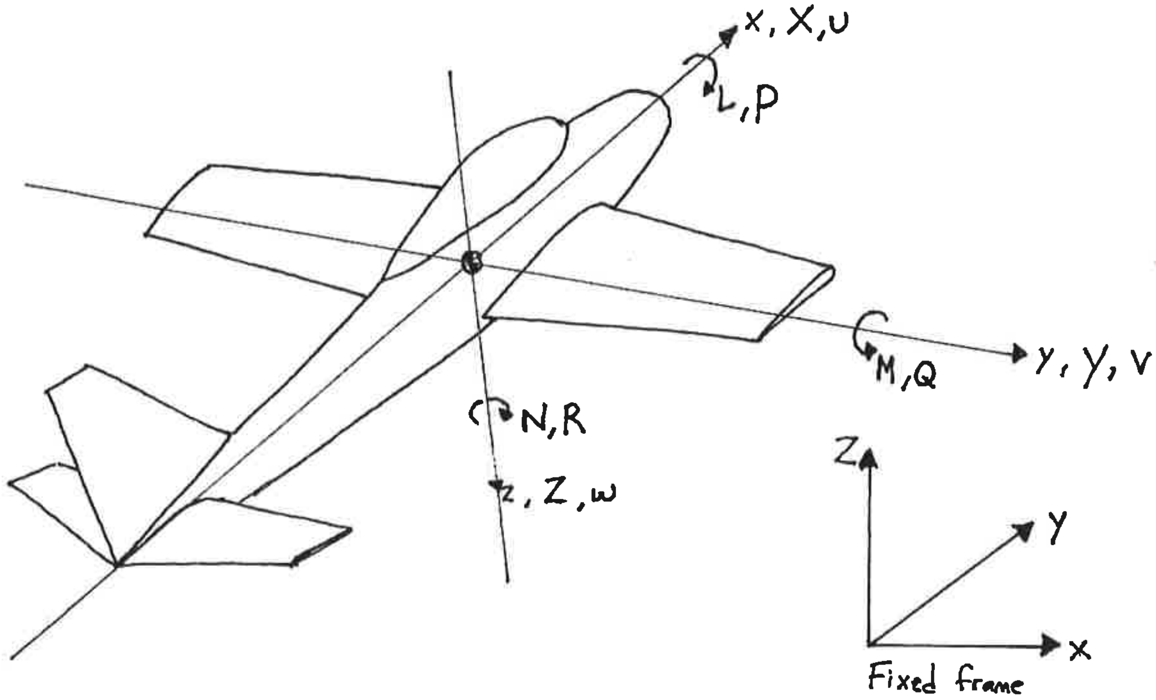
AEM 368

Flight Dynamics and Control 1

Stability and Control Experiment

Aircraft Coordinate System



x, y, z	Aircraft "stability" frame location
X, Y, Z	Forces in stability frame
u, v, w	Velocity in stability frame
L, M, N	moment in stability frame
P, Q, R	Angular velocities (roll, pitch, yaw)
ϕ, θ, ψ	Euler angles (orientation)

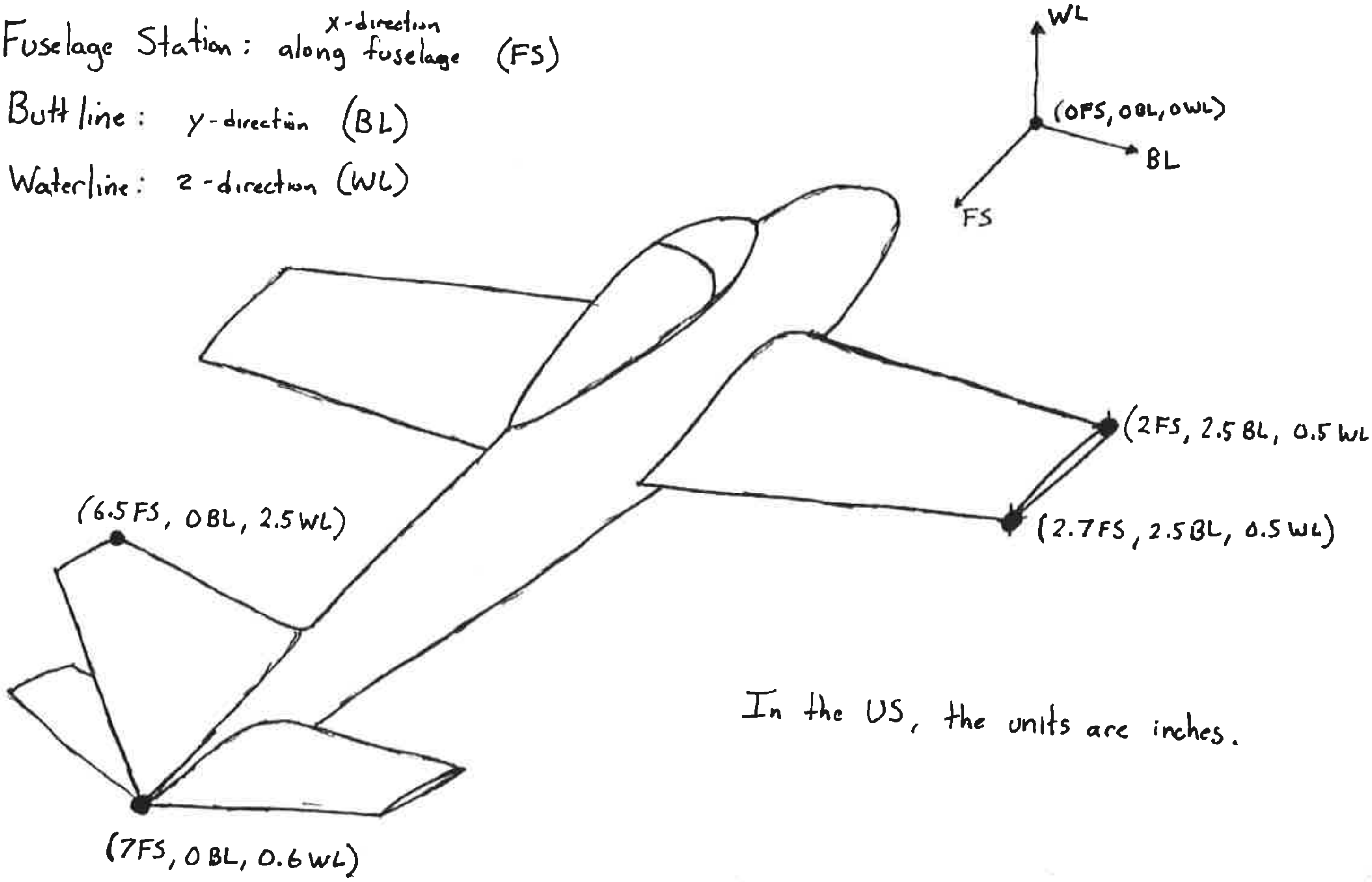
Warning:

Outside of aerodynamics, the x axis is usually down the length of the a/c.

$$X_{loft} = -X_{aero\ stability}$$
$$Y_{loft} = Y_{aero\ stability}$$
$$Z_{loft} = -Z_{aero\ stability}$$

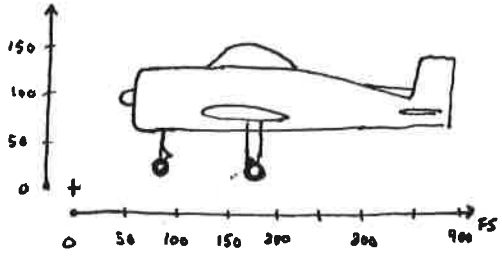
Locations on an Aircraft (Aircraft Station Coordinates)

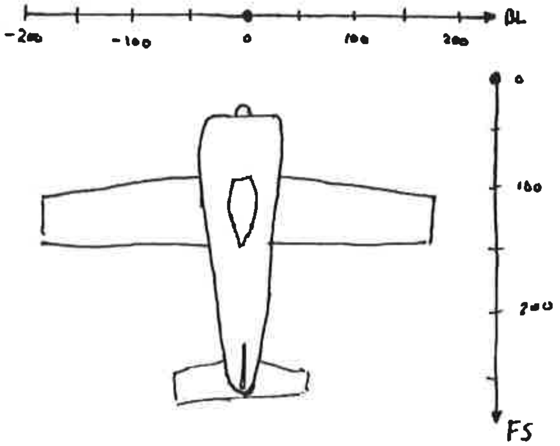
- Fuselage Station: along $\overset{\text{x-direction}}{\text{fuselage}}$ (FS)
- Butt line: y-direction (BL)
- Waterline: z-direction (WL)



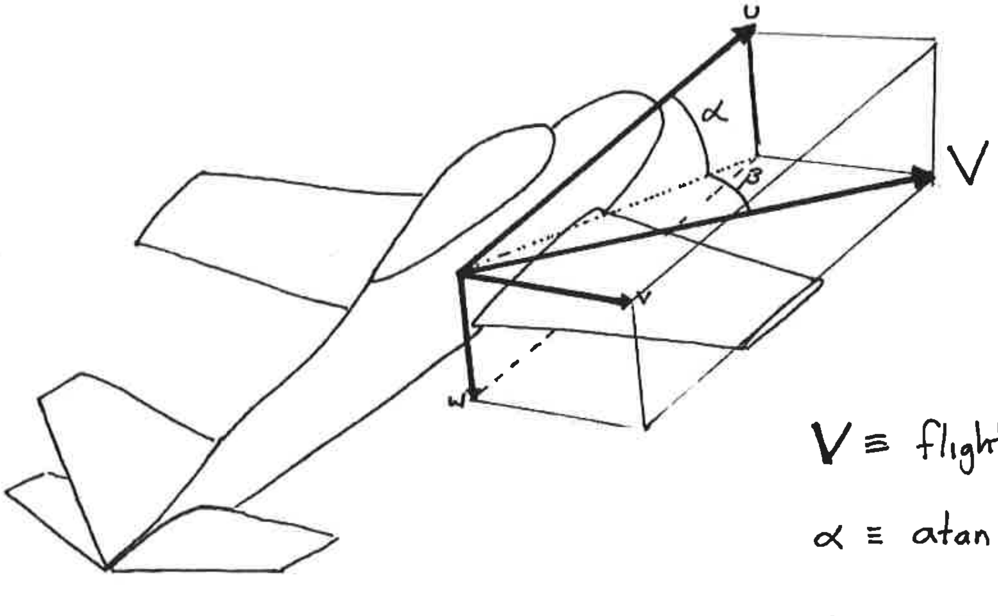
In the US, the units are inches.

- Zero FS is usually not the most forward location of the aircraft. Rather the origin is placed arbitrarily forward such that negative FS does not occur.
- Zero BL is usually along the centerline
- Zero WL is usually placed such that all values are positive





Angle of Attack and Sideslip



$V \equiv$ flight velocity vector

$\alpha \equiv \operatorname{atan}\left(\frac{w}{U}\right)$ Angle of Attack

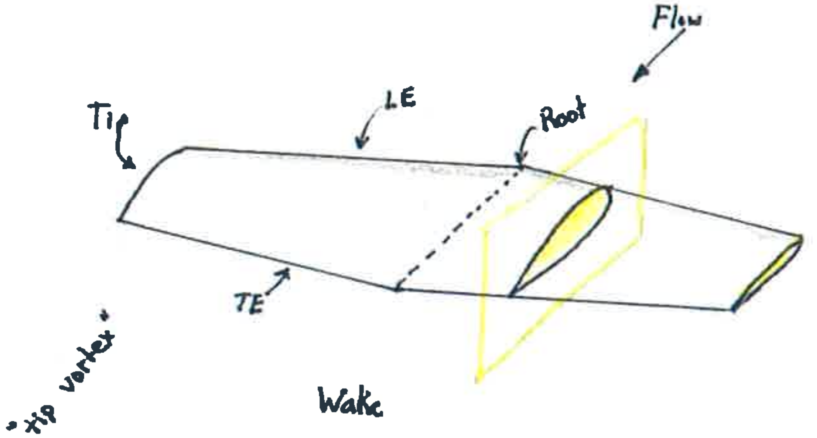
$\beta \equiv \operatorname{asin}\left(\frac{v}{V}\right)$ Sideslip

α is defined wrt the projection of V onto the body fram (i.e. U)

β is defined wrt the v projection and V.

