0.312 \text{ rad}$$

$$= (0.312)(57.3) = 17.9°$$

$\boxed{\delta_r \approx 18°}$