Wing:

- Three dimensional closed surface
- Generates aerodynamic force
- Cross sections are airfoils

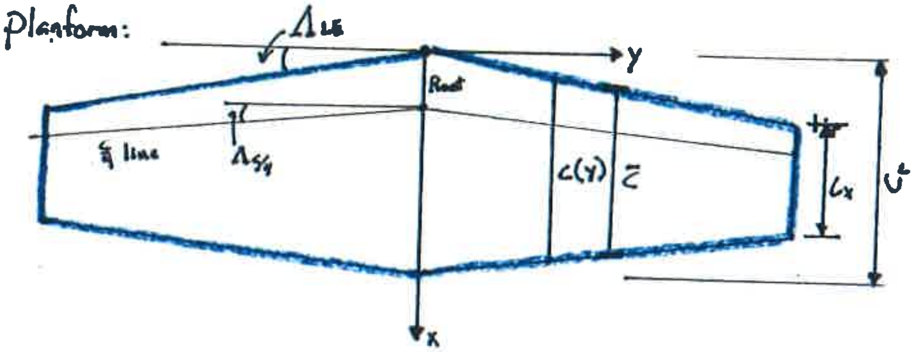


Etymology:

wing English
↓
wenge middle English
↓
vent (english) Vaenger Norse
↓ ↓
vent (Latin) we-ingjaz Old German
↘ ↓
we (to blow) Indo-European

Interestingly, not from Indo-European word for "fly" which is "peth" from which we get "feather" and "pen".

Planform:



$S \equiv$ Wing Area $[L^2]$

$b \equiv$ Span $[L]$

$c \equiv$ chord

$AR \equiv$ Aspect Ratio $\equiv \frac{b^2}{S}$

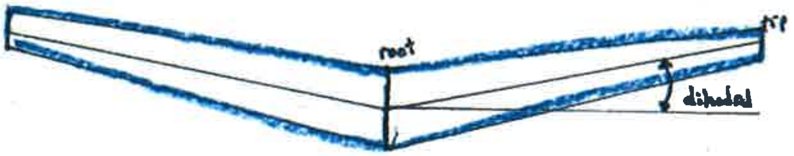
$\lambda \equiv$ taper ratio $\equiv \frac{c_t}{c_r}$, $\left(\frac{\text{tip}}{\text{root}}\right)$

MAC $\equiv$ Mean Aerodynamic Chord $\equiv$

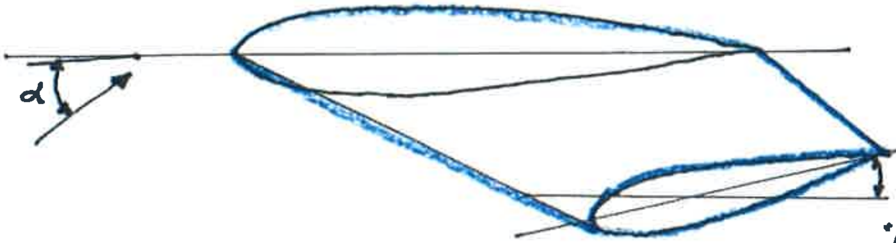
$$\frac{1}{S}\int_{-b/2}^{b/2} c(y)^2\,dy$$

$\bar{c} \equiv$ Average Chord

Front view



Side view



wash out $\equiv$ decrease in incidence angle at tip

"wash in" is a positive increase ... usually a bad idea

The atmosphere on Earth

Dry Air (= Atmospheric air − H_2O − contaminants (dust, pollen, ⋯))

	Mole Fraction	Molecular Weight $\frac{kg}{kmol} = \frac{lbm}{lbmol}$	
Nitrogen	78.08%	28.02	$N \equiv N$ strong triple bond!
Oxygen	20.95%	32.0	$O = O$ double bond
Argon	0.93%	39.94	Greek αργον "inactive" Ar no bond
Carbon Dioxide	0.03%	44.01	$O = C = O$ linear shape double bond
Other	0.01%		
	100 %		

Apparent Molecular Weight $= \sum a_i M_i$

$$M \approx 28.02 \cdot 0.7808 + 32.0 \cdot 0.2095 + 39.94 \cdot 0.0093 + 44.01 \cdot 0.0003$$

$$\approx 28.97 \frac{lbm}{lbmol} = 28.97 \frac{kg}{mol}$$

Gas Constant for air

$$R = \frac{\bar{R}}{M} = \frac{1545.34\ ft\ lbf}{R\ \cancel{lbmol}} \left| \frac{\cancel{lbmol}}{28.97\ \cancel{lbm}} \right| \frac{32.174\ \cancel{lbm}}{slug} = 1716.5 \frac{ft\ lbf}{R\ slug}$$

$$= 53.35 \frac{ft\ lbf}{R\ lbm}$$

Air density SSL (14.696 psi, 59°F)

$$\rho = \frac{PM}{\bar{R}T} = \frac{14.696\ \cancel{psi}}{} \left| \frac{28.97\ \cancel{lbm}}{\cancel{lbmol}} \right| \frac{\cancel{R}\ \cancel{lbmol}}{1545.34\ ft\ \cancel{lbf}} \left| \frac{}{518.67\ \cancel{R}} \right| \frac{slug}{32.174\ \cancel{lbm}} \left| \frac{144\ \cancel{in^2}}{ft^2} \right| \frac{\cancel{lbf}}{\cancel{psi}\ \cancel{in^2}}$$

$$= 0.00237 \frac{slug}{ft^3}$$

Wet air

The addition of water vapor changes the properties of "air":

$N \equiv N$ $O=C=O$ $N \equiv N$

Ar $N \equiv N$

$^{H}O^{H}$ $O=O$

Water vapor behaves as an ideal gas, thus we can model the mixture as an IG.

$$\rho = \frac{P_{dry}}{R_{dry} T_{dry}} + \frac{P_{vapor}}{R_{vapor} T_{vapor}}$$

where the partial pressures add to total pres

$$P = P_{dry} + P_{vapor}$$

$$= \frac{P_{dry} M_{dry}}{R\, T_{dry}} + \frac{P_{vapor} M_{vapor}}{R\, T_{vapor}}$$

Temps are identical

$$= \frac{P_{dry} M_{dry} + P_{vapor} M_{vapor}}{RT}$$

Partial pressure of vapor

$$P_{vapor} = \phi P_{sat}$$

ϕ = relative humidity

$$= \frac{(P - P_{vapor}) M_{dry} + P_{vapor} M_{vapor}}{RT}$$

$$= \frac{(P - \phi P_{sat}) M_{dry} + \phi P_{sat} M_{vapor}}{RT}$$

$$= \frac{P M_{dry} + \phi P_{sat} (M_{vapor} - M_{dry})}{RT}$$

M_{dry} = 28.97

M_{vapor} = 18.0

Thus, $M_{vapor} - M_{dry}$ is <u>negative</u>

Increasing the water vapor <u>decreases</u> air density

Impact of Wet Air.

					%SSL
Standard Sea Level (SSL) Std-Day	59°F	0% rh	0^{ft}_{MSL}	$\rho = 0.00237 \frac{slug}{ft^3}$	100
Alabama Summer (hot + humid)	90°F	90% rh	$\approx 0^{ft}_{MSL}$	$\rho = 0.002206 \frac{slug}{ft^3}$	93%
" " Dry	90°F	0% rh		$\rho = 0.00224 \frac{slug}{ft^3}$	94%
Alabama Winter (wet)	40°F	90% rh		$\rho = 0.00246 \frac{slug}{ft^3}$	104%
" " Dry	40°F	0% rh		$\rho = 0.00246 \frac{slug}{ft^3}$	104%
Antarctica (cold + dry)	-126°F	0% rh		$\rho = 0.00369 \frac{slug}{ft^3}$	155%
Denver, CO (std day)	40°F	0% rh	5000^{ft}_{MSL}	$\rho \approx 0.00205 \frac{slug}{ft^3}$	86%

ASHRAE Chart Comparison

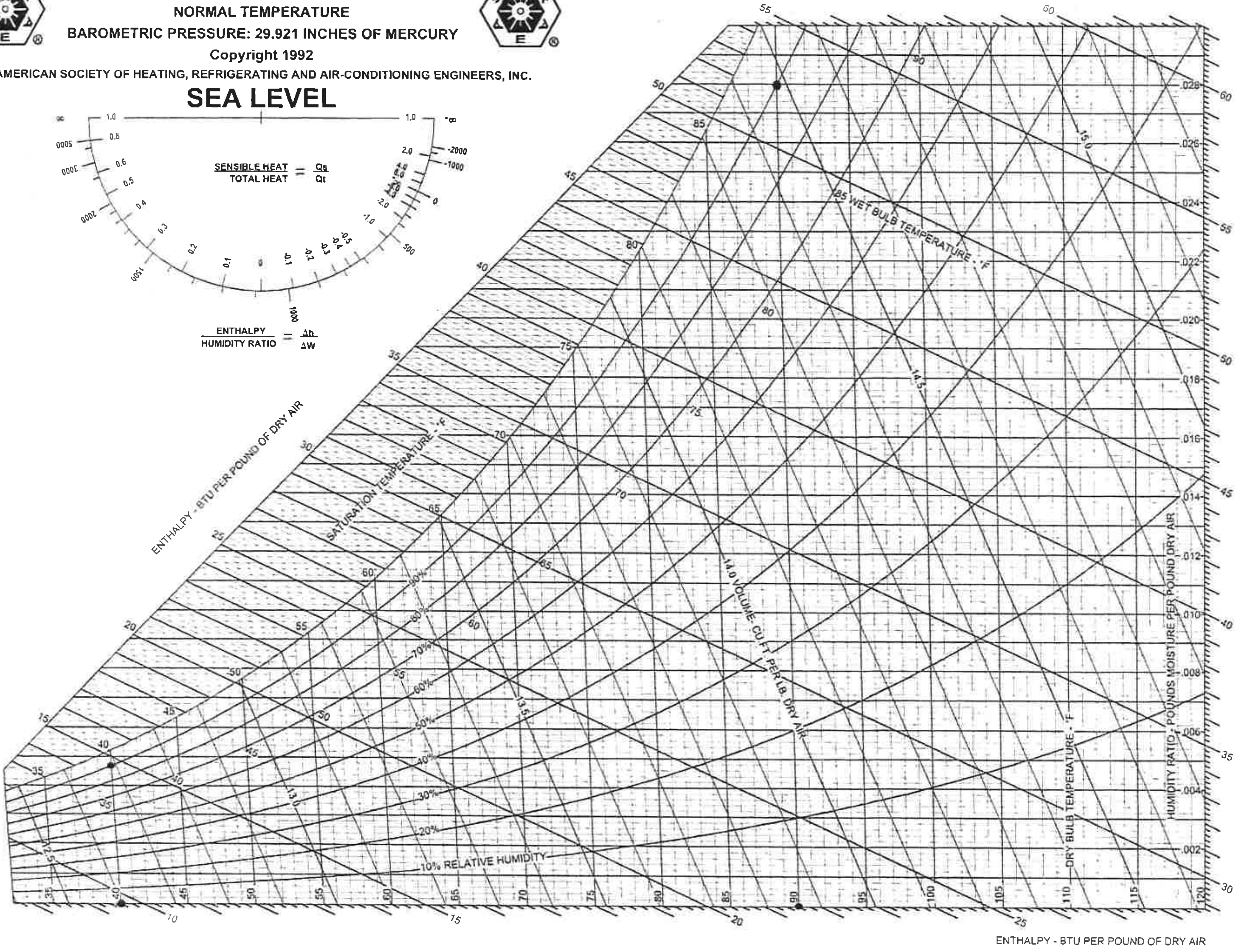
ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

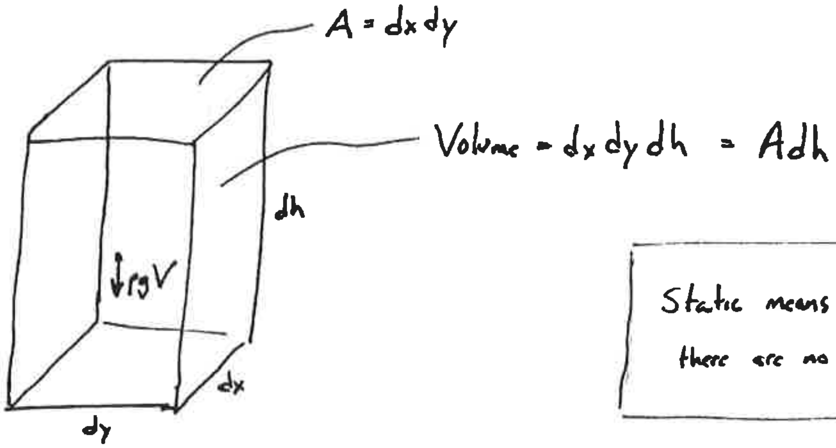
BAROMETRIC PRESSURE: 29.921 INCHES OF MERCURY

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.

SEA LEVEL



Static Column of Fluid



Static means no velocity, so no $\frac{du_i}{dx_j}$, thus there are no viscous forces.

Summation of forces in h direction

$$P_{bottom} A = P_{top} A + \rho g V$$

Taylor Series expansion for $P_{top} = P_{bottom} + \frac{dP}{dh} dh$

$$PA = \left(P + \frac{dP}{dh} dh\right) A + \rho g A\,dh$$

Reduce (divide by A, cancel PA terms)

$$\frac{dP}{dh} dh + \rho g\,dh = 0$$

Divide by dh

$$\frac{dP}{dh} + \rho g = 0$$

Gov Equ

$$\boxed{dP = -\rho g_o\,dh}$$

Atmosphere (continued)

$$dp = -\rho g_0 \, dh$$

Ideal Gas is $p = \rho R T$, substitute for ρ

R = 1716.5 $\frac{\text{ft lbf}}{\text{R slug}}$

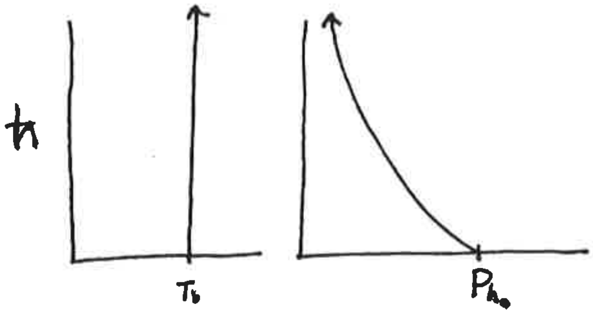
$$dp = -\frac{p}{RT} g_0 dh \Rightarrow \frac{dp}{p} = -\frac{g_0 \, dh}{RT}$$

- Isothermal
  Zero lapse rate $(\Delta T \neq f(h))\,(T = T_0)$

$$\frac{dp}{p} = \frac{-g_0}{RT_0} dh \qquad \text{Integrate} \qquad \ln p \Big|_{p_{h_0}}^{p_{h_1}} = \frac{-g_0}{RT_0} h \Big|_{h_0}^{h_1}$$

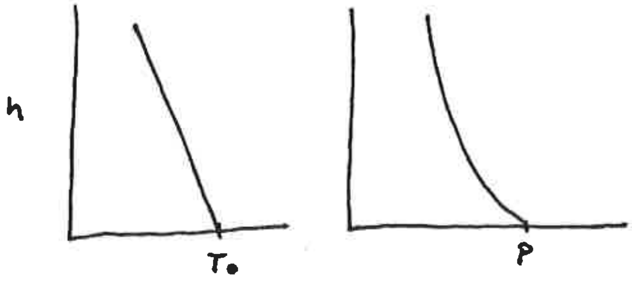
$$\underbrace{\ln p_{h_1} - \ln p_{h_0}}_{\ln \frac{p_{h_1}}{p_{h_0}}} = \frac{-g_0}{RT_0}(h_1 - h_0)$$

$$\Rightarrow p_{h_1} = p_{h_0} e^{\frac{-g_0 (h_1 - h_0)}{RT_0}}$$



- Linear lapse rate $(T = T_0 - \lambda(h_1 - h_0))$

$$\frac{dp}{p} = \frac{-g_0}{R(T_0 - \lambda(h_1 - h_0))} dh \qquad \text{Integrate (slightly involved)} \qquad \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{-g_0}{R\lambda}}$$



Lesson 2

Non standard Atmosphere

MIL - STD - 210A

Hydrostatic Equation (review)

$$dp = -\rho g_0\, dh$$

P is pressure
h is geopotential altitude
g_0 is gravity
ρ is density

With the ideal gas equation of state

$$P = \rho R T$$

So,

$$dp = -\frac{P}{RT} g_0\, dh$$

But $T = T(h)$ (a function of height)

We could solve analytically (as in lesson 1) or numerically.

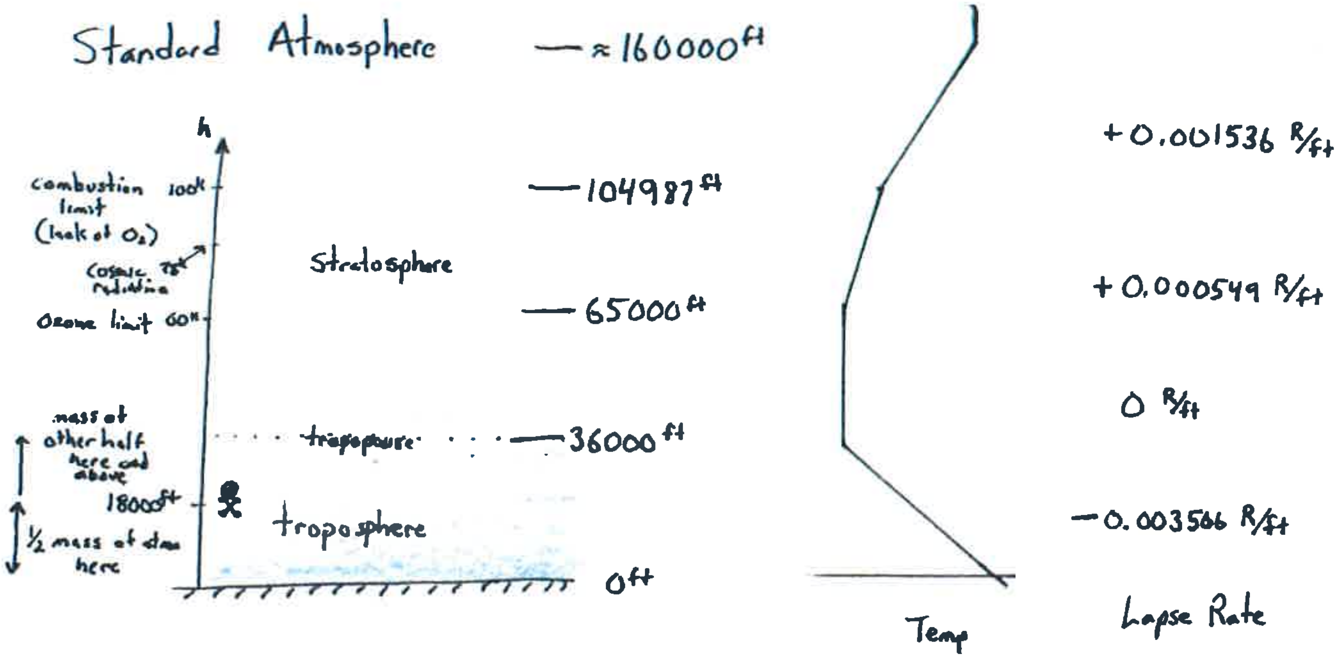
$$\Delta p = -\frac{P}{RT} g_0\, \Delta h \quad \Rightarrow \quad \underbrace{P(h_1) = P(h_0) - \frac{(P(h_1)-P(h_0))}{R\,(T(h_1)-T(h_0))} g_0\,(h_1-h_0)}_{\text{one possible integration method}}$$

Units:

$$dp = -\frac{P}{RT} g_0\, dh$$

$$\underbrace{\frac{lbf}{in^2}}_{dp} = \underbrace{\frac{\cancel{lbf}}{in^2}}_{P}\ \underbrace{\frac{\cancel{R}\ slug}{1716.5\ \cancel{ft}\ \cancel{lbf}}}_{\frac{1}{R}}\ \underbrace{\frac{}{\cancel{R}}}_{1/T}\ \underbrace{\frac{32.174\ \cancel{ft}}{\cancel{s^2}}}_{g_0}\ \underbrace{\frac{ft}{}}_{dh}\ \frac{lbf\ \cancel{s^2}}{slug\ \cancel{ft}} \quad \checkmark$$

Standard Atmosphere



We are lucky to live at the bottom of an ocean of protective air.

- Death zone for humans begins around $20^{k\,ft}$ or so...
- Half the mass of air is below $18^{k\,ft}$
- Ozone above $60^{k\,ft}$ prevents use of outside air for humans.
- Cosmic radiation becomes significant around $75^{k\,ft}$.
- Normal jet combustion fails around $100^{k\,ft}$. (Not enough O_2)
- The positive lapse rate around $65^{k\,ft}$ makes the atmosphere stable. Convection is minimal.

Non-dimensional P, T, ρ ratios

$$\delta = \frac{P}{P_{ssl}} \qquad \theta = \frac{T}{T_{ssl}} \qquad \sigma = \frac{\rho}{\rho_{ssl}} = \frac{\delta}{\theta}$$

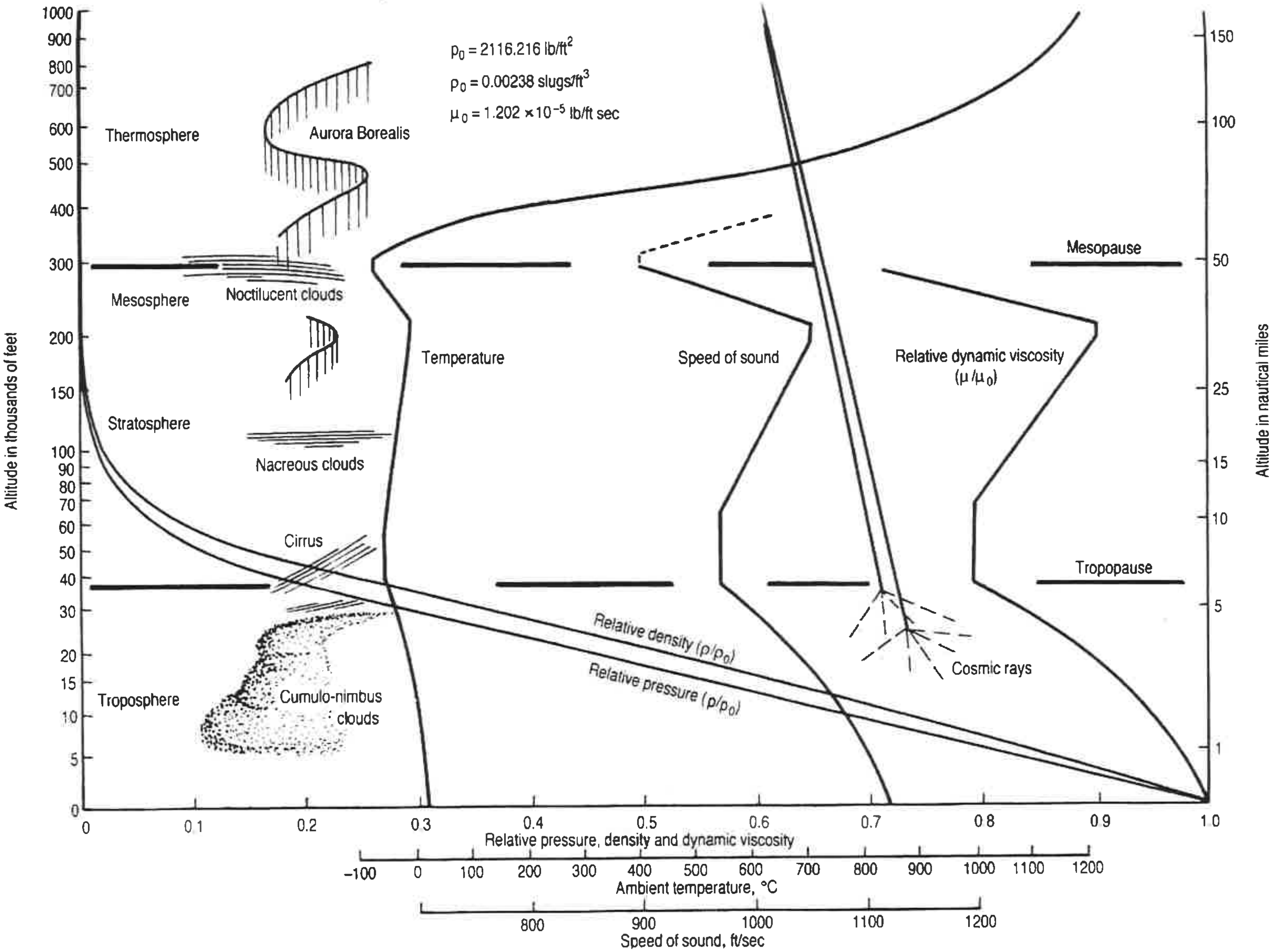
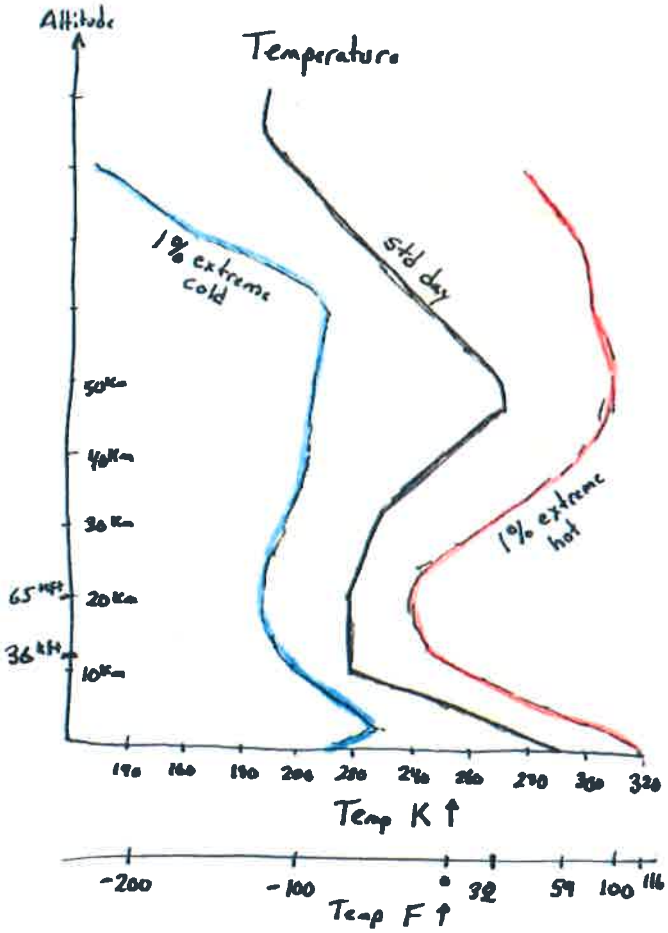


Fig. 1.2 General characteristics of the atmosphere (based upon ICAO and US Standard Atmosphere 1962).

Source: The Anatomy of the Airplane
Stanton

The atmosphere is not, has not, and never will be standard.



See:

U.S. Standard Atmosphere 1976

(NASA-TM-X-74335)

for details (241 pages!)

Rankine:

R = F + 459.67

Kelvin:

K = C + 273.1

Non Std Atmosphere Models. (MIL-STD-210A)

Std

h [ft]	λ [R/ft]
0	- 0.003566
36089	0
65617	+ 0.000549
104987	+ 0.001536
154199	0

$T_0 = 59$°F

Hot

h [ft]	λ [R/ft]
0	- 0.003840
39370	+ 0.000439
67 257	+ 0.000768

$T_0 = 103.28$°F

Tropic

h [ft]	λ [R/ft]
0	- 0.003840
52493	+ 0.002085
68898	+ 0.001361

$T_0 = 90.086$°F

Cold

0	+ 0.013716
3281	0
9843	- 0.003292
31 168	0
42651	+ 0.004872
50853	0
60696	+ 0.002524
73819	- 0.000425

$T_0 = -59$°F

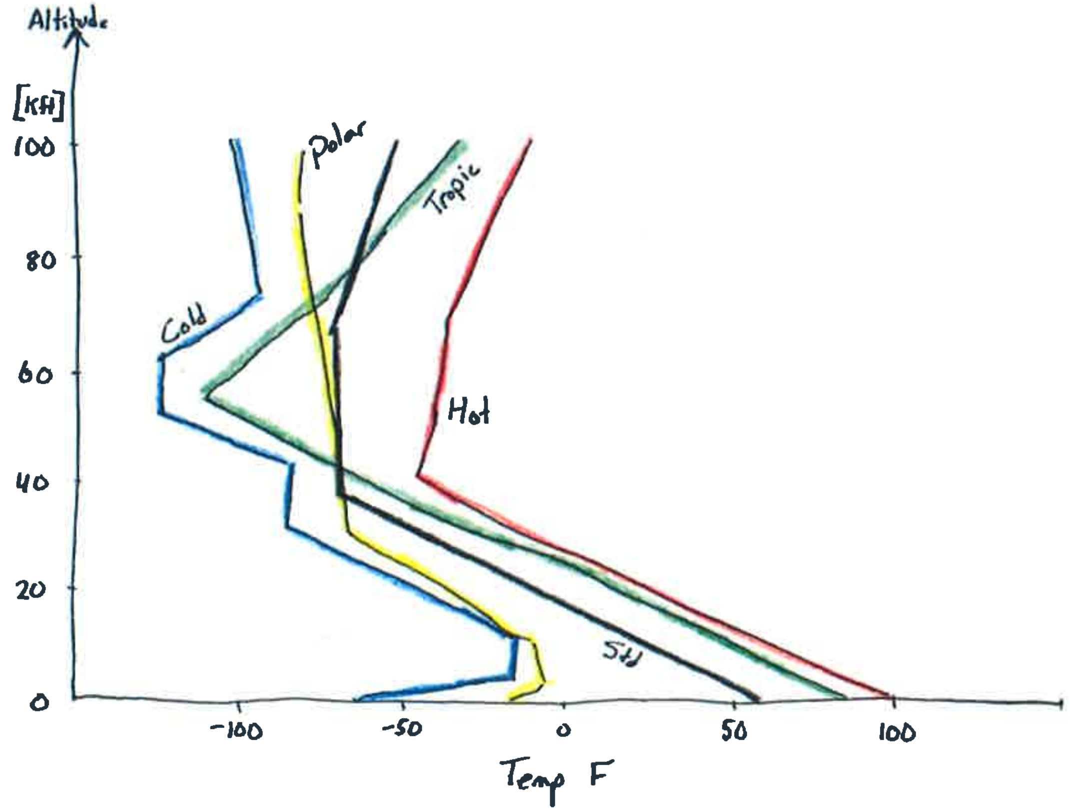
Polar (warning: reverse engineered from data!)

0	0.003
3281	- 0.00055
9843	- 0.0028
31 168	- 0.0003
88000	0

$T_0 = -15.67$°F

ISA + X

Add X to temp at Std day



Examples:

What is the temperature at 10000 ft on a standard day?

$$T(10^{kft}) = T(SSL) + \lambda \cdot h$$

$$= 59°F - 0.003566\ {}^{R}/_{ft} \cdot 10000\ ft$$

$$\boxed{T = 23.3^{F}}$$

$$\theta = \frac{T}{T_{SSL}} = \frac{23.3^{F} + 459.67}{59 + 459.67} = \boxed{0.93 = \theta}$$

what is the temperature at 10000 ft on an ISA + 20 day?

$$T_{10000ft\ std} = 23.3^{F}$$

$$\boxed{T_{10kft\ ISA+20} = 43.3^{F}}$$

Just add

What is the pressure at 10000 ft on a std day?

Linear lapse rate $\lambda = -0.003566\ {}^{R}/_{ft}$

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{-g_0}{R\lambda}}$$

$$= (0.93)^{-\frac{32.174\ ft}{s^2} \left| \frac{slug\ R}{1716.5\ ft\ lbf} \right| \frac{ft}{-0.003566\ R} \left| \frac{lbf\ s^2}{slug\ ft}\right.}$$

$$\boxed{\delta = 0.6828}$$

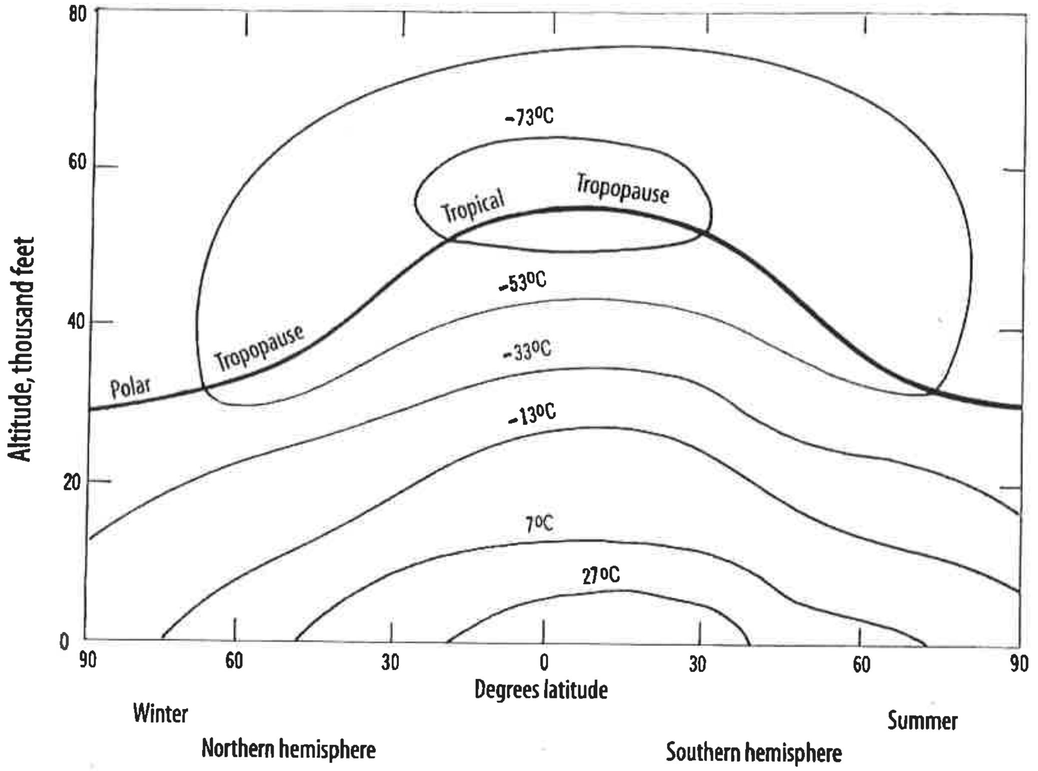


Fig. 1.1 Typical variation in atmospheric temperature along a meridian of longitude, summer in the Southern Hemisphere.

Source: TA of the Airplane, Stinton

Fact: Cold temperature testing of a/c is often done at high altitudes in the tropics!

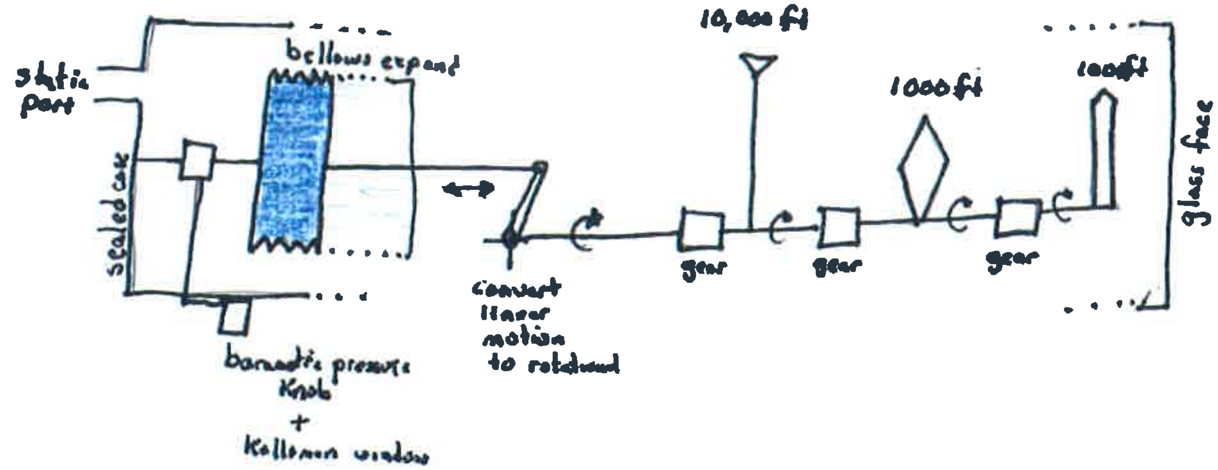
Why? We need to know some meteorology.

Altimeter (pressure type)

A sensor used to measure altitude. Uses local static pressure

Reading 13 700 ft

Inside (cartoon, not to scale)



Calibrated to SSL and Standard Atmosphere.

- Given a standard day, the altimeter reads a geometric altitude
- Given a non standard day, the altimeter reads pressure altitude (if 29.92 setting)
- The knob corrects for local pressure differences such that the altimeter reads the airport's elevation. Obviously, a non standard pressure profile combined with airport approach and departures makes knowing the correct Kollsman window setting critical. "Altimeter setting two niner eight two"

Flight Level (FL):

When set to 29.92 in Hg, the altimeter's reading is a convenient reference for high altitude aircraft. Read as hundreds of feet. FL350 = 35000 ft

Q: why not low altitude?

Q: Why is a reference needed?

RVSM (Reduced Vertical Separation Minima)

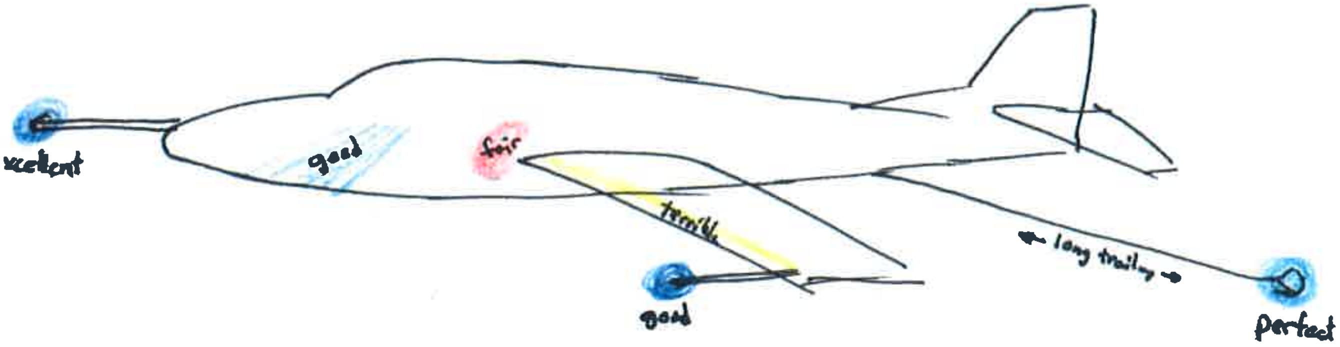
To increase the number of aircraft operating (safely) between FL 290 and FL410, the verticle spacing between aircraft was reduced from 2000 ft to 1000 ft.

Aircraft operating in that region require special certification. In particular, the static pressure source becomes critical.

Easy Right?!? No. Placing the static port(s) can be a challenge.

Given that an aircraft creates a pressure distribution to generate lift and that a fuselage impacts the pressure distribution, the static sources must be placed in a region such that minimum deviation occurs during $\underbrace{\text{cruise}}_{\text{low } \alpha}$ and $\underbrace{\text{App/Dep}}_{\text{high } \alpha}$.

So, where?! Usually...



Lesson 3

Aerodynamics

Lagrangian and Eulerian Frames

$$\underbrace{\frac{D(\,)}{Dt}}_{\substack{\text{particle}\\ \text{frame}\\ \text{"Lagrangian"}}} = \underbrace{\frac{\partial(\,)}{\partial t} + V \cdot \nabla(\,)}_{\substack{\text{Eulerian frame}\\ \text{fixed location}}} = \frac{\partial(\,)}{\partial t} + \frac{d(U)_i}{dx_j}\frac{dx_i}{dt}$$

Example:

If the lapse rate is -5° per 1000^{ft} and an aircraft is climbing at 1500 ft/min, what is the rate of change in temperature on the aircraft?

A: We want the temperature along the aircraft moving at 1500 ft/m. This is a Lagrangian frame.

$$\frac{DT}{Dt} = \underbrace{\cancelto{0}{\frac{\partial T}{\partial t}}}_{\substack{\text{no change}\\ \text{in temp}\\ \text{at a const}\\ \text{height } \bar{w} \text{ time}}} + \cancelto{1500\frac{ft}{mn}\cdot\frac{dT}{dh}}{\frac{\partial (U)_i}{dx_j}\frac{dx_j}{dt}} = 1500\frac{ft}{min}\cdot\frac{-5°}{1000ft} = -7.5°/_{min}$$

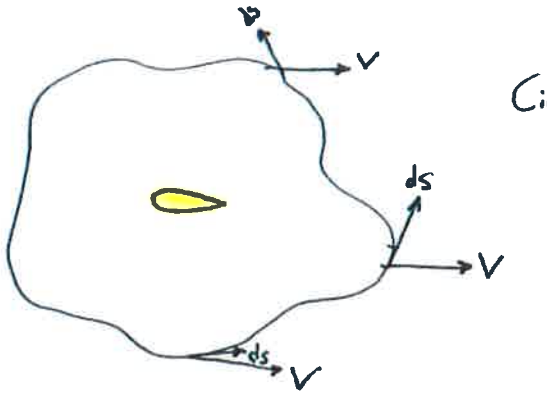
Q: If the atmosphere is now cooling at 1°/min, what is the rate of change in temperature on the aircraft?

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{dT}{dh}\cdot\frac{dh}{dt} = -1°/_{min} + \frac{-5°}{1000ft}\cdot\frac{1500\,ft}{min} = -8.5°/_{min}$$

Use the frame that simplifies the problem

vs

Circulation and Lift



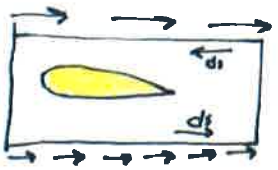
$$\text{Circulation} \equiv \Gamma = -\oint V \cdot ds$$

proportional to the velocity component tangent to a closed curve

Lift is $L = \rho V \Gamma$ ← circulation.
ρ ← density; V ← freestream velocity

No circulation .... no lift.

So on average, an airfoil generating lift has a net negative $V \cdot ds$. One way this can happen is if the flow above the airfoil is moving faster than the flow below

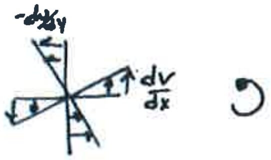


$V \cdot ds$ is negative and larger
$V \cdot ds$ is positive and smaller
} $L > 0$

Vorticity:

The local velocity derivatives represent the pointwise vorticity

$$\omega = \nabla \times V \quad \text{in 2D rect' coord'} = \frac{dv}{dx} - \frac{du}{dy}$$



You can measure vorticity with an object in a flow by watching the rotation.

Vorticity and Circulation are related by

$$\Gamma = -\oint V \cdot ds = -\iint \omega \cdot \hat{n}\, dA$$

Since Lift requires Circulation .
and
Circulation is an integrated form of vorticity

Units:

$$\frac{\left[\frac{ft}{s}\right]}{ft} = \frac{1}{s}$$

Lift requires vorticity ... somewhere

Lift

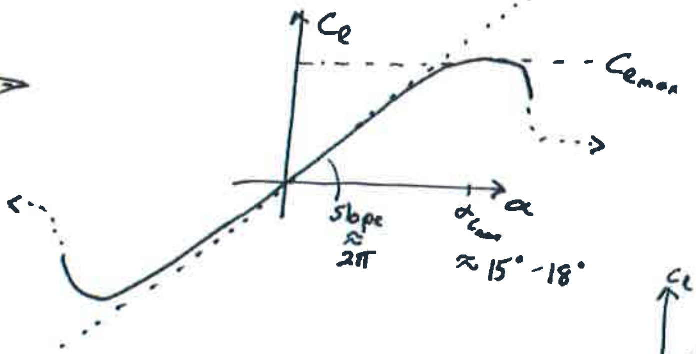
$\boxed{2\pi \text{ corrected for aspect ratio}}$

$$\text{Lift} = \underbrace{C_L}_{\text{lift coeffi..}} \underbrace{q}_{\text{dynam pressur}} \underbrace{S}_{\text{area}}$$

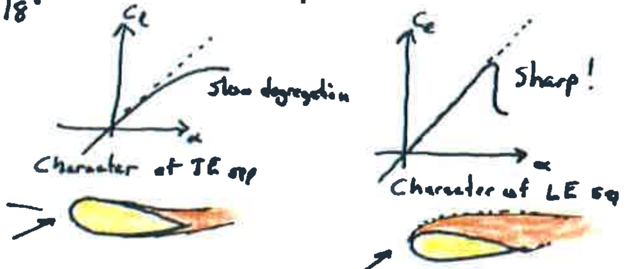
2-Dimensional airfoil

$C_{\ell_\alpha} \approx 2\pi$

$C_{\ell_\alpha} \equiv \frac{dC_\ell}{d\alpha}$ in $\frac{1}{\text{radians}}$



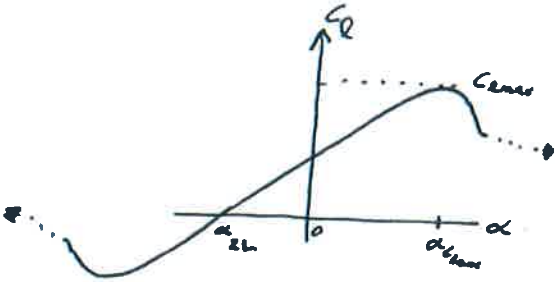
Lower C_ℓ deviation from $2\pi\alpha$ as α increases is due to flow separation.



Camber

Mean chord line is through the forward ~~and~~ LE and rear TE pts.

Adding camber offsets the C_ℓ vs α curve such that zero lift occurs at α_{ZL}

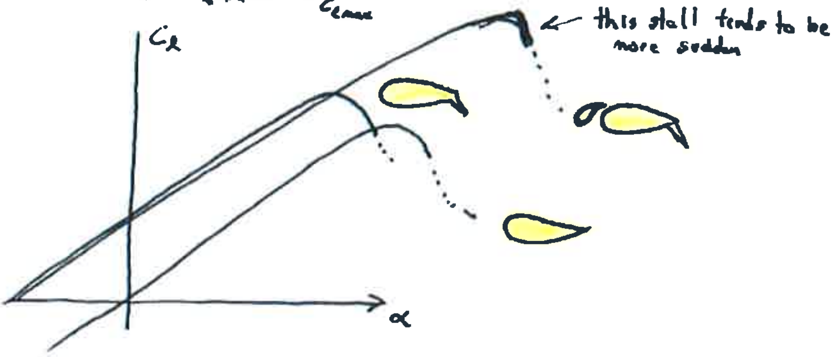


Knowing $C_{\ell_\alpha} \approx 2\pi$ and α_{ZL}, what is a good estimate for $C_\ell(\alpha=0)$?

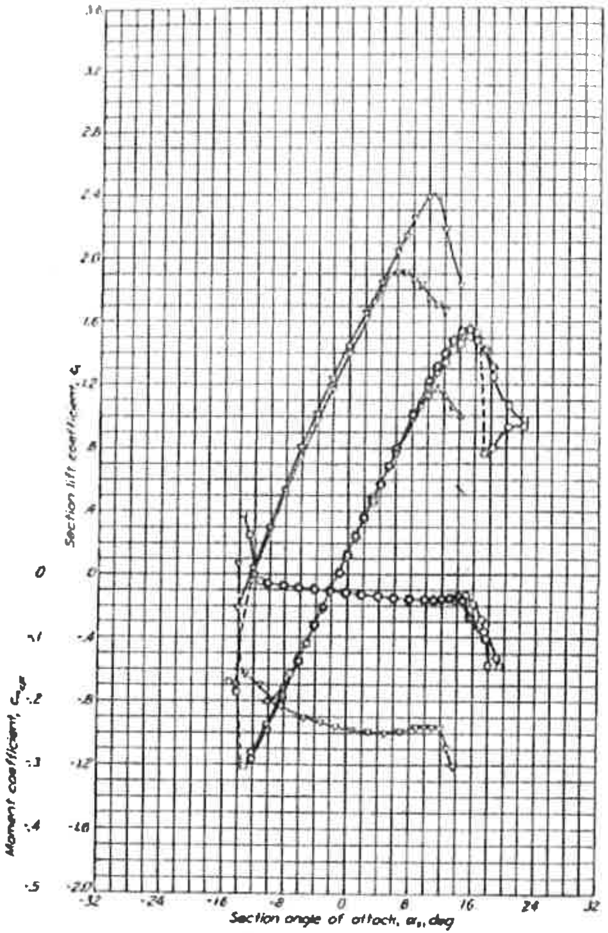
$$C_\ell(\alpha=0) \approx 2\pi \cdot (-\alpha_{ZL})$$

Slats and Flaps

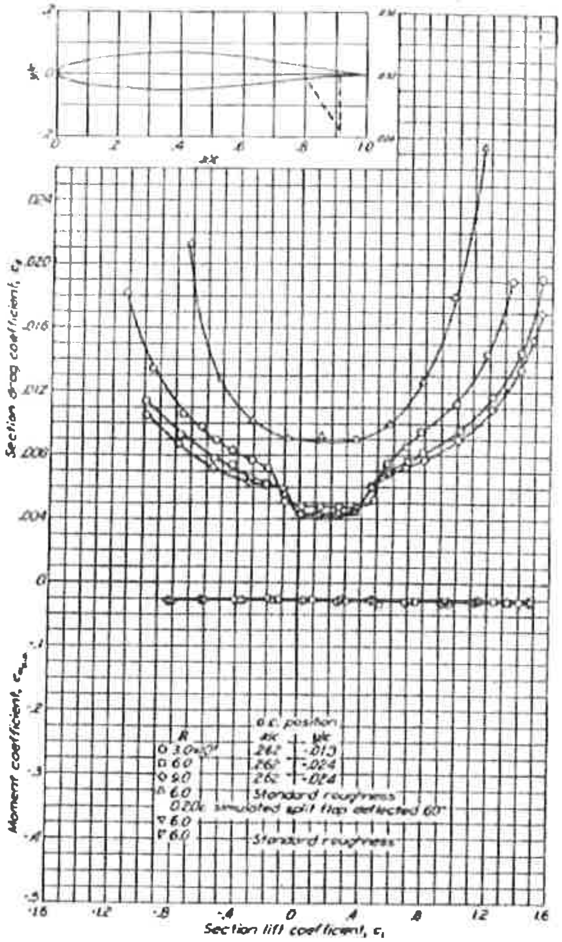
Flaps shift the C_ℓ vs α curve upward. Slats extend $\alpha_{C_{\ell_{max}}}$



this stall tends to be more sudden



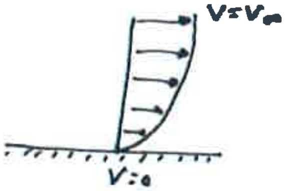
NACA 64_1-212 Wing Section



NACA 64_1-212 Wing Section (Continued)

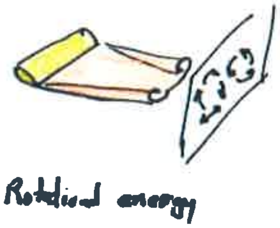
Drag

Surface Friction



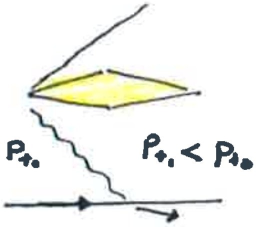
+

Induced Drag



+

Wave Drag



In general, drag is complicated to estimate for arbitrary configurations

See:

Fluid-Dynamic Drag, Hoerner

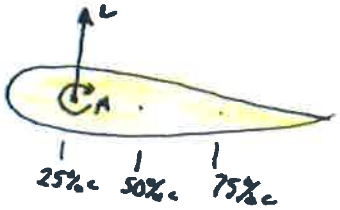
Integrated Forces and Moments on 2D airfoils.

$$L = q C_L S$$

$$D = q C_D S$$

$$M = q C_M S \bar{c}$$

Subsonic Airfoils



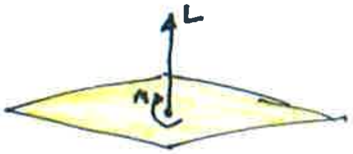
Lift and moment act at the aerodynamic center.

25% chord

$$A.C. \equiv \frac{dC_m}{dC_L} = 0$$

Thus $C_m(\alpha) = C_{m_0}$

Supersonic Airfoil



A.C. at 50% chord

See AEM 413

Transonic

Rapid and non-monotonic shift in a.c.

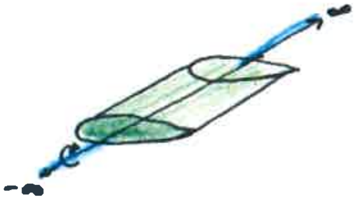
Strongly depends on airfoil shape

3D Wings

From physics, an inviscid vortex must start and end on a solid surface or form a closed loop.

Possibilities:

1)



Vortex extends off wingtips forever.

$$C_L = \int C_\ell \, dy \to \infty$$

Impossible

X

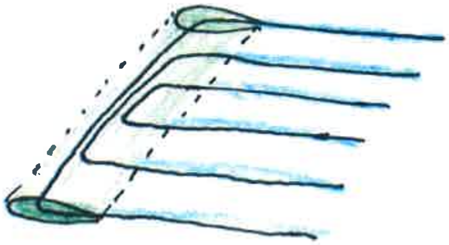
2)

- Finite lift
- Velocity at tip is infinite

$$V \propto \Gamma \cdot \frac{1}{r}$$

Reasonable mental model, but still wrong!

3)



- Discrete vortices distributed along span
- Velocity at tip = 0 since $\Gamma_{tip} = 0$
- Trailing vortices induce α at wing which varies with span location.

4)

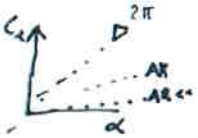
notice that the tip vortex has strength of zero!

- Continuous distribution of vorticity along span

✓

Analysis of Prandtl Lifting Line Theory:

The elliptical lift distribution is optimal for minimizing induced drag.



$$C_{L_\alpha} = \frac{C_{l_\alpha}}{1 + \frac{C_{l_\alpha}}{\pi AR}}$$

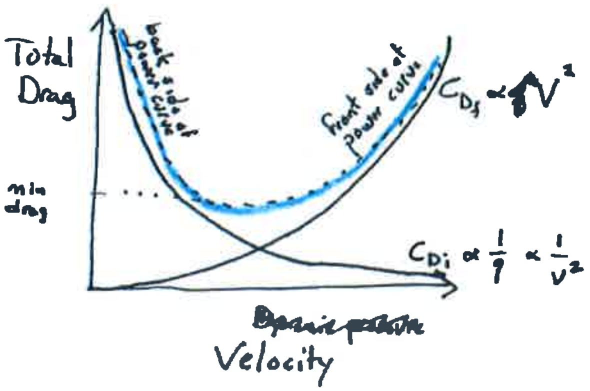
Reducing AR reduces lift slope to away from 2D value.

$$C_D = \frac{C_L^2}{\pi AR e}$$

where e=1 for elliptic only

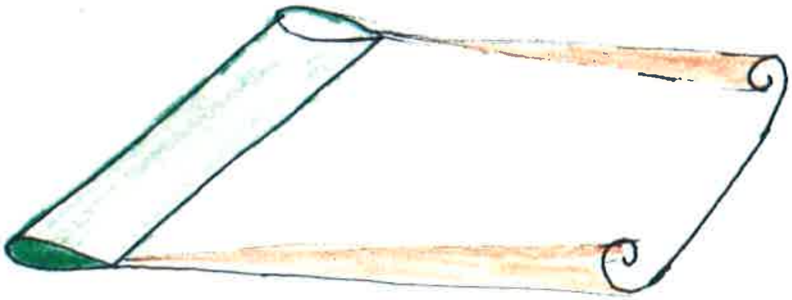
Induced drag depends on lift squared and the inverse of AR.

$$= \frac{\left(\frac{L}{b}\right)^2}{q^2 \pi e} = C \frac{(\text{Span loading})^2}{(\text{dynamic pressure})^2}$$



Non-Elliptical distributions have an "e" value are essentially a ratio of actual to elliptical performance

$$e_{\text{non-elliptical}} < e_{\text{elliptical}} = 1$$



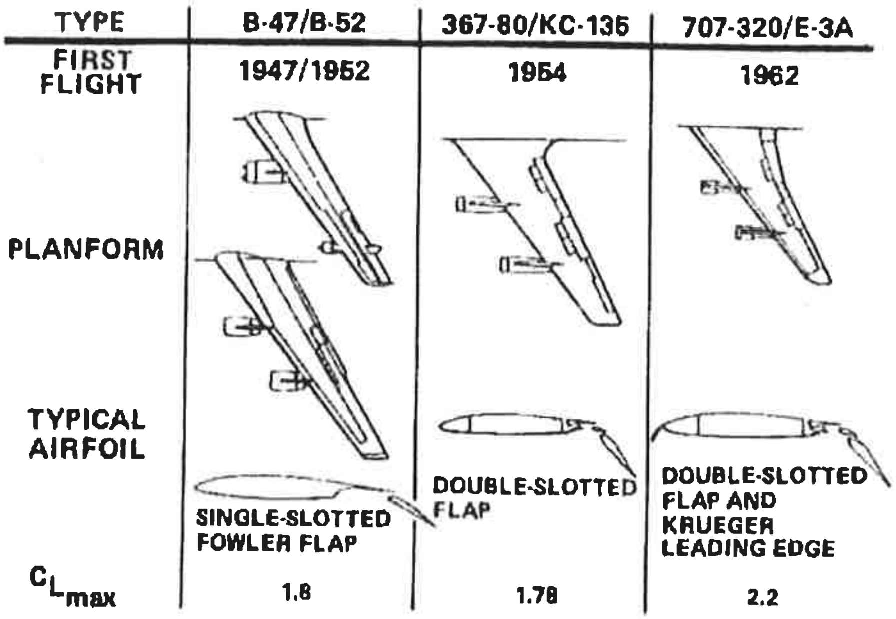


Figure 26.4 - Trends in Boeing transport high-lift development.
Source: AGARD CP-365, paper no. 9

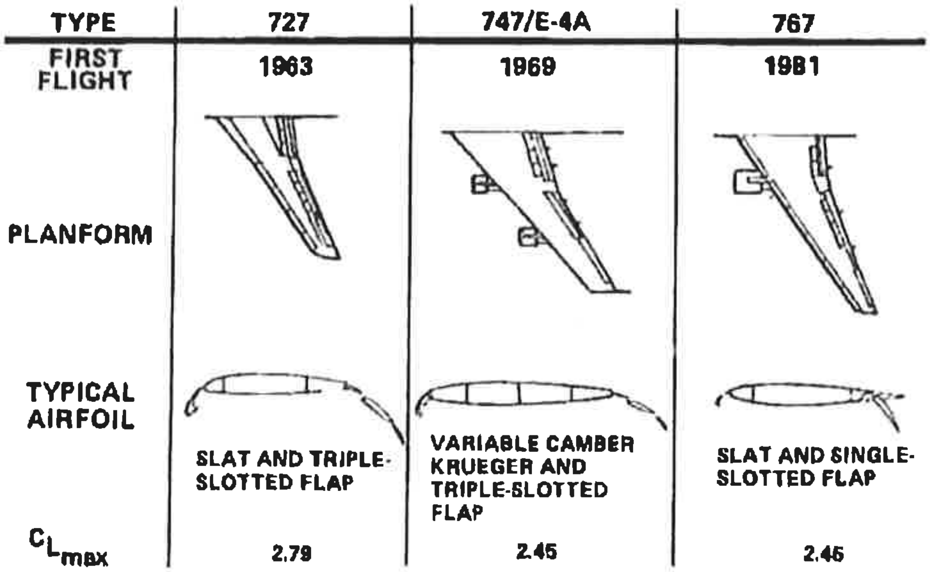
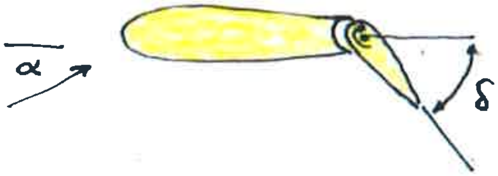


Figure 26.5 - Trends in Boeing transport high-lift development - continued
Source: AGARD CP-365, paper no. 9

Hinge Moments

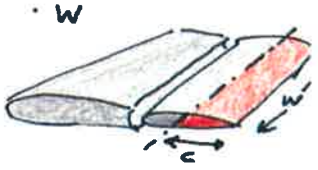
(Necessary for the upcoming flight control system portion of the class)



A moment is necessary to maintain the control surface at δ.

$$C_h = f(\alpha, \delta, Re, \text{gap}, \text{etc.}) \quad \text{and possibly time}$$

$$H = C_h \cdot q \cdot \underset{\text{area aft of hinge}}{S} \cdot \underset{\text{chord aft of hinge}}{c} = C_h \cdot q \cdot c^2 \cdot w$$



The pilot is connected to the surface in some way

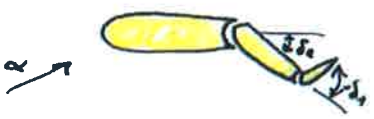
$$\delta = f(\underset{\delta_s}{\text{stick angle}})$$

$$F\ell_s\delta_e = H\delta_e \Rightarrow F = \underbrace{\left(\frac{\delta_e}{\ell_s\delta_s}\right)}_{\text{Gearing Ra}} H_e$$

How much force can a pilot exert? How long? $H \propto V^2$; pilot is limited human!

Constraints? Stick position, structural stresses, . . .

Trim Tab



$$C_h = C_{h_0} + \underbrace{C_{h_\alpha}}_{\frac{dC_h}{d\alpha}}\alpha + \underbrace{C_{h_{\delta_e}}}_{\frac{dC_h}{d\delta_e}}\delta_e + \underbrace{C_{h_{\delta_t}}}_{\frac{dC_h}{d\delta_t}}\delta_t$$

For the pilot to have zero stick force, $C_h = 0$

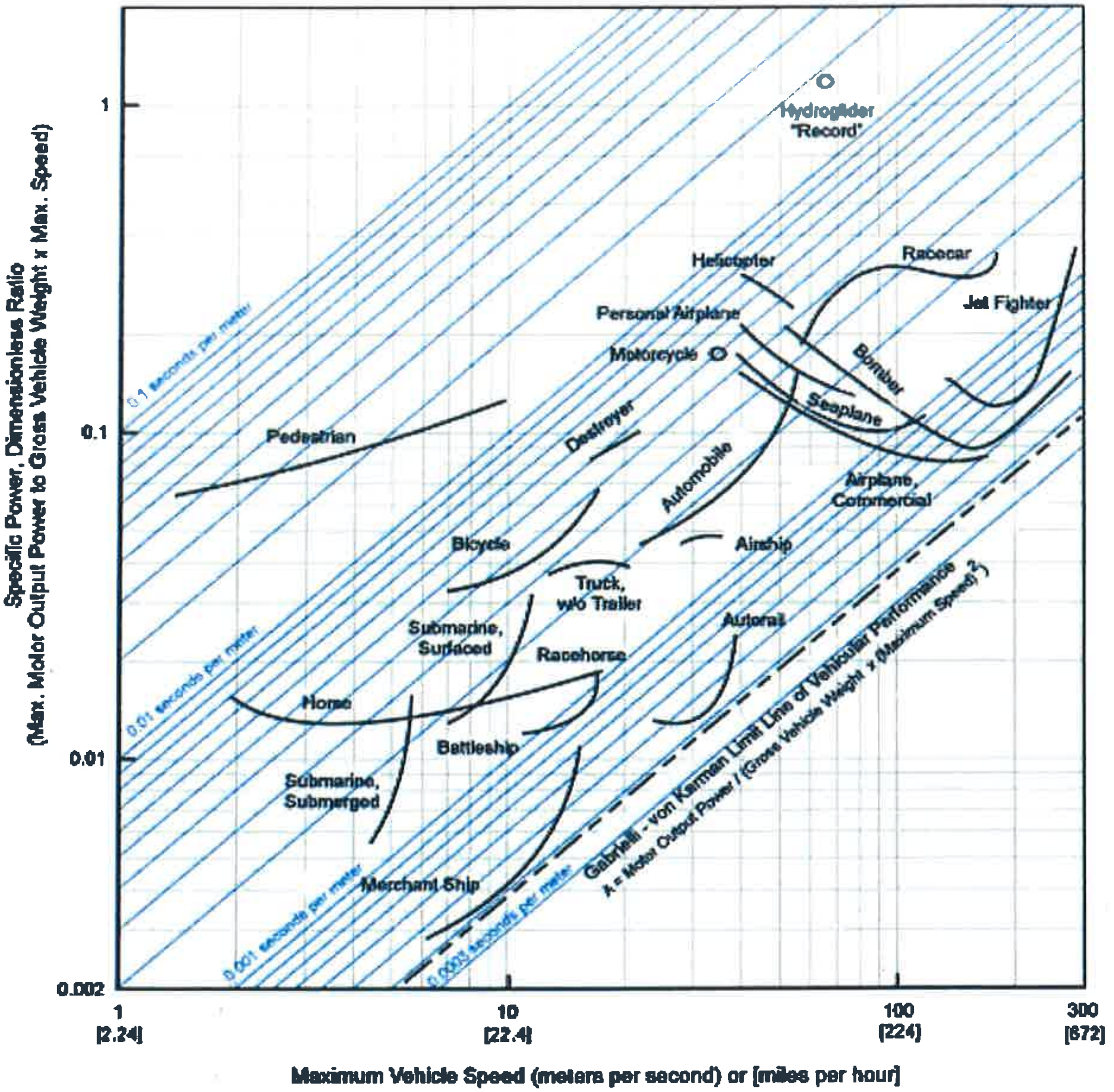
We (as pilots) can adjust δ_t to ensure $C_h = 0$ for a particular δ_e required at a particular α.

Stick free:

$$C_h = 0 = \left(\cancelto{\overset{\text{trimmed}}{0}}{C_{h_0} + C_{h_{\delta_t}}\delta_t}\right) + \underbrace{C_{h_\alpha}}_{\text{usually negative}}\alpha + \underbrace{C_{h_{\delta_e}}}_{\text{usually negative}}\delta_e \Rightarrow \delta_e = -\frac{C_{h_\alpha}}{C_{h_{\delta_e}}}\alpha$$

The elevator "floats".
TEU $\bar{w}$ α increas

Von Karmen – Gabrielli Limit



Int J Energy Environ Eng
DOI 10.1007/s40095-015-0160-6

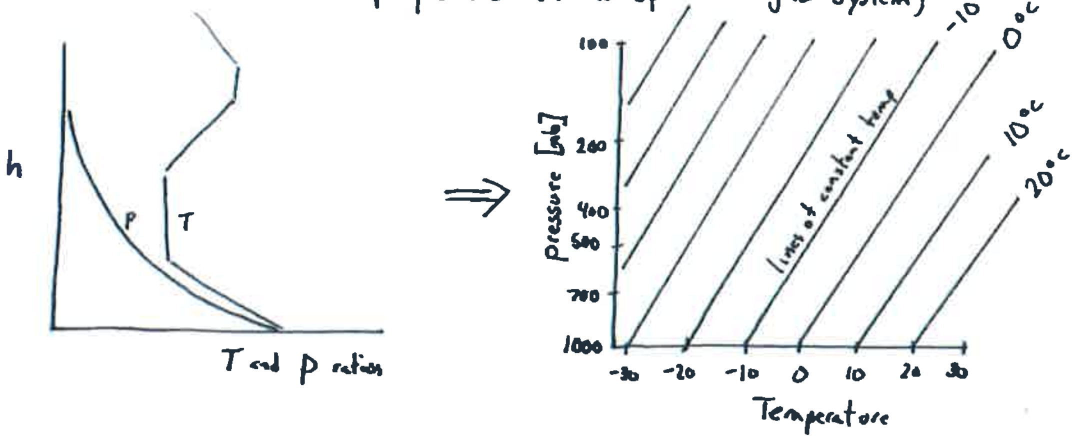
Lesson 4

Clouds

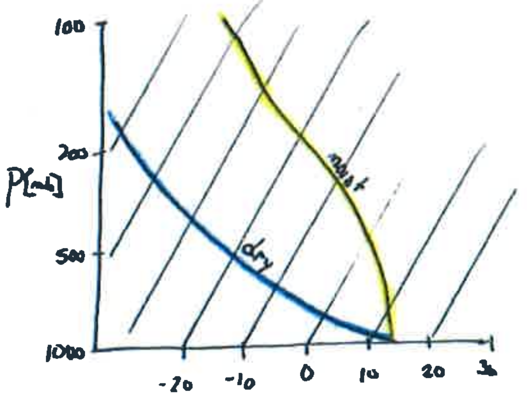
Why Clouds Form:

So far, we discussed only the dry-air lapse rate. Actual air has moisture.

SkewT - Log p (draw $\overset{\text{height}}{\text{Pressure}}$ and Temp profiles on a special grid system)



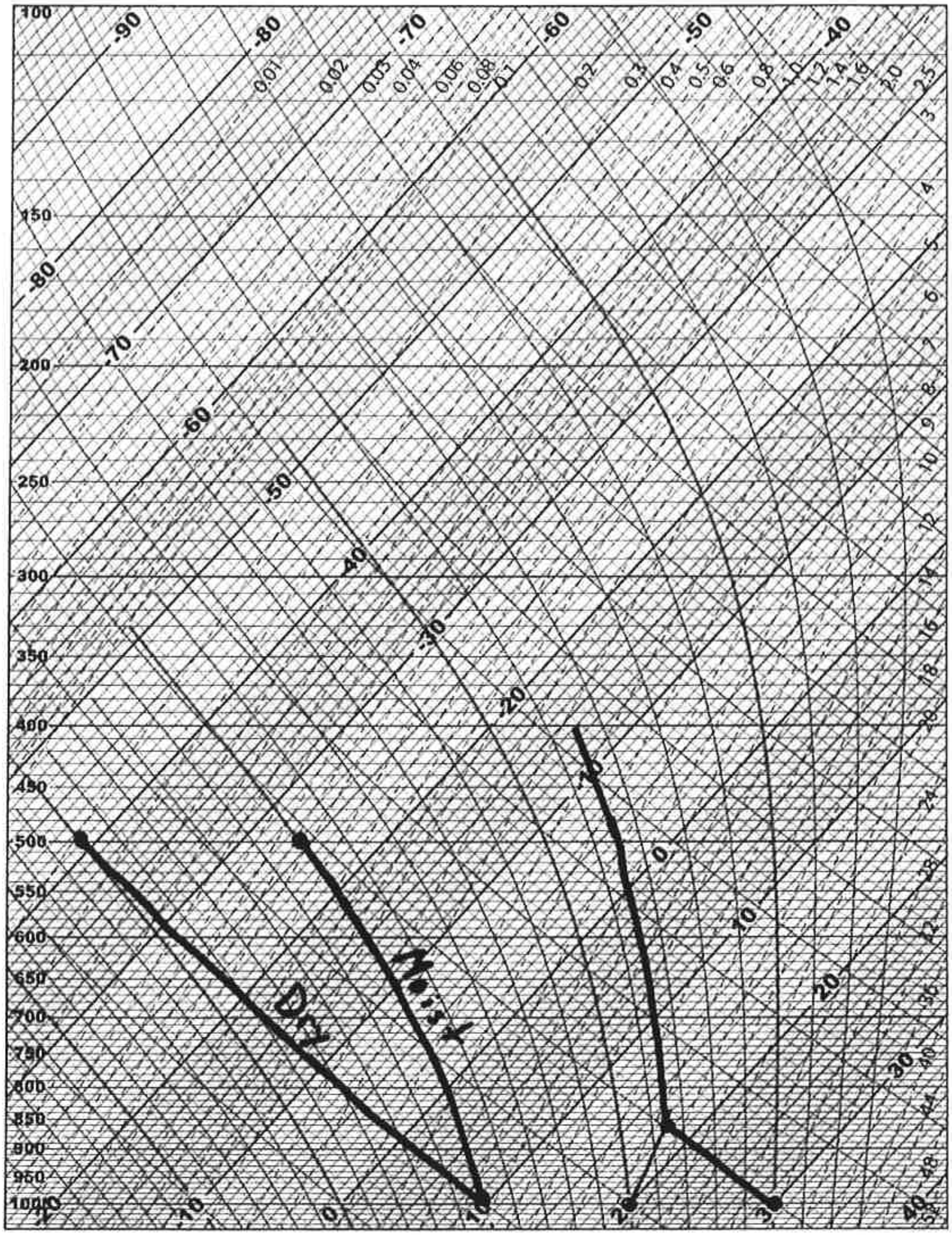
Using a skewT - log p plot, lets look at the standard ~~day~~ ~~lapse~~ rate (dry)



Changing the height to a log scale of pressure and tilting the temperature grid/coordinate allows us to see more of the atmosphere

Now, with water, the latent heat of water moving from $\overset{\text{vapor}}{\text{~~liquid~~}}$ to $\overset{\text{liquid}}{\text{~~vapor~~}}$ allows a slower cooling rate (Moist/Saturated lapse rate)
Once the moisture is condensed out (see previous Arden-Buck eqn) as the pressure drops, the moist lapse rate approaches the dry lapse rate.

Air ascends at the dry lapse rate until it becomes saturated (condenses), at which it ascends at the moist lapse rate



Skew-T / log p diagram for use with *A First Course in Atmospheric Thermodynamics* ©2008 by G.W. Petty

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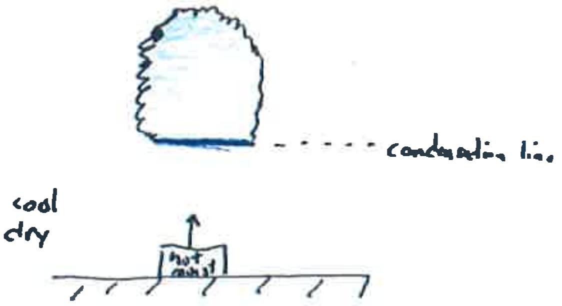
Example ① Lift moist vs dry air from $\underbrace{1000\text{ mb}}_{\approx SSL}$ to $\underbrace{500\text{ mb}}_{18000\text{ft}}$

Example ② Surface temp 30°C, Dew point 20°C
Clouds form when condensation begins
at intersection of dewpoint and dry lapse
around 850 mb ≈ 4500 ft

Atmospheric Stability

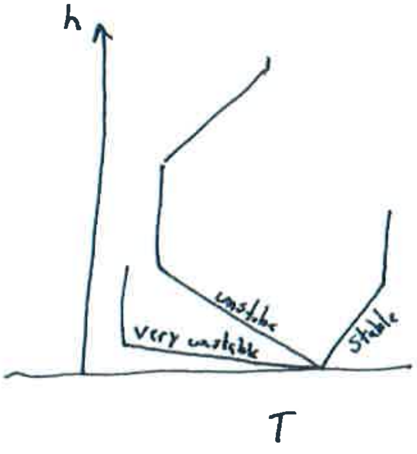
The driving force for cloud generation is density induced buoyancy. The hot moist air has a lower density and tends to rise within a cooler dry mass of air.

We saw that moist air rising has more energy (warmer) than dry air moved to the same height. This becomes an unstable system; As the moist air rises it tends to increase the driving force (density lowers).

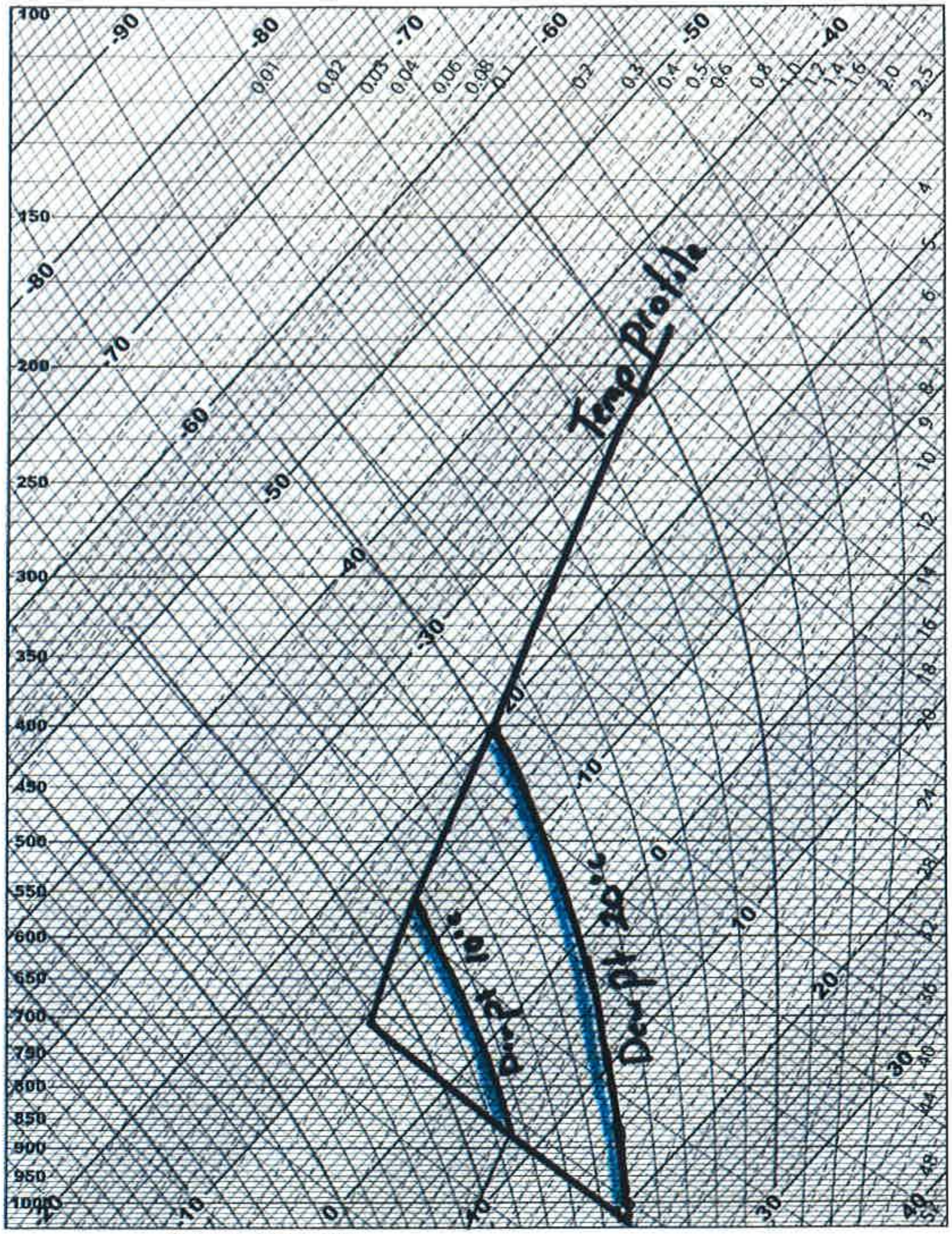


For the standard atmosphere, where does the cloud stop rising? When the lapse rate changes to be more positive and eventually the density variation (and momentum) go to zero. The std atmosphere is naturally unstable to at least 36000 ft!

In reality, the convection/motion from the surface warms the upper atmosphere and reduces the lapse driving rate. Clouds are a visible sign of moving energy up!



tiny.cc/AEM368Boiling



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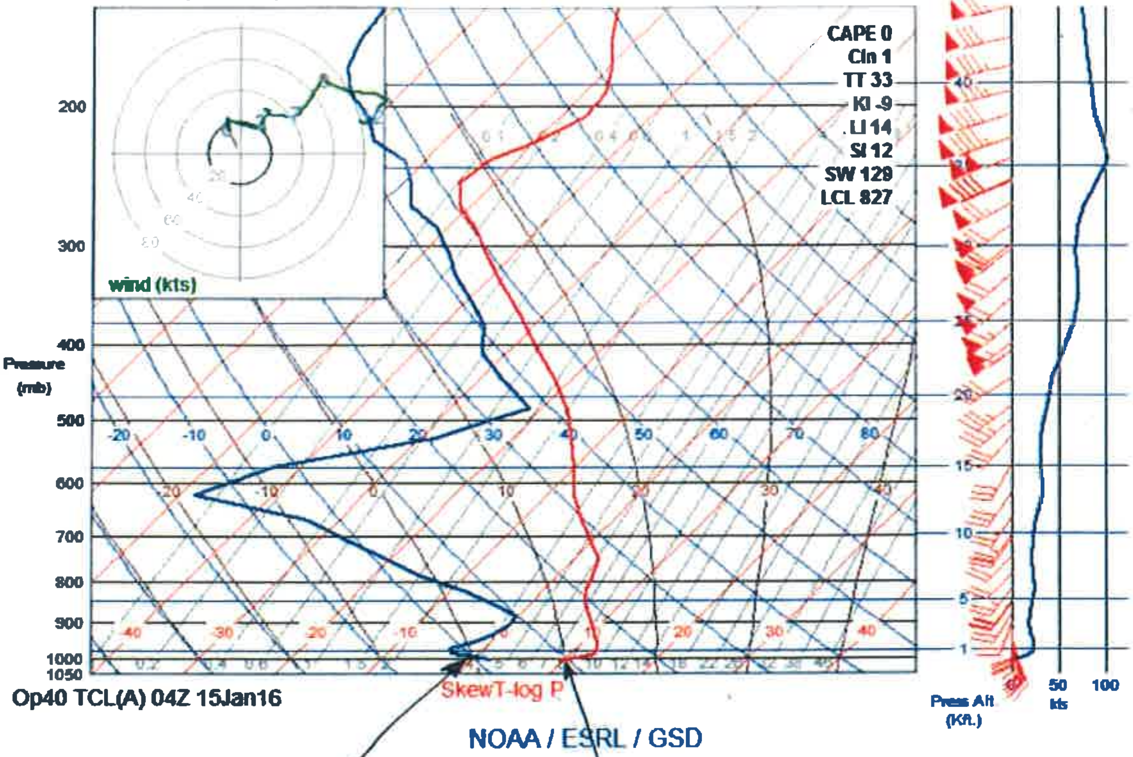
Tuscaloosa

Op40 Analysis, valid 15-Jan-2016 04:00:00 (10.3nm/355° from TCL)



Dew Point °C

Temp °C